ESTIMATION AND CONFIDENCE INTERVALS:

11.1. Single Mean:

Q1. An electrical firm manufacturing light bulbs that have a length of life that is normally distributed with a standard deviation of 30 hours. A sample of 50 bulbs were selected randomly and found to have an average of 750 hours. Let μ be the population mean of life lengths of all bulbs manufactured by this firm.

(1) Find a point estimate for μ.

(2) Construct a 94% confidence interval for μ.

Solution:

$\hat{\mu} = 750$

$$\bar{X} - Z_{1-\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + Z_{1-\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}$$

$1-\alpha = 0.94,\ \ \alpha = 0.06,\qquad \frac{\alpha}{2} = 0.03$

$Z_{1-\frac{\alpha}{2}} = Z_{0.97} = 1.88$

$$750 - 1.88\frac{30}{\sqrt{50}} < \mu < 750 + 1.88\frac{30}{\sqrt{50}}$$

$742 < \mu < 758$

Q2. Suppose that we are interested in making some statistical inferences about the mean, μ, of a normal population with standard deviation σ=2.0. Suppose that a random sample of size *n*=49 from this population gave a sample mean $\bar{X}$ =4.5.

(1) The distribution of $\bar{X}$ is

(A) N(0,1) (B) t(48) (C) N(μ, 0.2857) (D) N(μ, 2.0) (E) N(μ, 0.3333)

(2) A good point estimate of μ is

(A) 4.50 (B) 2.00 (C) 2.50 (D) 7.00 (E) 1.125

(3) The standard error of $\bar{X}$ is $\frac{2}{7} = 0.2857$

(A) 0.0816 (B) 2.0 (C) 0.0408 (D) 0.5714 (E) 0.2857

(4) A 95% confidence interval for μ is

(A) (3.44,5.56) (B) (3.34,5.66) (C) (3.54,5.46) (D) (3.94,5.06) (E) (3.04,5.96)

(5) If the upper confidence limit of a confidence interval is 5.2, then the lower confidence limit is

(A) 3.6 (B) 3.8 (C) 4.0 (D) 3.5 (E) 4.1

(6) The confidence level of the confidence interval (3.88, 5.12) is

(A) 90.74% (B) 95.74% (C) 97.74% (D) 94.74% (E) 92.74%

(7) If we use $\bar{X}$ to estimate μ, then we are 95% confident that our estimation error will not exceed

(A) e=0.50 (B) E=0.59 (C) e=0.58 (D) e=0.56 (E) e=0.51

(8) If we want to be 95% confident that the estimation error will not exceed e=0.1 when we use $\bar{X}$ to estimate μ, then the sample size *n* must be equal to

(A) 1529 (B) 1531 (C) 1537 (D) 1534 (E) 1530

Solution:

4) $\bar{X} - Z_{1-\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + Z_{1-\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}$

$1-\alpha = 0.95,\ \ \alpha = 0.05,\qquad \frac{\alpha}{2} = 0.025$

$Z_{1-\frac{\alpha}{2}} = Z_{0.975} = 1.96$

$$4.5 - 1.96\frac{2}{\sqrt{49}} < \mu < 4.5 + 1.96\frac{2}{\sqrt{49}}$$
$$3.94 < \mu < 5.06$$
7) $Z_{1-\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} = 1.96\frac{2}{\sqrt{49}} = 0.56$

8) $n = \left(\frac{Z_{1-\frac{\alpha}{2}}\sigma}{e}\right)^2 = \left(\frac{1.96*2}{0.1}\right)^2 = 1536.64 = 1537$

Q3. The following measurements were recorded for lifetime, in years, of certain type of machine: 3.4, 4.8, 3.6, 3.3, 5.6, 3.7, 4.4, 5.2, and 4.8. Assuming that the measurements represent a random sample from a normal population, then a 99% confidence interval for the mean life time of the machine is

(A) $-5.37 \le \mu \le 3.25$ (B) $4.72 \le \mu \le 9.1$

(C) $4.01 \le \mu \le 5.99$ (D) $3.37 \le \mu \le 5.25$

Solution:

$\bar{X} = 4.31, S^2 = 0.7087$

$t_{n-1,\frac{\alpha}{2}} = t_{8,0.005} = 3.355$

$$\bar{X} - t_{n-1,\frac{\alpha}{2}}\frac{S}{\sqrt{n}} < \mu < \bar{X} + t_{n-1,\frac{\alpha}{2}}\frac{S}{\sqrt{n}}$$
$$4.31 - 3.355\frac{\sqrt{0.7087}}{\sqrt{9}} < \mu < 4.31 + 3.355\frac{\sqrt{0.7087}}{\sqrt{9}}$$
$$3.37 < \mu < 5.25$$

Q4.H.WA researcher wants to estimate the mean lifespan of a certain light bulbs. Suppose that the distribution is normal with standard deviation of 5 hours.

1. Determine the sample size needed on order that the researcher will be 90% confident that the error will not exceed 2 hours when he uses the sample mean as a point estimate for the true mean.
2. Suppose that the researcher selected a random sample of 49 bulbs and found that the sample mean is 390 hours.
   (i) Find a good point estimate for the true mean μ.
   (ii) Find a 95% confidence interval for the true mean μ.

Q5.H.W The amount of time that customers using ATM (Automatic Teller Machine) is a random variable with a standard deviation of 1.4 minutes. If we wish to estimate the population mean μ by the sample mean $\bar{X}$, and if we want to be 96% confident that the sample mean will be within 0.3 minutes of the population mean, then the sample size needed is

(A) 98 (B) 100 (C) 92 (D) 85

Q6 H.W A random sample of size n=36 from a normal quantitative population produced a mean $\bar{X} = 15.2$ and a variance $S^2 = 9$.

(a) Give a point estimate for the population mean μ.
(b) Find a 95% confidence interval for the population mean μ.

Q7.H.WA group of 10 college students were asked to report the number of hours that they spent doing their homework during the previous weekend and the following results were obtained:

7.25, 8.5, 5.0, 6.75, 8.0, 5.25, 10.5, 8.5, 6.75, 9.25

$\sum X = 75.75, \sum X^2 = 600.563\}$

It is assumed that this sample is a random sample from a normal distribution with unknown variance σ^2. Let μ be the mean of the number of hours that the college student spend doing his/her homework during the weekend.

(a) Find the sample mean and the sample variance.
(b) Find a point estimate for μ.
(c) Construct a 80% confidence interval for μ.

Q8.H.WAn electronics company wanted to estimate its monthly operating expenses in thousands riyals (μ). It is assumed that the monthly operating expenses (in thousands riyals) are distributed according to a normal distribution with variance σ^2=0.584.

(I) Suppose that a random sample of 49 months produced a sample mean $\overline{X} = 5.47$.

(a) Find a point estimate for μ.
(b) Find the standard error of $\overline{X}$.
(c) Find a 90% confidence interval for μ.

(II) Suppose that they want to estimate $\mu$by $\overline{X}$. Find the sample size (n) required if they want
their estimate to be within 0.15 of the actual mean with probability equals to 0.95.

Q9. The tensile strength of a certain type of thread is approximately normally distributed with standard deviation of 6.8 kilograms. A sample of 20 pieces of the thread has an average tensile strength of 72.8 kilograms. Then,

(a) A point estimate of the population mean of the tensile strength (μ) is:

(A) 72.8 (B) 20 (C) 6.8 (D) 46.24 (E) None of these

(b) Suppose that we want to estimate the population mean (μ) by the sample mean $(\overline{X})$. To be95% confident that theerror of our estimate of the mean of tensile strength will beless than 3.4 kilograms, the minimum sample size should be at least:

(A) 4 (B) 16 (C) 20 (D) 18 (E) None of these

(c) For a 98% confidence interval for the mean of tensile strength, we have the lower bound equal to:

(A) 68.45 (B) 69.26 (C) 71.44 (D) 69.68 (E) None of these

(d) For a 98% confidence interval for the mean of tensile strength, we have the upper boundequal to:

(A) 74.16 (B) 77.15 (C) 75.92 (D) 76.34 (E) None of these

Solution:

$\bar{X} = 72.8, \quad \sigma = 6.8, \quad n = 20$

a) $\hat{\mu} = \bar{X} = 72.8$

b) $Z_{1-\frac{\alpha}{2}} = Z_{0.975} = 1.96$

$$n = \left(\frac{Z_{1-\frac{\alpha}{2}}\sigma}{e}\right)^2 = \left(\frac{1.96 * 6.8}{3.4}\right)^2 = 15.3664 \approx 16$$

c&d) $\bar{X} - Z_{1-\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + Z_{1-\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}$

$1 - \alpha = 0.98, \quad \alpha = 0.02, \quad \frac{\alpha}{2} = 0.01$

$Z_{1-\frac{\alpha}{2}} = Z_{0.99} = 2.055$

$$72.8 - 2.055\frac{6.8}{\sqrt{20}} < \mu < 72.8 + 2.055\frac{6.8}{\sqrt{20}}$$
$$69.675 < \mu < 75.924$$

9.2 An electrical firm manufactures light bulbs that have a length of life that is approximately normally distributed with a standard deviation of 40 hours. If a sample of 30 bulbs has an average life of 780 hours, find a 96% confidence interval for the population mean of all bulbs produced by this firm.

Solution:

$$\bar{X} = 780, \quad \sigma = 40, \quad n = 30$$
$$\bar{X} - Z_{1-\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + Z_{1-\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}$$
$$1 - \alpha = 0.96, \quad \alpha = 0.04, \quad \frac{\alpha}{2} = 0.02$$
$$Z_{1-\frac{\alpha}{2}} = Z_{0.98} = 1.755$$
$$780 - 1.755\frac{40}{\sqrt{30}} < \mu < 780 + 1.755\frac{40}{\sqrt{30}}$$
$$767.12 < \mu < 792.87$$

9.6 How large a sample is needed in Exercise 9.2 if we wish to be 96% confident that our sample mean will be within 10 hours of the true mean?

$$1 - \alpha = 0.96, \quad \alpha = 0.04, \quad \frac{\alpha}{2} = 0.02$$
$$Z_{1-\frac{\alpha}{2}} = Z_{0.98} = 1.755$$
$$n = \left(\frac{Z_{1-\frac{\alpha}{2}}\sigma}{e}\right)^2 = \left(\frac{1.755 * 40}{10}\right)^2 = 49.3 \approx 50$$

9.4 The heights of a random sample of 50 college students showed a mean of 174.5 centimeters and a standard deviation of 6.9 centimeters.

(a) Construct a 95% confidence interval for the mean height of all college students.

(b) What can we assert with 95% confidence about the possible size of our error if we estimate the mean height of all college students to be 174.5 centimeters?

Solution:

$$\bar{X} = 174.5, \quad S = 6.9, \quad n = 50$$
a) $\bar{X} - Z_{1-\frac{\alpha}{2}}\frac{S}{\sqrt{n}} < \mu < \bar{X} + Z_{1-\frac{\alpha}{2}}\frac{S}{\sqrt{n}}$
$$1 - \alpha = 0.95, \quad \alpha = 0.05, \quad \frac{\alpha}{2} = 0.025$$
$$Z_{1-\frac{\alpha}{2}} = Z_{0.975} = 1.96$$
$$174.5 - 1.96\frac{6.9}{\sqrt{50}} < \mu < 174.5 + 1.96\frac{6.9}{\sqrt{50}}$$
$$172.54 < \mu < 176.46$$

b) $e = 1.96\frac{6.9}{\sqrt{50}} = 1.91$

H.W: 9.5 , 9.8

11.2. Two Means:

Q1.(I) The tensile strength of type I thread is approximately normally distributed with standard deviation of 6.8 kilograms. A sample of 20 pieces of the thread has an average tensile strength of 72.8 kilograms. Then,

1) To be 95% confident that the error of estimating the mean of tensile strength by the sample mean will be less than 3.4 kilograms, the minimum sample size should be:

(A) 4 (B) 16 (C) 20 (D) 18 (E) None of these

2) The lower limit of a 98% confidence interval for the mean of tensile strength is

(A) 68.45 (B) 69.26 (C) 71.44 (D) 69.68 (E) None of these

3) The upper limit of a 98% confidence interval for the mean of tensile strength is

(A) 74.16 (B) 77.15 (C) 75.92 (D) 76.34 (E) None of these

Q1.(II). The tensile strength of type II thread is approximately normally distributed with standard deviation of 6.8 kilograms. A sample of 25 pieces of the thread has an average tensile strength of 64.4 kilograms. Then for the 98% confidence interval of the difference in tensile strength means between type I and type II , we have:

1) the lower bound equals to:

(A) 2.90 (B) 4.21 (C) 3.65 (D) 6.58 (E) None of these

2) the upper bound equals to:

(A) 13.90 (B) 13.15 (C) 12.59 (D) 10.22 (E) None of these

Solution:

I) $\overline{X_1} = 70,\ \sigma_1 = 6.8,\ n_1 = 20$

II) $\overline{X_2} = 64.4,\ \sigma_2 = 6.8,\ n_1 = 25$

$$(\overline{X_1} - \overline{X_2}) - Z_{1-\frac{\alpha}{2}}\sqrt{\frac{{\sigma_1}^2}{n_1} + \frac{{\sigma_2}^2}{n_2}} < \mu_1 - \mu_2 < (\overline{X_1} - \overline{X_2}) + Z_{1-\frac{\alpha}{2}}\sqrt{\frac{{\sigma_1}^2}{n_1} + \frac{{\sigma_2}^2}{n_2}}$$

$1 - \alpha = 0.98,\ \ \alpha = 0.02,\qquad \frac{\alpha}{2} = 0.01$

$Z_{\frac{\alpha}{2}} = Z_{0.01} = 2.055$

$$(70 - 64.4) - 2.055\sqrt{\frac{6.8^2}{20} + \frac{6.8^2}{25}} < \mu_1 - \mu_2 < (70 - 64.4) + 2.055\sqrt{\frac{6.8^2}{20} + \frac{6.8^2}{25}}$$

$4.208 < \mu_1 - \mu_2 < 12.5922$

Q2.H.W Two random samples were independently selected from two normal populations with equal variances. The results are summarized as follows.

	First Sample	Second Sample
sample size (n)	12	14
sample mean ($\overline{X}$)	10.5	10.0
sample variance (S^2)	4	5

Let μ_1 and μ_2 be the true means of the first and second populations, respectively.

1. Find a point estimate for $\mu_1 - \mu_2$..

2. Find 95% confidence interval for $\mu_1 - \mu_2$.

Q3.H.WA researcher was interested in comparing the mean score of female students, μ_f, with the mean score of male students, μ_m, in a certain test. Two independent samples gave the following results:

Sample	Observations							mean	Variance
Scores of Females	89.2	81.6	79.6	80.0	82.8			82.63	15.05
Scores of Males	83.2	83.2	84.8	81.4	78.6	71.5	77.6	80.04	20.79

Assume the populations are normal with equal variances.

(1) The pooled estimate of the variance s_p^2 is

(A) 17.994 (B) 17.794 (C) 18.094 (D) 18.294 (E) 18.494

(2) A point estimate of $\mu_f - \mu_m$ is

(A) 2.63 (B) −2.59 (C) 2.59 (D) 0.00 (E) 0.59

(3) The lower limit of a 90% confidence interval for $\mu_f - \mu_m$ is

(A) −1.97 (B) −1.67 (C) 1.97 (D) 1.67 (E) −1.57

(4) The upper limit of a 90% confidence interval for $\mu_f - \mu_m$ is

(A) 6.95 (B) 7.45 (C) −7.55 (D) 7.15 (E) 7.55

Q4.H.W A study was conducted to compare to brands of tires A and B. 10 tires of brand A and 12 tires of brand B were selected randomly. The tires were run until they wear out. The results are:

Brand A: $\overline{X}_A$ = 37000 kilometers $S_A = 5100$

Brand B: $\overline{X}_B$ = 38000 kilometers $S_B = 6000$

Assuming the populations are normally distributed with equal variances,

(1) Find a point estimate for $\mu_A - \mu_B$.

(2) Construct a 90% confidence interval for $\mu_A - \mu_B$.

Q5.H.WThe following data show the number of defects of code of particular type of software program made in two different countries (assuming normal populations with unknownequal variances)

Country	observations							mean	standard dev.
A	48	39	42	52	40	48	54	46.143	5.900
B	50	40	43	45	50	38	36	43.143	5.551

(a) A point estimate of $\mu_A - \mu_B$ is

(A) 3.0 (B) −3.0 (C) 2.0 (D) −2.0 (E) None of these

(b) A 90% confidence interval for the difference between the two population means $\mu_A - \mu_B$ is

(A) $-2.46 \le \mu_A - \mu_B \le 8.46$ (B) $1.42 \le \mu_A - \mu_B \le 6.42$

(C) $-1.42 \le \mu_A - \mu_B \le -0.42$ (D) $2.42 \le \mu_A - \mu_B \le 10.42$

Q6.DELET A study was made by a taxi company to decide whether the use of new tires (A) instead of the present tires (B) improves fuel economy. Six cars were equipped with tires (A) and driven over a prescribed test course. Without changing drivers and cares, test course was made with tires (B). The gasoline consumption, in kilometers per liter (km/L), was recorded as follows: (assume the populations are normal with equal unknown variances)

Car	1	2	3	4	5	6
Type (A)	4.5	4.8	6.6	7.0	6.7	4.6

Type (B)	3.9	4.9	6.2	6.5	6.8	4.1

(a) A 95% confidence interval for the true mean gasoline consumption for brand A is:

(A) $4.462 \le \mu_A \le 6.938$ (B) $2.642 \le \mu_A \le 4.930$

(C) $5.2 \le \mu_A \le 9.7$ (D) $6.154 \le \mu_A \le 6.938$

(b) A 99% confidence interval for the difference between the true means consumption of type (A) and type (B) ($\mu_A - \mu_B$) is:

(A) $-1.939 \le \mu_A - \mu_B \le 2.539$ (B) $-2.939 \le \mu_A - \mu_B \le 1.539$

(C) $0.939 \le \mu_A - \mu_B \le 1.539$ (D) $-1.939 \le \mu_A - \mu_B \le 0.539$

Q7.H.WA geologist collected 20 different ore samples, all of the same weight, and randomly divided them into two groups. The titanium contents of the samples, found using two different methods, are listed in the table:

Method (A)	Method (B)
1.1 1.3 1.3 1.5 1.4	1.1 1.6 1.3 1.2 1.5
1.3 1.0 1.3 1.1 1.2	1.2 1.7 1.3 1.4 1.5
$\overline{X}_1 = 1.25$, $S_1 = 0.1509$	$\overline{X}_2 = 1.38$, $S_2 = 0.1932$

(a) Find a point estimate of $\mu_A - \mu_B$ is

(b) Find a 90% confidence interval for the difference between the two population means $\mu_A - \mu_B$. (Assume two normal populations with equal variances).

From book:

9.35 A random sample of size n_1= 25, taken from a normal population with a standard deviation σ_1= 5, has a mean $\overline{X_1}$= 80. A second random sample of size n_2 = 36, taken from a different normal population with a standard deviation σ_2 = 3, has a mean $\overline{X_2}$= 75. Find a 95% confidence interval for $\mu_1 - \mu_2$.

Solution:

I) $\overline{X_1} = 80,\ \sigma_1 = 5,\ n_1 = 25$

II) $\overline{X_2} = 75,\ \sigma_2 = 3,\ n_1 = 36$

$$(\overline{X_1} - \overline{X_2}) - Z_{1-\frac{\alpha}{2}}\sqrt{\frac{{\sigma_1}^2}{n_1} + \frac{{\sigma_2}^2}{n_2}} < \mu_1 - \mu_2 < (\overline{X_1} - \overline{X_2}) + Z_{1-\frac{\alpha}{2}}\sqrt{\frac{{\sigma_1}^2}{n_1} + \frac{{\sigma_2}^2}{n_2}}$$

$1 - \alpha = 0.95,\ \alpha = 0.05,\quad \frac{\alpha}{2} = 0.025$

$Z_{1-\frac{\alpha}{2}} = Z_{0.975} = 1.96$

$$(80 - 75) - 1.96\sqrt{\frac{5^2}{25} + \frac{3^2}{36}} < \mu_1 - \mu_2 < (80 - 75) + 1.96\sqrt{\frac{5^2}{25} + \frac{3^2}{36}}$$

$2.808 < \mu_1 - \mu_2 < 7.191$

9.41 The following data represent the length of time, in days, to recovery for patients randomly treated with one of two medications to clear up severe bladder infections:

Medication1	Medication2
.n_1= 14	.n_2=16
$\overline{X}_1 = 17$	$\overline{X}_2 = 19$
$S_1^2 = 1.5$	$S_2^2 = 1.8$

Find a 99% confidence interval for the difference $\mu_2 - \mu_1$ in the mean recovery times for the two medications, assuming normal populations **with equal variances**.

Solution:

I) $\overline{X_1} = 80,\ \sigma_1 = 5,\ n_1 = 25$

II) $\overline{X_2} = 75,\ \sigma_2 = 3,\ n_1 = 36$

$$(\overline{X_1} - \overline{X_2}) - t_{n_1+n_2-2,\frac{\alpha}{2}} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} < \mu_1 - \mu_2 < (\overline{X_1} - \overline{X_2}) + t_{n_1+n_2-2,\frac{\alpha}{2}} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$1 - \alpha = 0.99,\ \ \alpha = 0.01,\ \ \ \ \frac{\alpha}{2} = 0.005$

$t_{n_1+n_2-2,\frac{\alpha}{2}} = t_{28,0.005} = 2.763$

$$S_p{}^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2} = \frac{(13)1.5 + (15)1.8}{28} = 1.6607$$

$S_p = 1.289$

$$(17 - 19) - 2.763 * 1.289 \sqrt{\frac{1}{14} + \frac{1}{16}} < \mu_1 - \mu_2 < (17 - 19) + 2.763 * 1.289 \sqrt{\frac{1}{14} + \frac{1}{16}}$$

$-3.303 < \mu_1 - \mu_2 < -0.697$

H.W: 9.36 , 9.43

11.3. Single Proportion:

Q1. A random sample of 200 students from a certain school showed that 15 students smoke. Let p be the proportion of smokers in the school.

1. Find a point Estimate for p.
2. Find 95% confidence interval for p.

Solution:

1) $\hat{p} = \frac{15}{200} = 0.075$

2) $\hat{p} - Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}}$

$1 - \alpha = 0.95, \ \ \alpha = 0.05, \qquad \frac{\alpha}{2} = 0.025$

$Z_{1-\frac{\alpha}{2}} = Z_{0.975} = 1.96$

$$0.075 - 1.96\sqrt{\frac{0.075 * 0.925}{200}} < p < 0.075 + 1.96\sqrt{\frac{0.075 * 0.925}{200}}$$

$0.1115 < p < 0.0385$

Q2. A researcher was interested in making some statistical inferences about the proportion of females (p) among the students of a certain university. A random sample of 500 students showed that 150 students are female.

(1) A good point estimate for p is

(A) 0.31 (B) 0.30 (C) 0.29 (D) 0.25 (E) 0.27

(2) The lower limit of a 90% confidence interval for p is

(A) 0.2363 (B) 0.2463 (C) 0.2963 (D) 0.2063 (E) 0.2663

(3) The upper limit of a 90% confidence interval for p is

(A) 0.3337 (B) 0.3137 (C) 0.3637 (D) 0.2937 (E) 0.3537

Solution:

1) $\hat{p} = \frac{150}{500} = 0.3$

2&3) $\hat{p} - Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}}$

$1 - \alpha = 0.90, \ \ \alpha = 0.10, \qquad \frac{\alpha}{2} = 0.05$

$Z_{1-\frac{\alpha}{2}} = Z_{0.95} = 1.645$

$$0.3 - 1.645\sqrt{\frac{0.3 * 0.7}{500}} < p < 0.3 + 1.645\sqrt{\frac{0.3 * 0.7}{500}}$$

$0.2663 < p < 0.3337$

Q3. H.W In a random sample of 500 homes in a certain city, it is found that 114 are heated by oil. Let p be the proportion of homes in this city that are heated by oil.

1) Find a point estimate for p.
2) Construct a 98% confidence interval for p.

Q4. H.W In a study involved 1200 car drivers, it was found that 50 car drivers do not use seat belt.

(1) A point estimate for the proportion of car drivers who do not use seat belt is:

(A) 50 (B) 0.0417 (C) 0.9583 (D) 1150 (E) None of these

(2) The lower limit of a 95% confidence interval of the proportion of car drivers not using seat belt is

(A) 0.0322 (B) 0.0416 (C) 0 .0304 (D) –0.3500 (E) None of these

(3) The upper limit of a 95% confidence interval of the proportion of car drivers not using seatbelt is

(A) 0.0417 (B) 0.0530 (C) 0.0512 (D) 0.4333 (E) None of these

Q5. H.W A study was conducted to make some inferences about the proportion of female employees (p) in a certain hospital. A random sample gave the following data:

Sample size	250
Number of females	120

(a) Calculate a point estimate ($\hat{p}$) for the proportion of female employees (p).

(b) Construct a 90% confidence interval for p.

Q6. H.W In a certain city, the traffic police was interested in knowing the proportion of car drivers who do not use seat built. In a study involved 1200 car drivers it was found that 50 car drivers do not use seat belt.

(a) A point estimate for the proportion of car drivers who do not use seat built is:

(A) 50 (B) 0.0417 (C) 0.9583 (D) 1150 (E) None of these

(b) A 95% confidence interval of the proportion of car drivers who do not use seat built has the lower bound equal to:

(A) 0.0322 (B) 0.0416 (C) 0 .0304 (D) –0.3500 (E) None of these

(c) A 95% confidence interval of the proportion of car drivers who do not use seat built has the upper bound equal to:

(A) 0.0417 (B) 0.0530 (C) 0.0512 (D) 0.4333 (E) None of these

From book :

9.51 In a random sample of 1000 homes in a certain city, it is found that 228 are heated by oil. Find 99% confidence intervals for the proportion of homes in this city that are heated by oil using both methods presented on page 297.

Solution:

$$\hat{p} = \frac{228}{1000} = 0.228, \qquad \hat{q} = 0.772$$

$$\hat{p} - Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$1-\alpha = 0.99, \;\; \alpha = 0.01, \qquad \frac{\alpha}{2} = 0.005$$

$$Z_{1-\frac{\alpha}{2}} = Z_{0.995} = 2.575$$

$$0.228 - 2.575\sqrt{\frac{0.228 * 0.772}{1000}} < p < 0.228 + 2.575\sqrt{\frac{0.228 * 0.772}{1000}}$$

$$0.1938 < p < 0.2621$$

9.56 A geneticist is interested in the proportion of African males who have a certain minor blood disorder. In a random sample of 100 African males, 24 are found to be afflicted.

(a) Compute a 99% confidence interval for the proportion of African males who have this blood disorder.
(b) What can we assert with 99% confidence about the possible size of our error if we estimate the proportion of African males with this blood disorder to be 0.24.

Solution:

a) $\hat{p} = \frac{24}{100} = 0.24, \ \hat{q} = 0.76$

$$\hat{p} - Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$1 - \alpha = 0.99, \ \ \alpha = 0.01, \qquad \frac{\alpha}{2} = 0.005$$

$$Z_{1-\frac{\alpha}{2}} = Z_{0.995} = 2.575$$

$$0.24 - 2.575\sqrt{\frac{0.24 * 0.76}{100}} < p < 0.24 + 2.575\sqrt{\frac{0.24 * 0.76}{100}}$$

$$0.13 < p < 0.35$$

b) $n = \frac{Z_{1-\frac{\alpha}{2}}{}^{2}\hat{p}\hat{q}}{e^2}$

$$n = \frac{2.575^2 * 0.24 * 0.76}{0.24^2} = 20.99 \approx 21$$

H.W: 9.58 , 9.59

11.4. Two Proportions:

Q1. A survey of 500 students from a college of science shows that 275 students own computers. In another independent survey of 400 students from a college of engineering shows that 240 students own computers.

(a) a 99% confidence interval for the true proportion of college of science's student who own computers is

(A)	$-0.59 \le p_1 \le 0.71$	(B)	$0.49 \le p_1 \le 0.61$
(C)	$2.49 \le p_1 \le 6.61$	(D)	$0.3 \le p_1 \le 0.7$

(29) a 95% confidence interval for the difference between the proportions of students owning computers in the two colleges is

(A)	$0.015 \le p_1 - p_2 \le 0.215$	(B)	$-0.515 \le p_1 - p_2 \le 0.215$
(C)	$-0.450 \le p_1 - p_2 \le -0.015$	(D)	$-0.115 \le p_1 - p_2 \le 0.015$

Solution

$$n_1 = 500, \quad \hat{p}_1 = \frac{275}{500} = 0.55, \qquad \hat{q}_1 = 0.45$$

$$n_2 = 400, \quad \hat{p}_2 = \frac{240}{400} = 0.6, \qquad \hat{q}_2 = 0.4$$

a)

$$\hat{p}_1 - Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1}} < p_1 < \hat{p}_1 + Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1}}$$

$$1-\alpha = 0.99, \quad \alpha = 0.01, \qquad \frac{\alpha}{2} = 0.005$$

$$Z_{1-\frac{\alpha}{2}} = Z_{0.995} = 2.575$$

$$0.55 - 2.575\sqrt{\frac{0.55 * 0.45}{500}} < p < 0.55 + 2.575\sqrt{\frac{0.55 * 0.45}{500}}$$

$$0.4927 < p < 0.6073$$

$$(\hat{p}_1 - \hat{p}_2) - Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} < p_1 - p_2 < (\hat{p}_1 - \hat{p}_2) + Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

$$1-\alpha = 0.95, \quad \alpha = 0.05, \qquad \frac{\alpha}{2} = 0.025$$

$$Z_{1-\frac{\alpha}{2}} = Z_{0.975} = 1.96$$

$$(0.55 - 0.6) - 1.96\sqrt{\frac{0.55 * 0.45}{500} + \frac{0.4 * 0.6}{400}} < p_1 - p_2$$

$$< (0.55 - 0.6) + 1.96\sqrt{\frac{0.55 * 0.45}{500} + \frac{0.4 * 0.6}{400}}$$

$$-0.1148 < p_1 - p_2 < 0.0148$$

Q2. A food company distributes "smiley cow" brand of milk. A random sample of 200 consumers in the city (A) showed that 80 consumers prefer the "smiley cow" brand of milk. Another independent random sample of 300 consumers in the city (B) showed that 90 consumers prefer "smiley cow" brand of milk. Define:

p_A = the true proportion of consumers in the city (A) preferring "smiley cow" brand.
p_B = the true proportion of consumers in the city (B) preferring "smiley cow" brand.

(a) A 96% confidence interval for the true proportion of consumers preferring brand (A) is:

(A)	$0.328 \le p_A \le 0.375$	(B)	$0.228 \le p_A \le 0.675$
(C)	$0.328 \le p_A \le 0.475$	(D)	$0.518 \le p_A \le 0.875$

(b) A 99% confidence interval for the difference between proportions of consumers preferring brand (A) and (B) is:

(A)	$0.0123 \le p_A - p_B \le 0.212$	(B)	$-0.2313 \le p_A - p_B \le 0.3612$
(C)	$-0.0023 \le p_A - p_B \le 0.012$	(D)	$-0.0123 \le p_A - p_B \le 0.212$

Solution:

$n_A = 200, \quad \hat{p}_A = \frac{80}{200} = 0.4, \qquad \hat{q}_A = 0.6$

$n_B = 300, \quad \hat{p}_B = \frac{90}{300} = 0.3, \qquad \hat{q}_B = 0.7$

a) $\hat{p}_A - Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\hat{p}_A\hat{q}_A}{n_A}} < p_A < \hat{p}_A + Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\hat{p}_A\hat{q}_A}{n_A}}$

$1-\alpha = 0.96, \quad \alpha = 0.04, \qquad \frac{\alpha}{2} = 0.02$

$Z_{1-\frac{\alpha}{2}} = Z_{0.98} = 2.055$

$$0.4 - 2.055\sqrt{\frac{0.4*0.6}{200}} < p_A < 0.4 - 2.055\sqrt{\frac{0.4*0.6}{200}}$$

$0.3288 < p_A < 0.4712$

b) $(\hat{p}_A - \hat{p}_B) - Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\hat{p}_A\hat{q}_A}{n_A} + \frac{\hat{p}_B\hat{q}_B}{n_B}} < p_A - p_B < (\hat{p}_A - \hat{p}_B) + Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\hat{p}_A\hat{q}_A}{n_A} + \frac{\hat{p}_B\hat{q}_B}{n_B}}$

$$(0.4-0.3) - 2.055\sqrt{\frac{0.4*0.6}{200} + \frac{0.3*0.7}{300}} < p_1 - p_2$$

$$< (0.4-0.3) + 2.055\sqrt{\frac{0.4*0.6}{200} + \frac{0.3*0.7}{300}}$$

$0.0122 < p_A - p_B < 0.2122$

Q3.H.W A random sample of 100 students from school "A" showed that 15 students smoke. Another independent random sample of 200 students from school "B" showed that 20 students smoke. Let p_1 be the proportion of smokers in school "A" and p_2 is the proportion of smokers in school "B".

(1) Find a point Estimate for $p_1 - p_2$.
(2) Find 95% confidence interval for $p_1 - p_2$.

From book:

9.66 Ten engineering schools in the United States were surveyed. The sample contained 250 electrical engineers, 80 being women; 175 chemical engineers, 40 being women. Compute a 90% confidence interval for the difference between the proportions of women in these two fields of engineering. Is there a significant difference between the two proportions?

Solution:

$$n_1 = 250, \quad \hat{p}_1 = \frac{80}{250} = 0.32, \qquad \hat{q}_1 = 0.68$$

$$n_2 = 175, \quad \hat{p}_2 = \frac{40}{175} = 0.229, \qquad \hat{q}_2 = 0.771$$

$$(\hat{p}_1 - \hat{p}_2) - Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} < p_1 - p_2 < (\hat{p}_1 - \hat{p}_2) + Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

$$1 - \alpha = 0.9, \qquad \alpha = 0.1 \ \ \frac{\alpha}{2} = 0.05 \Longrightarrow Z_{1-\frac{\alpha}{2}} = Z_{0.95} = 1.645$$

$$(0.32 - 0.229) - 1.645\sqrt{\frac{0.32 * .68}{250} + \frac{0.229 * 0.771}{250}} < p_1 - p_2$$

$$< (0.32 - 0.229) + 1.645\sqrt{\frac{0.32 * .68}{250} + \frac{0.229}{250}}$$

$$0.0917 < p_1 - p_2 < 0.1623$$

Homework : 9.67