Chapter 23

Electric Fields

Coulomb's Law, Equation

- Mathematically,

$$F_e = k_e \frac{|q_1||q_2|}{r^2}$$

- The SI unit of charge is the **coulomb** (C)
- k_e is called the **Coulomb constant**
  - $k_e = 8.9875 \times 10^9$ N·m²/C² $= 1/(4\pi e_o)$
  - e_o is the **permittivity of free space**
  - $e_o = 8.8542 \times 10^{-12}$ C^2 / N·m²

Hydrogen Atom Example

- The electrical force between the electron and proton is found from Coulomb's law
  - $F_e = k_e q_1 q_2 / r^2 = 8.2 \times 10^8$ N
- This can be compared to the gravitational force between the electron and the proton
  - $F_g = G m_e m_p / r^2 = 3.6 \times 10^{-47}$ N

The Superposition Principle

- The resultant force on any one charge equals the vector sum of the forces exerted by the other individual charges that are present
  - Remember to add the forces *as vectors*
- The resultant force on q_1 is the vector sum of all the forces exerted on it by other charges: $\mathbf{F}_1 = \mathbf{F}_{21} + \mathbf{F}_{31} + \mathbf{F}_{41}$

Electric Field – Definition

- The electric force is a field force
- An **electric field** is said to exist in the region of space around a charged object
  - This charged object is the **source charge**
- When another charged object, the **test charge**, enters this electric field, an electric force acts on it
- The test charge serves as a detector of the field

Electric Field – Definition, cont

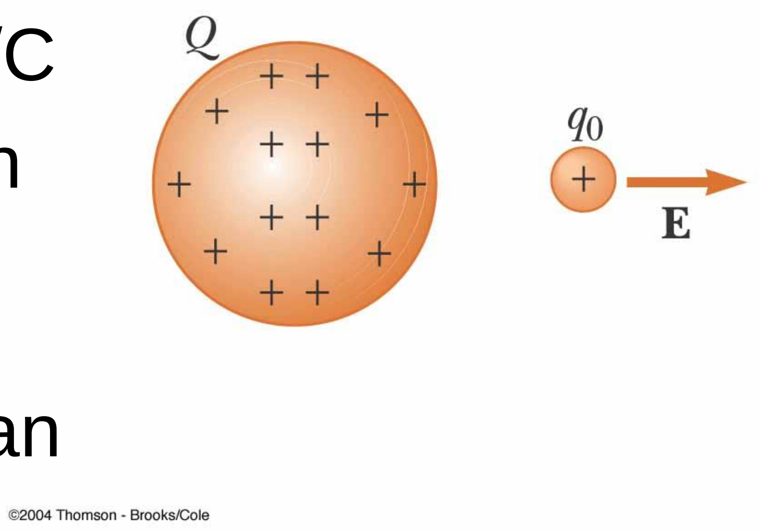
- The electric field is defined as the electric force on the test charge per unit charge
- The electric field vector, **E**, at a point in space is defined as the electric force **F** acting on a positive test charge, q_o placed at that point divided by the test charge: $\mathbf{E} = \mathbf{F}_e / q_o$

Relationship Between **F** and **E**

- $\mathbf{F}_e = q\mathbf{E}$
  - This is valid for a point charge only
  - One of zero size
  - For larger objects, the field may vary over the size of the object
- If *q* is positive, **F** and **E** are in the same direction
- If *q* is negative, **F** and **E** are in opposite directions

Electric Field Notes, Final

- The direction of **E** is that of the force on a positive test charge
- The SI units of **E** are N/C
- We can also say that an electric field exists at a point if a test charge at that point experiences an electric force



Electric Field, Vector Form

- Remember Coulomb's law, between the source and test charges, can be expressed as

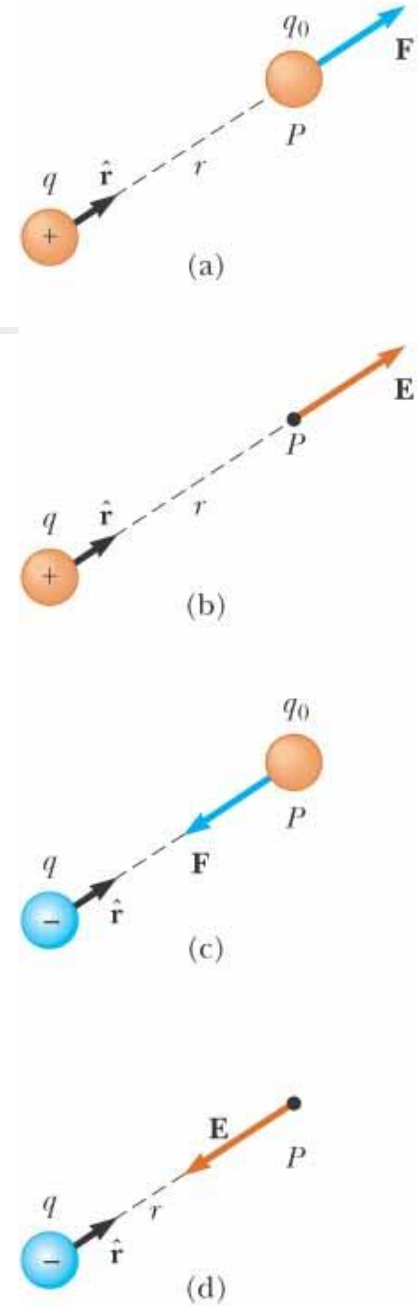
$$\mathbf{F}_e = k_e \frac{q q_o}{r^2} \hat{\mathbf{r}}$$

- Then, the electric field will be

$$\mathbf{E} = \frac{\mathbf{F}_e}{q_o} = k_e \frac{q}{r^2} \hat{\mathbf{r}}$$

More About Electric Field Direction

- a) *q* is positive, **F** is directed away from *q*
- b) The direction of **E** is also away from the positive source charge
- c) *q* is negative, **F** is directed toward *q*
- d) **E** is also toward the negative source charge



Superposition with Electric Fields

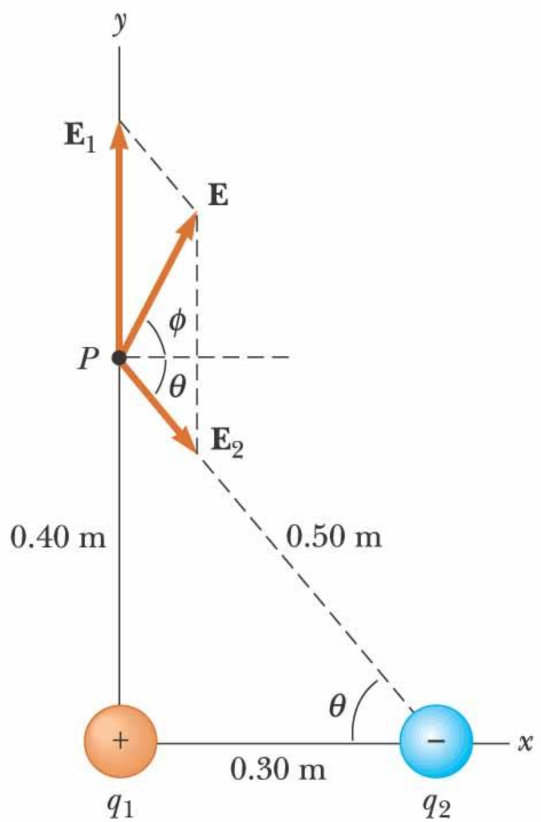
- At any point *P*, the total electric field due to a group of source charges equals the vector sum of electric fields of all the charges

$$\mathbf{E} = k_e \sum_i \frac{q_i}{r_i^{\,2}} \hat{\mathbf{r}}_i$$

Example 23.5

EXAMPLE 23.5 **Electric Field Due to Two Charges**

A charge $q_1 = 7.0\ \mu C$ is located at the origin, and a second charge $q_2 = -5.0\ \mu C$ is located on the x axis, 0.30 m from the origin (Fig. 23.13). Find the electric field at the point P, which has coordinates (0, 0.40) m.



Example 23.5

Solution First, let us find the magnitude of the electric field at P due to each charge. The fields $\mathbf{E}_1$ due to the 7.0-μC charge and $\mathbf{E}_2$ due to the -5.0-μC charge are shown in Figure 23.13. Their magnitudes are

$$E_1 = k_e \frac{|q_1|}{r_1^{\,2}} = \left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right) \frac{(7.0 \times 10^{-6}\ \text{C})}{(0.40\ \text{m})^2}$$

$$= 3.9 \times 10^5\ \text{N/C}$$

$$E_2 = k_e \frac{|q_2|}{r_2^{\,2}} = \left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right) \frac{(5.0 \times 10^{-6}\ \text{C})}{(0.50\ \text{m})^2}$$

$$= 1.8 \times 10^5\ \text{N/C}$$

Example 23.5

The vector $\mathbf{E}_1$ has only a y component. The vector $\mathbf{E}_2$ has an x component given by $E_2 \cos\theta = \frac{3}{5}E_2$ and a negative y component given by $-E_2 \sin\theta = -\frac{4}{5}E_2$. Hence, we can express the vectors as

$$\mathbf{E}_1 = 3.9 \times 10^5 \mathbf{j} \text{ N/C}$$

$$\mathbf{E}_2 = (1.1 \times 10^5 \mathbf{i} - 1.4 \times 10^5 \mathbf{j}) \text{ N/C}$$

The resultant field $\mathbf{E}$ at P is the superposition of $\mathbf{E}_1$ and $\mathbf{E}_2$:

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = (1.1 \times 10^5 \mathbf{i} + 2.5 \times 10^5 \mathbf{j}) \text{ N/C}$$

From this result, we find that $\mathbf{E}$ has a magnitude of 2.7×10^5 N/C and makes an angle ϕ of 66° with the positive x axis.

Exercise Find the electric force exerted on a charge of 2.0×10^{-8} C located at P.

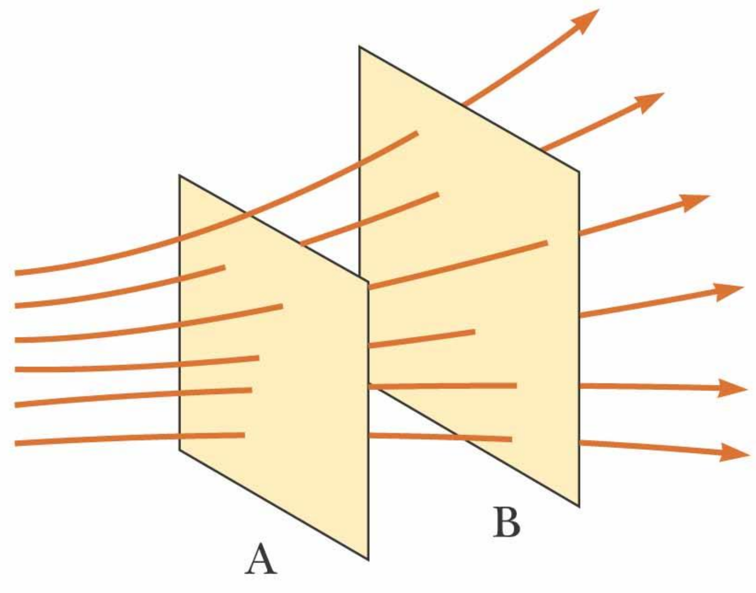
Answer 5.4×10^{-3} N in the same direction as $\mathbf{E}$.

Electric Field Lines

- Field lines give us a means of representing the electric field pictorially
- The electric field vector **E** is tangent to the electric field line at each point
  - The line has a direction that is the same as that of the electric field vector
- The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of the electric field in that region

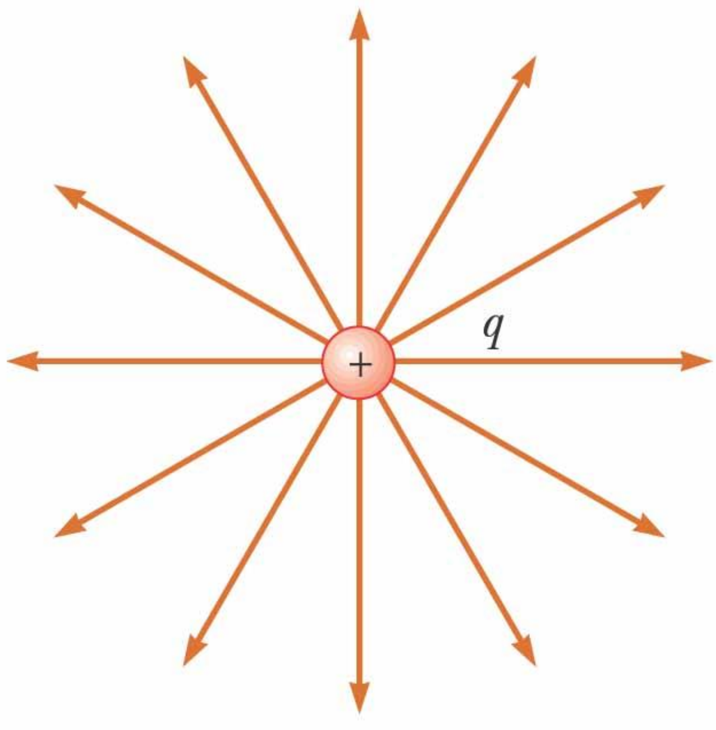
Electric Field Lines, General

- The density of lines through surface A is greater than through surface B
- The magnitude of the electric field is greater on surface A than B
- The lines at different locations point in different directions
  - This indicates the field is non-uniform



Electric Field Lines, Positive Point Charge

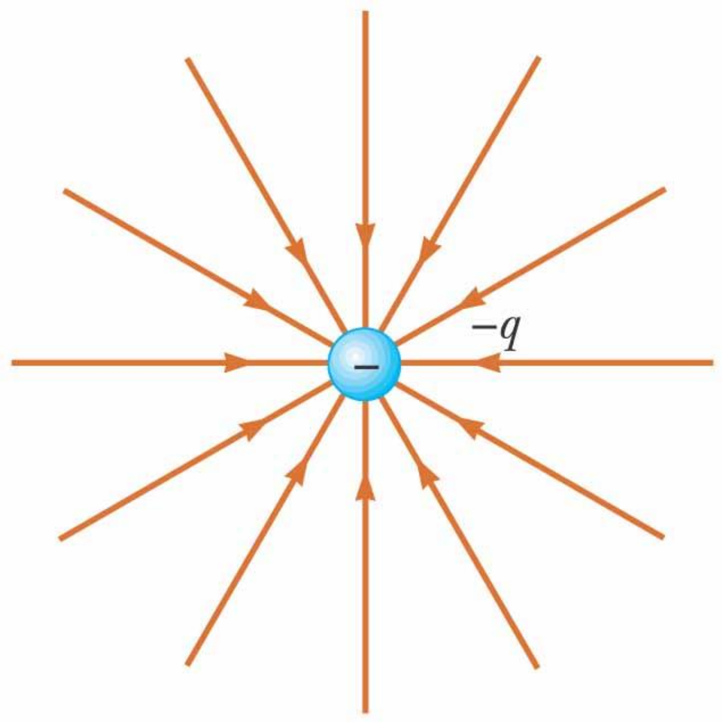
- The field lines radiate outward in all directions
  - In three dimensions, the distribution is spherical
- The lines are directed away from the source charge
  - A positive test charge would be repelled away from the positive source charge



(a)

Electric Field Lines, Negative Point Charge

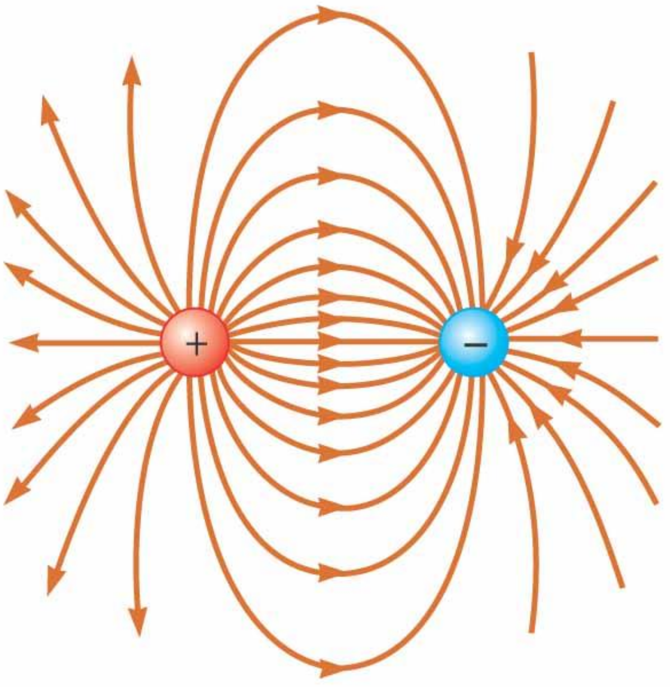
- The field lines radiate inward in all directions
- The lines are directed toward the source charge
  - A positive test charge would be attracted toward the negative source charge



(b)

Electric Field Lines – Dipole

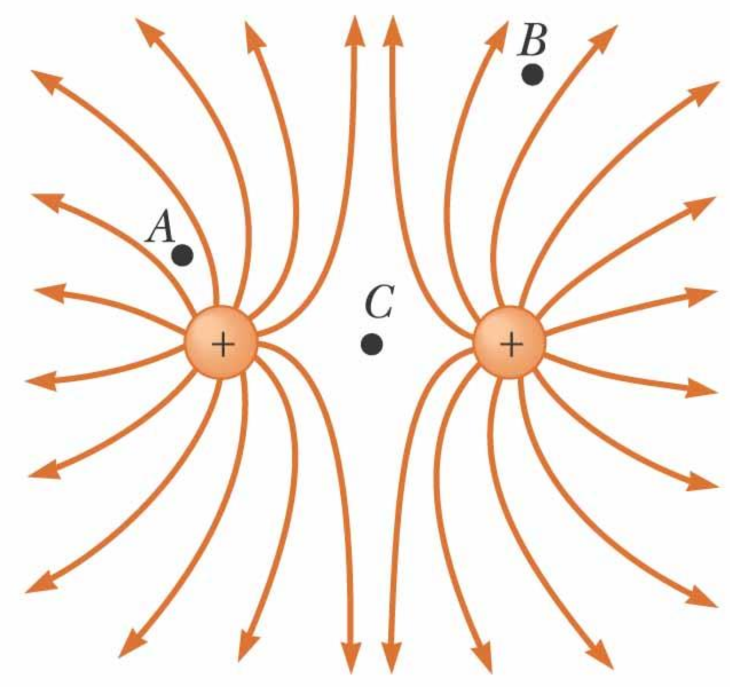
- The charges are equal and opposite
- The number of field lines leaving the positive charge equals the number of lines terminating on the negative charge



(a)

Electric Field Lines – Like Charges

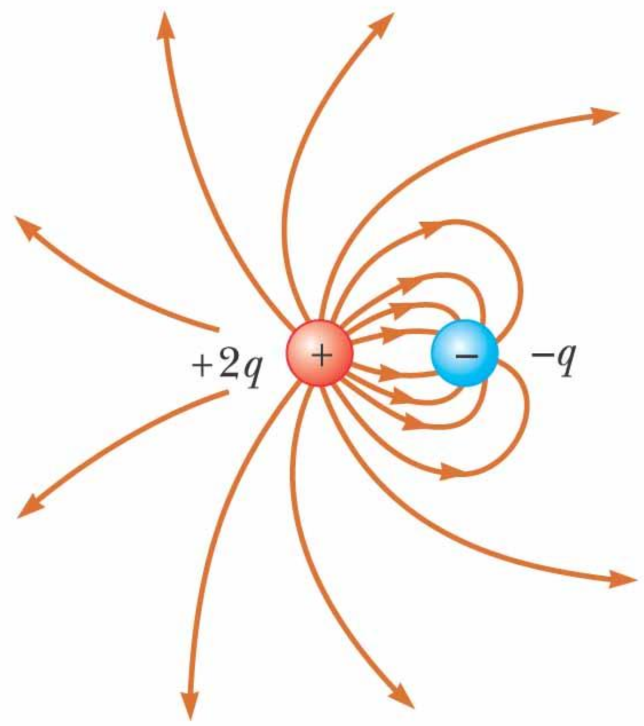
- The charges are equal and positive
- The same number of lines leave each charge since they are equal in magnitude
- At a great distance, the field is approximately equal to that of a single charge of $2q$



(a)

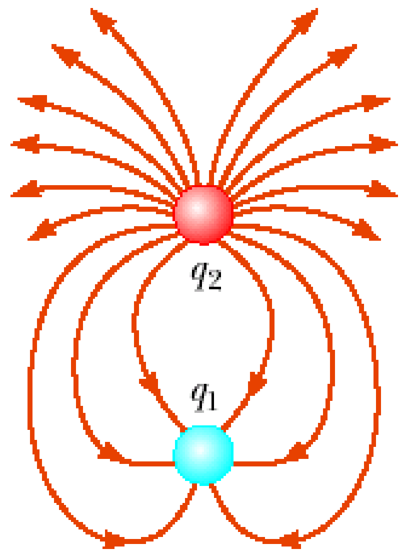
Electric Field Lines, Unequal Charges

- The positive charge is twice the magnitude of the negative charge
- Two lines leave the positive charge for each line that terminates on the negative charge
- At a great distance, the field would be approximately the same as that due to a single charge of $+q$



Question

- The Figure shows the electric field lines for two point charges separated by a small distance. (a) Determine the ratio q_1/q_2. (b) What are the signs of q_1 and q_2?



Motion of Charged Particles

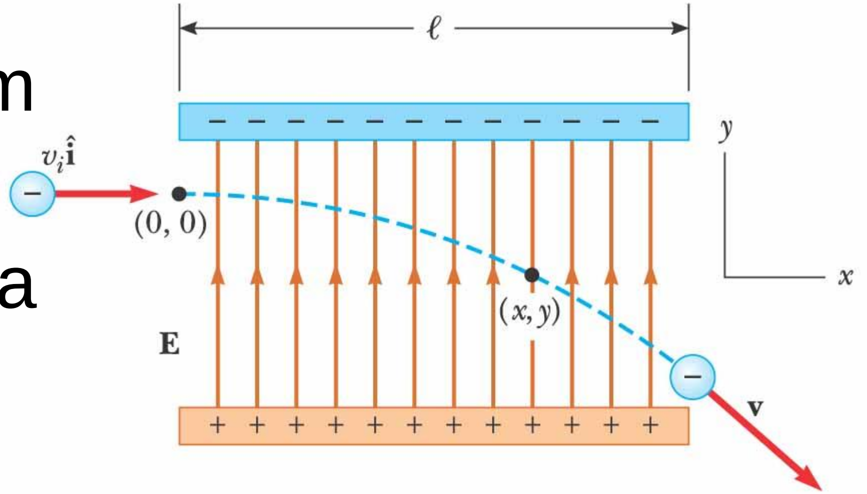
- When a charged particle is placed in an electric field, it experiences an electrical force
- If this is the only force on the particle, it must be the net force
- The net force will cause the particle to accelerate according to Newton's second law

Motion of Particles, cont

- $\mathbf{F}_e = q\mathbf{E} = m\mathbf{a}$
- If **E** is uniform, then **a** is constant
- If the particle has a positive charge, its acceleration is in the direction of the field
- If the particle has a negative charge, its acceleration is in the direction opposite the electric field
- Since the acceleration is constant, the kinematic equations can be used

Electron in a Uniform Field, Example

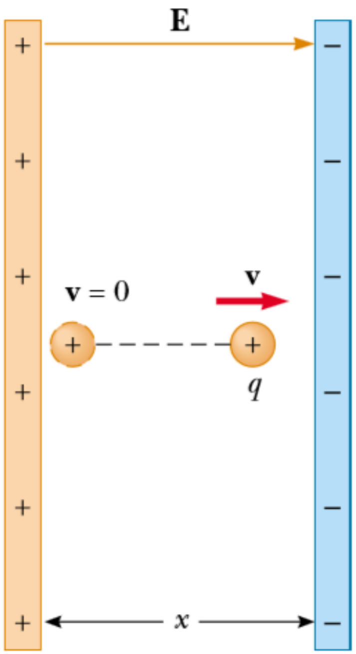
- The electron is projected horizontally into a uniform electric field
- The electron undergoes a downward acceleration
  - It is negative, so the acceleration is opposite **E**
- Its motion is parabolic while between the plates



Example 23.10

EXAMPLE 23.10 **An Accelerating Positive Charge**

A positive point charge q of mass m is released from rest in a uniform electric field **E** directed along the x axis, as shown in Figure 23.24. Describe its motion.



Example 23.10

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2$$

$$v_{xf} = v_{xi} + a_x t$$

$${v_{xf}}^2 = {v_{xi}}^2 + 2a_x(x_f - x_i)$$

Taking $x_i = 0$ and $v_{xi} = 0$, we have

$$x_f = \tfrac{1}{2}a_x t^2 = \frac{qE}{2m}\,t^2$$

$$v_{xf} = a_x t = \frac{qE}{m}\,t$$

$${v_{xf}}^2 = 2a_x x_f = \left(\frac{2qE}{m}\right)x_f$$

The kinetic energy of the charge after it has moved a distance $x = x_f - x_i$ is

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\left(\frac{2qE}{m}\right)x = qEx$$

Example 23.11

EXAMPLE 23.11 **An Accelerated Electron**

An electron enters the region of a uniform electric field as shown in Figure 23.25, with $v_i = 3.00 \times 10^6$ m/s and $E = 200$ N/C. The horizontal length of the plates is $\ell = 0.100$ m. (a) Find the acceleration of the electron while it is in the electric field.

Solution The charge on the electron has an absolute value of 1.60×10^{-19} C, and $m = 9.11 \times 10^{-31}$ kg. Therefore, Equation 23.8 gives

$$\mathbf{a} = -\frac{eE}{m}\mathbf{j} = -\frac{(1.60 \times 10^{-19}\ \text{C})(200\ \text{N/C})}{9.11 \times 10^{-31}\ \text{kg}}\mathbf{j}$$

$$= -3.51 \times 10^{13}\,\mathbf{j}\ \text{m/s}^2$$

Example 23.11

(b) Find the time it takes the electron to travel through the field.

$$t = \frac{\ell}{v_i} = \frac{0.100 \text{ m}}{3.00 \times 10^6 \text{ m/s}} = 3.33 \times 10^{-8} \text{ s}$$

(c) What is the vertical displacement y of the electron while it is in the field?

$$y = \tfrac{1}{2} a_y t^2 = \tfrac{1}{2}(-3.51 \times 10^{13} \text{ m/s}^2)(3.33 \times 10^{-8} \text{ s})^2$$
$$= -0.019\,5 \text{ m} = -1.95 \text{ cm}$$

Exercise Find the speed of the electron as it emerges from the field.

Answer 3.22×10^6 m/s.

The Cathode Ray Tube (CRT)

- A CRT is commonly used to obtain a visual display of electronic information in oscilloscopes, radar systems, televisions, etc.
- The CRT is a vacuum tube in which a beam of electrons is accelerated and deflected under the influence of electric or magnetic fields

CRT, cont

- The electrons are deflected in various directions by two sets of plates
- The placing of charge on the plates creates the electric field between the plates and allows the beam to be steered