Measurement and Errors in Experimental Physics

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1 Introduction

Physics is fundamentally an experimental science, it is essentially based on observations of natures and modeling such observations with a theory. Or conducting experiments that either confirm or exclude a model that was set a priori. Although the recent scheme is recent and just few decades ago, observations were ahead of theoretical models.
Since physics is based on *observations* and *experiments* in which we measure particular quantities (could dynamical or non-dynamical), for example the velocity, momentum of some particle, or temperature of an object, that could vary in time. Or distances between galaxies that appear unchanging in time. The act of measurement itself is what gives us *data* in which we could analyse to extract information about some part of nature that we want to discover or some theory that we wish to test.

2 Measurement

There are two types of measurements, **direct** and **indirect**. We define the direct measurement is by measuring a physical quantity with a device that resembles the *same* quantity; like using a ruler to measure lengths. This type of measurement is very rare and most measurements in experimental physics are bases on the second type. The indirect measurement could be made by letting the measuring device or *apparatus* interact with the measured system. And btextased on some *model* we can extract the data regarding the quantity we want to measure, like measuring the area of a table by measuring its length L and width W Based on the model that this table is rectangular and the the area is then $L \times W$. Surly this 'model' assumes a priori that the table is rectangular, but there is no way to say it is indeed rectangular with infinite accuracy.
Another example of indirect measurement is the measurement of temperature by an Hg or alcohol thermometer. We measure the expansion of the liquid and use the model of liquid thermal expansion:

$$\frac{\Delta L}{L} = \alpha \Delta T \tag{1}$$

Hence if we know the initial length and the factor α we could associate the change of length of the liquid with the change of temperature.
In both cases of direct and indirect measurement we need to preform a **Calibration** procedure. That is to to establish a relation between the quantity we measure that we call a *signal* i with the quantity we wanted to originally measure q. Unfortunately, there is no guarantee that such

relation is easily established, but for many cases the relation could be approximates to being a proportionality relation, and the proportionality constant is called the Calibration coefficient, i.e.

$$R = \frac{i}{q} \tag{2}$$

Before preforming any experiment/ measurement with an apparatus, it is essential to calibrate that apparatus using the relation above- draw a graph using a known set of quantities $\{q_i\}$ and then compute the slope $= R$. Calibration is different from using a model as discussed above, here we could know nothing about the nature of interaction between the measured system and the apparatus that produces the signal i. However, we *only* know that for a certain values of q the relation (2) holds.
When we wish to measure a quantity, for example the intensity of light via a photoelectric cell We start from the initial intensity from the source E_i, and detect intensity E_d at the cell. Which is calculated from the relation (2) $E_d = i/R$. However, there is no guarantee that $E_d = E_i$ because the detected intensity could only be a fraction of the incoming one, due to reflection of the light from the cell's surface, scattering of light by the air and so on. We define the ratio

$$\Sigma = \frac{E_d}{E_i} \tag{3}$$

Which is known as the **sampling** of the signal. Once the sampling is known and the calibration, as well we can recover the incident intensity from the signal i using the relation:

$$E_i = \frac{i}{R\,\Sigma} \tag{4}$$

Sometimes, we detect only part of the incoming flux of the quantity (like the intensity) or sometimes momentum on purpose, to keep the system after measurement. Here the sample would be very small $\Sigma << 1$. The case where the sampling is close to one $\Sigma \sim 1$ we call the measurement a **destructive measurement.**

3 Errors and Uncertainties

When preforming a measurement as discussed above, the measurement procedure does not reflect the *exact* value of the quantity of study. It is *impossible* to make the measurement procedure and/or the model in study perfect! We are bound to have errors in our measurements, and it is essential to be able to define these errors and know their origin. It can be said that :

A measurement without uncertainties has no meaning at all

If we measured the length of an object and it was found to be $2,8cm$, this statement seems fine at first, but if we have a theory that predicts the length of that object to be $3,0$ cm. That measurement will be indeed meaningless, in its current form it tells us nothing about the correctness of the model we are studying. However, if we included the uncertainties / errors in the measurement and we find that the length is $2,8 \pm 4cm$, now you would immediately say that the measurement *confirms* the model within the experimental errors. Because $3,0$cm lies within the uncertainty we have put.
Thus, it is crucial for experimental physics to be able to define and calculate the errors, otherwise experimentation becomes useless. There are two types of errors that we deal with in experimentation, **systematic** and **random** (stexttatistical).

- **Systematic error** : is associated with the model used in the computation or the experimental setup. It could be an error due to imperfection of the apparatus used in the measurement or calibration ..etc .
- **Random error** : is associated with unknown or uncontrolled factors- like fluctuations in the system- or the fact that the system in study is fundamentally random or probabilistic (like quantum mechanical systems: such as radioactive decay).

Usually, random errors are easily improved in principle, by increasing the number of experiments preformed. However, systematic errors are much harder to correct and do not improve with repetition of experimentation. It may often be reduced by very carefully with standardized procedures. Part of the learning process in the various sciences is learning how to use standard instruments and protocols so as to minimize systematic error.

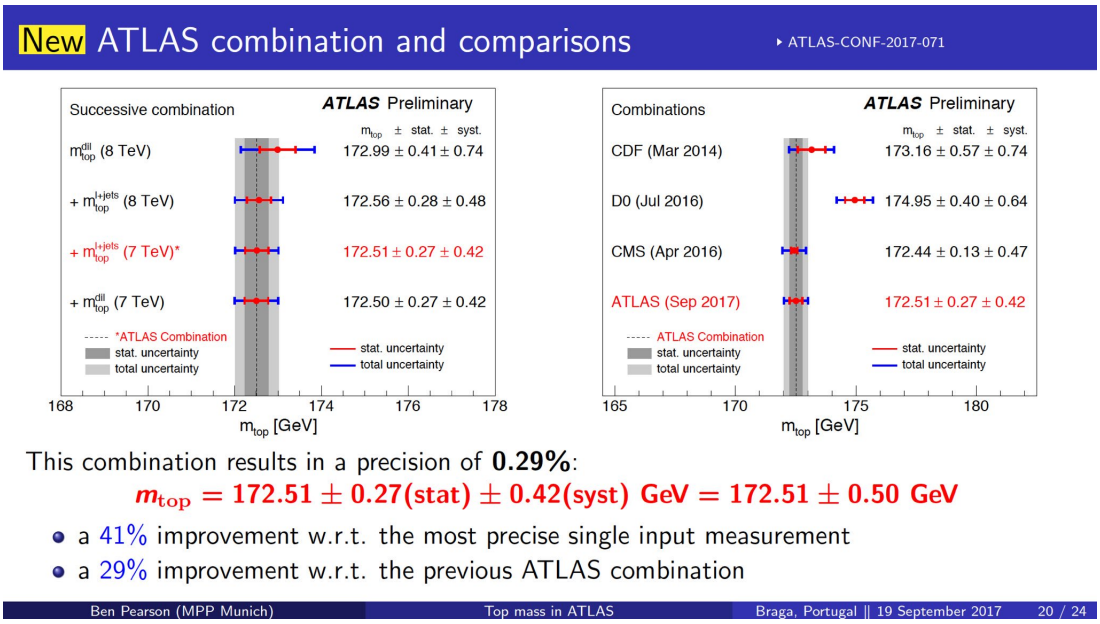

Figure 1: A new result by the ATLAS experiment at the Large hadron collider LHC, showing the mass of the 'top' quark.The major improvement in this result is the fact that they were able to reduce the systematic errors. Observe that the statistical errors and systematic errors are treated differently.

4 Statistics

The proper language to deal with the measured quantities having uncertainties/ errors is statistics. Where these quantities are treated as 'random' variables. X having their own **probability density distribution function** $f(X)$ telling us the likelihood of a measured quantity to take a certain

value X_0. Ideally, the function $f(X)$ takes a normal distribution form, or a Gaussian function

$$f(X) = \varphi_{\mu,\sigma}(X) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{-1}{2}\left(\frac{X-\mu}{\sigma}\right)^2\right] \tag{5}$$

We call $X_0 = \mu$ the mean value or expected value. And the quantity σ the standard deviation. Hence, when an experiment is preformed many times, we define the mean-value with the 'measured'

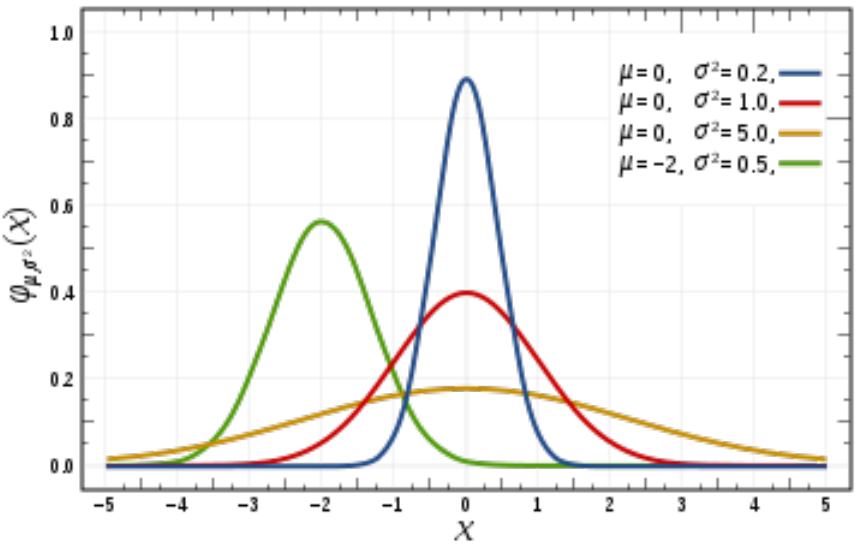

Figure 2: Normal distribution with different means and standard deviation

value , and the uncertainty with the standard deviation. This is of course the uncertainty for the random errors. while the uncertainty for systematic errors is calculated in a different way, and it shall not be discussed in this course. We can also define the full width half maximum or FWHM or simply the 'width'Γ as

$$\Gamma = 2\sqrt{2\ln 2}\,\sigma \tag{6}$$

Which plays an important role in many areas of physics, like particle physics for example.

Interpretation of the probability

We have demonstrated the proper language to use to describe the measured quantities in terms of probability (density) distribution $\varphi_{\mu,\sigma}$. But what is the meaning of this function ? How one could interpret the meaning of probability? This is not a purely philosophical question, rather, it is a necessity in order to draw conclusions from the collected data from measurement, this process is called **inference**. Mainly there are two approaches to state inference.

- **Frequentest approach** , a objective way to interpret the probability of an event A is that to take an infinite number of experiments $N \to \infty$ with identical conditions and see the

'frequency' of which the event A occurs, $N(A)$, i.e.

$$P(A) = \lim_{N\to\infty} \frac{N(A)}{N}$$

The main difficulties of this approach is that it is impossible to preform an infinite number of measurements/ experiments and we could not usually produce identical conditions. Hence, we try to make as many measurements as possible and the make the experimentation as controlled as possible to approach this interpretation

- **Bayèsienne approach** , in this approach we collect data about a phenomenon and from this data we could define the probability as the *degree of belief* that a certain value is more likely than others. As we can see this is completely subjective approach, but its is inadvisable if one could not preform any number of experiments or create identical ones. We shall not discuss the bayèsienne school of statistics as almost always in physics the frequentest school is followed.

Since we have agreed that the distribution $\varphi_{\mu,\sigma}$ is interpreted using the frequentest school, we could compute the mean μ and the uncertainty σ from the following laws: Let X_i denote the i th measurement in an experiment measuring the variable X. We define the expected value of the mean value of the variable X as

$$\mu = \bar{X} = \frac{\sum_{i=1}^{N} X_i}{N} \tag{7}$$

Where N is the number of measurements preformed. And the standard deviation or the uncertainty as:

$$\sigma_X = \Delta X = \sqrt{\bar{X^2} - \bar{X}^2} \tag{8}$$

If we have a quantity X_0 that is known before and we with to compute the **absolute error** we use the formula

$$\epsilon = |X_0 - \bar{X}| \tag{9}$$

and the **relative error**

$$\eta = \frac{\epsilon}{X_0} \tag{10}$$

These quantities ϵ and η are called *approximation errors*. We may also define the **per-cent error** as $\delta = 100\% \times \eta$. There will tell us about the systematic errors we had in the experiments. Since X_0 is usually a quantity that is measured already by scientists with high precision. There are many other types of errors and their ways to be analyzed statistically, the reader is strongly advised to consult a reference in statistics and experimental physics for further information