

KING SAUD UNIVERSITY

COLLEGE OF COMPUTER & INFORMATION SCIENCES
DEPT OF COMPUTER SCIENCE

CSC281 Discrete Mathematics

Second Semester 1427/1428 AH

First Mid-term Examination: Monday 21.3.1428 A.H./9.4.2007 C.E. (duration = 2 hours)

Instructor: Dr. Aqil Azmi

Instructions:

- ☐ Write your name and id.
- ☐ Answer all the questions.
- ☐ Write your final answer neatly in the designated space. Try showing your computation as much as possible.
- ☐ Use back sheet for scribbling/scratching.

S/N:

Name:

ID:

1. [Marks 10]

Assume that set $S = \{a\}$, write down the elements of the set $P(P(S))$ where $P(\cdot)$ denotes the powerset of the given set.

$P(S) = \{\emptyset, a\}$ $\{ \emptyset, a, \{\emptyset, a\} \}$

2. [Marks 13]

Use mathematical induction to prove that $1 + x + x^2 + \dots + x^n = (x^{n+1} - 1)/(x - 1)$, where $x \neq 1$.

3. [Marks 10]

Let $f(x) = e^x + 1$ and $g(x) = \ln x$. Compute $(f \circ g)(x)$ and $(g \circ f)(x)$.

4. [Marks 15]

Solve the linear congruence equation $34x \equiv 19 \pmod{65}$.

5. [Marks 10]

Use proof by contradiction to prove that the sum of an irrational number and a rational number is irrational.

6. [Marks 12]

Prove that if $7 \nmid n$ then n^2 leaves a remainder of 1, 2 or 4 (only) when divided by 7.

Hint: use proof by cases.

7. [Marks 10]

Determine if the integer 667 is a prime. Show all your computation.

8. [Marks 20]

You are given the sequence $a_{23} = 374$, $a_{24} = 398$, $a_{25} = 423$ and $a_{26} = 449$. Evaluate

the summation $\sum_{i=10}^{30} a_i$. Useful formulas: $\sum_{i=1}^n i = n(n+1)/2$, and

$\sum_{i=1}^n i^2 = n(n+1)(2n+1)/6$. Hint: express all a_i in term of a_{23} .

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1. [Marks 10]

A box contains 10 pens and 10 pencils. A boy selects at random without looking at them.

- a. How many he must select to be sure of having 3 of same type (i.e. 3 pens or 3 pencils)? Explain.
- b. How many he must select to be sure of having 2 pens and 1 pencil? Explain.

2. [Marks 10]

What is the coefficient of x^{17} in the expansion of $(x^2 - \frac{1}{x})^{40}$.

3. Marks 40]

The English alphabet contains a total of 26 letters (21 consonants and 5 vowels). How many strings of 6 uppercase letters of the English alphabet are there

- a. that contain no vowels, if letters can be repeated?
- b. that contain no vowels, if letters are not repeated?
- c. that start with a vowel (rest are consonants), if letters can be repeated?
- d. that start with a vowel (rest are consonants), if letters are not repeated?
- e. that contain exactly one vowel, if letters can be repeated?
- f. that contain at least one vowel, if letters can be repeated?
- g. that contains at most two vowels (rest are consonants), no repeated letters in the string?
- h. that start with letter X and contain exactly one vowel, if letters are not repeated?
- i. is a palindrome?
- j. contains the substring ABC, rest of the string is composed of the letters D, E, ..., Z each occurring no more than once?

4. [Marks 20]

How many positive integers not exceeding 100 that is divisible either by 4 or by 6 or by 10? Note: $\text{lcm}(4, 6, 10) = 60$. Hint: recall the union of three sets.

5. [Marks 10]

Determine if $a_n = n5^n$ is a solution to the recurrence relation $a_n = 10a_{n-1} - 25a_{n-2}$.

6. [Marks 10]

Write the generating function in closed form for the infinite sequence: 1, 3, 1, 3, 1, 3, ...

Note: $1/(1-ax)$ generates the infinite sequence: 1, a , a^2 , a^3 , ...

Handwritten notes and calculations:

- Boxed 60
- Boxed $l(a, b, c)$
- Boxed 4, 6, 10
- Handwritten: $2, 2, 3$
- Handwritten: $2, 5$
- Handwritten: $n5^n$
- Handwritten: $10(n-1)5^{n-1} - 25(n-2)5^{n-2}$
- Handwritten: $5(n-2)5^{n-2}$
- Handwritten: $5^{n-2} [10(n-1) - 25(n-2) + 25]$
- Handwritten: $5^{n-2} [10n - 10 + 25n - 50 + 25]$
- Handwritten: $5^{n-2} [35n - 35]$
- Handwritten: $5^{n-2} \cdot 35(n-1)$