COURSE DESCRIPTIONS

Ordinary differential equations (4 credits)
Prerequisite: Advanced calculus

1- **Introduction.** Some basic concepts; physical origins.
2- **First order differential equations.** Existence and uniqueness of solution; solution by separation of variables; homogenous equations; exact equations; linear equations; some special equations; substitutions; geometric and physical applications.
3- **Higher order linear differential equations.** Boundary-value problems; solutions of linear equations; homogeneous linear equations with constant coefficients; nonhomogeneous linear equations; variation of parameters; applications of second order equations to motion.
4- **Differential equations with variable coefficients.** The Cauchy-Euler equation; solution by power series; singular points and the Frobenius method; some special equations.

Textbook. Differential *Equations with Boundary-Value Problems* by Dennis G. Zill, PWS publishers, 1986.

Real Analysis I (4 credits)
Prerequisite: Advanced calculus

1- **Real number system**. Field and order properties; integers and rational numbers; completeness axiom; countable sets.
2- **Sequences**. Convergence; properties of convergent sequences; monotone sequences; Cauchy criterion, open and closed sets.
3- **Limit of a function**. Definitions and basic theorems; extensions of the limit; monotone functions.
4- **Continuity**. Definitions and properties, continuity on an interval; uniform continuity; compact sets and continuity.
5- **Differentiation**. The derivative of a function; the mean value theorem; L'Hopital's rule; Taylor's theorem.

Textbook. *Elements of Real Analysis* by M.A. Al-Gwaiz and S.A. Elsanousi, Chapman and Hall, 2006.
Reference. *Introduction to Real Analysis* by R.G. Bartle and D.R. Sherbert, 3rd edition, John Wiley, 2000.

Real Analysis II (4 credits)
Prerequisite: Real analysis I

1- **The Riemann integral**. Riemann integrability; Darboux's theorem and Riemann sums; properties of the Riemann integral; fundamental theorem of calculus; improper integrals.
2- **Sequences and series of functions**. Sequences of functions and their modes of convergence (pointwise and uniform); properties of uniform convergence; series of functions; power series.
3- **Lebesgue** measure. Classes of subsets of $\mathbb{R}$; Lebesgue outer measure; Lebesgue measure; measurable functions.
4- **Lebesgue integration**. Definition and properties of the Lebesgue integral; Lebesgue integral and pointwise convergence (monotone convergence theorem, Fatou's lemma, dominated convergence theorem); Lebesgue and Riemann integrals.

Textbook. *Elements of* Real *Analysis* by M.A. Al-Gwaiz and S.A. Elsanousi, Chapman and Hall, 2006.
Reference. *Introduction to Real Analysis* by R.G. Bartle and D.R. Sherbert, 3rd edition, John Wiley, 2000.

Complex Analysis (4 credits)
Prerequisite: Advanced calculus

1- **Complex numbers**. Cartesian and polar representation of a complex number; powers and roots; function of a complex variable.
2- **Limits and continuity**. Sequence of complex numbers and its convergence; continuity of a function of a complex variable; sequence of functions of a complex variable.
3- **Analytic functions**. Cauchy-Riemann equations; harmonic functions; polynomials and rational functions.
4- **Exponential function.** Definition by infinite series; logarithmic, trigonometric, and hyperbolic functions.
5- **Complex integration**. Contour integrals; Cauchy's theorem; Cauchy's integral formula; properties of analytic functions (Liouville theorem and the maximum and minimum modulus theorem).
6- **Laurent Series**. Zeros and singularities of analytic functions; representation by Laurent series; residue theorem; applications to the evaluation of real integrals.

References. *Complex Analysis* by John M. Howie, Springer-Verlag, 2003.
Complex Analysis by Lars V. Ahlfors, McGraw-Hill, 3rd edition, 1987.

Mathematical Methods (4 credits)
Prerequisite: Ordinary differential equations, linear algebra

1- **Inner product space**. Vector space; inner product space; the space of square integrable functions L^2**;** convergence in L^2; orthogonal functions.
2- **The Sturm-Liouville theory**. Linear second-order equations; self-adjoint differential operator; the Sturm-Liouville problem; existence and completeness of the eigenfunctions; regular and singular problems.
3- **Fourier series**. The fundamental theorem of Fourier series in L^2; pointwise theory of Fourier series; applications to boundary-value problems.
4- **Orthogonal polynomials**. Legendre, Hermite, and Laguerre polynomials as solutions of certain singular Sturm-Liouville problems; their orthogonality and completeness properties; generalized Fourier expansions.
5- **Bessel functions**. The gamma function; Bessel's equation and Bessel functions of the first kind; orthogonality properties.
6- **Fourier transformation**. The fourier transform; the Fourier integral; properties and applications in PDE.

Textbook. *Sturm-Liouville Theory & its Applications* by M.A. Al-Gwaiz, Springer, 2008.

Introduction to PDE (4 credits)
Prerequisite: Mathematical methods

1- **Basic concepts**. Partial differential equations; classification; solutions; sources (6 lectures).
2- **First-order equations**. Linear and quasi-linear equations; Lagrange method for solving quasi-linear equations; Cauchy's problem.
3- **Linear second-order equations**. classification into elliptic, parabolic, and hyperbolic types; solution by operator method and by separation of variables; Cauchy's problem.
4- **Laplace's equation**. Properties of harmonic functions and the maximum modulus principle; boundary-value problems (Dirichlet, Neumann, mixed); uniqueness of the solution; boundary-value problems in 2 and 3 dimensions; solution by separation of variables and Fourier series using Cartesian, polar, cylindrical, and spherical coordinates; Poisson's integral representation for the solution of Dirichlet's problem in a circle.

5- **The wave equation**. Mathematical model of a vibrating string; solution by separation of variables; D'Alenbert's solution; problems in 2 space dimensions; vibrations under friction and gravity.
6- **The heat equation**. Physical derivation using the laws of heat transfer; homogeneous and non-homogeneous boundary conditions; solution by separation of variables and Fourier series; boundary-value problems involving special functions; heat transfer in an infinite bar; representation of the solution by a Fourier integral.

Reference. *Partial Differential Equations: Methods & Applications* by Robert C. McOwen, 2nd edition, Pearson, 2003.

The following courses were offered in the graduate program of the department to M.Sc. and Ph.D. students:

Calculus of Variations (3 credits)
General variation of a functional; constrained extrema; Euler's equation. Weak and strong minima; the second variation and sufficient conditions for an extremum. The Hamilton-Jacobi equation and related topics. Lagrangian and Hamiltonian mechanics.

Theory of Distributions (3 credits)
The topological space of test functions *D*; distributions as continuous linear functionals on *D*; convergence and differentiation properties. Distributions with compact support; convolutions. Fourier transforms and tempered distributions; applications to differential equations. Sobolov spaces; applications to boundary-value problems.

Theory of linear partial differential equations (3 credits)
Prerequisite: Theory of distributions

The aim of this course is to give a systematic study of questions concerning existence, uniqueness, and regularity of solutions of linear partial differential equations and boundary-value problems. The restriction to linear equations with smooth coefficients is intended to highlight the role that distribution theory plays in the treatment of PDE. In particular the existence and uniqueness of linear equations with constant coefficients is investigated, and settled, by applying the Fourier transformation and thereby reducing the problem to one of division in the space of tempered distributions.

Reference: *The Analysis of Linear Partial Differential Operators*, vol. I & II, by L. Hormander, Springer-Verlag, 1983.