Big O

$t(n) \in O(g(n))$

if $\exists\, c > 0$ and n_0 such that:

$t(n) \le c\, g(n),\ n \ge n_0$

$t(n) = O(g(n))$

Q:

Does $100n+5 \in O(n^2)$?

$100n + 5 \le 100n + 5n$

$c = 105 \le 105n$

$n_0 = 1$

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Big Ω

$t(n) \in \Omega(g(n))$

if $\exists\, c$ /

$t(n) \ge c\, g(n),\ n \ge n_0$

Question:

Does $n^3 \in \Omega(n^2)$?

$n^3 \ge n^2$

$c = 1 \quad n_0 \ge 0$

Does $100n+5 \in \Omega(n^2)$?

Answer: No

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Big Θ

$t(n) \in \Theta(g(n))$

if $\exists\, c_1, c_2$ /

$c_1 g(n) \le t(n) \le c_2 g(n)$ for $n \ge n_0$

- Does $\frac{1}{2} n(n-1) \in \Theta(n^2)$?

$\frac{1}{2} n(n-1) = \frac{1}{2} n^2 - \frac{1}{2} n$

$7 - 3 < 7 \le \frac{1}{2} n^2$

$\frac{1}{2} n(n-1) \ge \frac{1}{4} n(n-1)$

$\underbrace{\frac{1}{4}}_{c_1} n^2 - \frac{n}{4} \le \underbrace{\frac{1}{2} n(n-1)} \le \underbrace{\frac{1}{2}}_{c_2} n^2$

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$f(n) \in O(g(n))$ and $g(n) \in O(h(n))$

$\Rightarrow f(n) \in O(h(n))$

$f(n) = O(g(n))$

$g(n) = \Omega(f(n))$

$n \in O(n^2)$

$O(n^3) = \{n^3, n^2, n, c\}$

$n \in O(n^2)$

$n^2 \in O(n^3)$

$\Rightarrow n \in O(n^3)$

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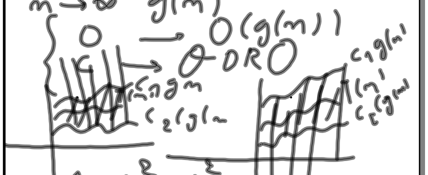
$\lim_{n \to \infty} \frac{T(n)}{g(n)}$

$0 \to O(g(n))$

$c \to \Theta$ OR O



$\frac{1}{2} n^2 = \Theta(n^2) = O(n^2) = \Omega(n^2)$

$\infty = \Omega$

$\frac{\frac{1}{2} n(n+1)}{n^2} = \frac{\frac{1}{2} n^2 + \frac{1}{2} n}{n^2}$

$\lim_{n \to \infty} \Theta(n^2) = \frac{\frac{1}{2} n^2}{n^2} = \frac{1}{2}$

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$1 < \log n < n < n \log n < n^2 < n^3 < n^4 \dots < 2^n < n!$

$\frac{n^2}{n^3} = \frac{1}{n} = 0$

Question

$n(n+1)/2 \in \begin{cases} O(n^3) & \text{True} \\ O(n^2) & \text{True} \\ \Theta(n^3) & \text{False} \\ \Omega(n) & \text{True} \end{cases}$

$\lim_{n \to \infty} \frac{\frac{n(n+1)}{2}}{n^3} = \lim_{n \to \infty} \frac{n^2}{n^3} = 0$

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```
for i:=0 to n-2
  for j:=i+1 to n-1
    if(A[i]=A[j]) return false
    end
  end
end
```

$$CT = \sum_{i=0}^{n-2}\left(\sum_{j=i+1}^{n-1} 1\right)$$

$$\sum_{j=i+1}^{n-1} 1 = (n-1)-(i+1)+1 = (n-i-1)$$

$$CT = \sum_{i=0}^{n-2}(n-i-1)$$

$$= \sum_{i=0}^{n-2} n - \sum_{i=0}^{n-2} i - \sum_{i=0}^{n-2} 1$$

$$(n-1)\,n - \frac{(n-1)(n-2)}{2} - (n-1)$$

$$\sum_{i=1}^{n} i = 1+2+3+\dots+n = \frac{n(n+1)}{2}$$

$$\sum_{i=0}^{n-2} i = 0+1+2+\dots+(n-2) = \frac{(n-1)(n-2)}{2}$$

$$CT = \Theta(n^2)$$

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```
for i:=0 to n-1
  for j:=0 to n-1
    for k:=0 to n-1
      operation
    end
  end
end
```

$$\sum_{i=0}^{n-1}\sum_{j=0}^{n-1}\sum_{k=0}^{n-1} 1 = \sum_{i=0}^{n-1}\sum_{j=0}^{n-1} n$$

$$= \sum_{i=0}^{n-1} n^2 = n^3$$

$$CT = O, \Theta(n^3).$$

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