function

$f : S \longrightarrow T$

$x \longrightarrow f(x)$

$S = Dom(f)$

$T = Img(f) = CoDomain(f)$

$T = \{ y = f(x) : x \in Dom(f) \}$

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Example:

$f(x) = \sin(x)$

$S = Dom(f) = \mathbb{R}$

$T = CoDomain = [-1, 1]$

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A function $f : S \longrightarrow T$ is called <u>one-to-one</u>

- if $x_1, x_2 \in S$ and $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$

$\Updownarrow$

if $x_1, x_2 \in S$ and $f(x_1) = f(x_2)$ then $x_1 = x_2$

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Given $f : S \longrightarrow T$

f maps <u>onto</u> $B \subseteq T$ iff:

for each $y \in T$, there is <u>at least</u> one $x \in S$

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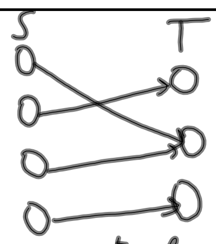

onto function

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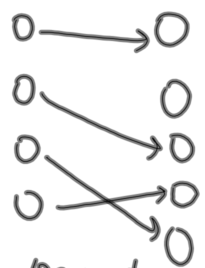

one-to one function

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A function $f : S \longrightarrow T$ is called <u>one to one correspondance</u> iff:
for each $y \in T$, there is exactly one $x \in S$

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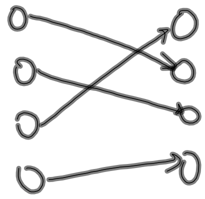

one-to-one correspondance
$\Longleftrightarrow$ onto and one to one

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Exaple:
$f : \mathbb{R} \longrightarrow \mathbb{R}$
$y \longrightarrow f(x) = 3x - 5$
f: one-to-one correspondance.
↳ 1. one-to one function
$x_1, x_2 \in \mathbb{R}$:
$f(x_1) = f(x_2) \Longrightarrow x_1 = x_2$

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$\cancel{3} x_1 - \cancel{5} = \cancel{3} x_2 - 5$
$\Downarrow$
$x_1 = x_2$
2- $y = 3x - 5$
$x = \frac{y+5}{3}$ (onto function)
Then: f is one to one correspondance

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$f : S \longrightarrow T$
$x \longrightarrow f(x)$
Inverse function
$f^{-1} : T \longrightarrow S$
$y \longrightarrow f^{-1}(y)$

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Composition function
$f : S \longrightarrow T$
$g : T \longrightarrow U$
$g \circ f : S \longrightarrow U$
$y \longrightarrow g(f(x))$

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Relations

$R = \{(x,y) : x \leq y\}$

$\leq$

Reflexive: $(x,x) \in R \ \forall x \in \mathbb{R}$

Antisymmetric

$(x,y) \in R$ and $(y,x) \in R$

$\Rightarrow x = y$

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transitive

$(x,y) \in R$ and $(y,z) \in R$

$\Rightarrow (x,z) \in R$

$\leq$

$R: \forall x \in \mathbb{R}: x \leq x$

$AS: \forall x,y \in \mathbb{R}: \left.\begin{matrix} x \leq y \\ \wedge \\ y \leq x \end{matrix}\right\} \Rightarrow x = y$

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$T: \forall x,y,z \in \mathbb{R}:$

$\left.\begin{matrix} x \leq y \\ \wedge \\ y \leq z \end{matrix}\right\} \Rightarrow x \leq z$

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$<$:

Anti-reflexive

$(x,x) \notin R$

$\left.\begin{matrix} x < y \\ y < x \end{matrix}\right\}$ Impossible.

Transitive: $x < y < z$

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symmetric

$(x,y) \in R \Rightarrow (y,x) \in R$

Exaple:

$m = n \bmod p$

mod

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$m \equiv n \bmod p$

mod $R = \{(m,n), .$

$R = \{(m,n) : m = n \bmod p\}$

Reflexive:

$m = m \bmod p$

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Symmetric

$m \equiv n \mod p$

$\Rightarrow n \equiv m \mod p$

$\left.\begin{array}{l} 4 \equiv 19 \mod 5 \\ 19 \equiv 4 \mod 5 \end{array}\right\}$ not symmetric

$\left.\begin{array}{l} m \equiv n \mod p \\ n \equiv r \mod p \end{array}\right\} m \equiv r \mod p$

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$m \equiv n \mod p \quad n \equiv r \mod p$

$4 \equiv 19 \mod 5 \quad 19 \equiv \_\_ \ 5$

Not transitive.

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A relation ~ is said to be equivalent if:

~ is reflexive

~ is symmetric

~ is transitive

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Induction

Let $p(m), p(m+1), \ldots p(n)$ be a finite sequence of propositions.

$\left\{\begin{array}{l} \text{if } p(m) \text{ is true} \\ \text{and: } p(k) \text{ is true } m \leq k \leq n \\ \text{we prove } P(k+1) \text{ is true.} \end{array}\right.$

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Exaple: $(S_n) = \sum_{i=1}^{n} i$

$1 + 2 + \cdots + n = \frac{n(n+1)}{2}$

Basic: $n = 1$

$1 = \frac{1(2)}{2} = 1$

Hypothesis

$1 + 2 + \ldots + k = \frac{k(k+1)}{2}$

$1 \leq k \leq n$

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$1 + 2 + \cdots + k + (k+1) \stackrel{?}{=} \frac{(k+1)(k+2)}{2}$

$\underbrace{1 + 2 + \ldots + k}_{\frac{k(k+1)}{2}} + (k+1)$

$\frac{k(k+1)}{2} + (k+1)$

$\frac{k(k+1) + 2(k+1)}{2} = \frac{(k+1)(k+2)}{2}$

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$\forall n \in \mathbb{N}$

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

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