KING SAUD UNIVERSITY
College of Science
Department of Mathematics

M-106

First Semester (1430/1431)

Correction Final-Exam

Name:	Number:
Name of Teacher:	Group No:

Max Marks: 50 Time: **Three hours**

Multiple Choice(1-20)	
Question # 21	
Question # 22	
Question # 23	
Question # 24	
Question # 25	
Question # 26	
Total	

Multiple Choice

Q.No:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
{a,b,c,d}	a	c	b	b	d	a	a	b	a	b	c	c	c	c	a	c	b	b	a	b

Q.No:1 If $\sum_{k=1}^{4}(k+a)=14$, then the value of a is equal to:

(a) 1 (b) 4 (c) -4 (d) -1

Q.No:2 The average value of the function $f(x)=\sin x\cos x$ on $[0,\frac{\pi}{4}]$ is equal to:

(a) $-\frac{1}{\pi}$ (b) $\frac{1}{4}$ (c) $\frac{1}{\pi}$ (d) $-\frac{1}{4}$

Q.No:3 If $F(x)=\int_{2x}^{x^2}\sin(t^3)dt$, then $F'(x)$ is equal to:

(a) $2x\sin(x^6)-\sin(8x^3)$ (b) $2x\sin(x^6)-2\sin(8x^3)$ (c) $2x\sin(x^6)-2\sin(6x^3)$
(d) $2x\sin(x^6)+2\sin(8x^3)$

Q.No:4 $\lim_{x\to 0}\frac{\int_0^x e^{t^2}dt}{x}$ is equal to:

(a) ∞ (b) 1 (c) 0 (d) -1

Q.No:5 To evaluate the integral $\int\frac{1}{\sqrt{x}+\sqrt[3]{x}}dx$, we use the substitution:

(a) $u=\sqrt{x}$ (b) $u=\sqrt[3]{x}$ (c) $u=\sqrt[4]{x}$ (d) $u=\sqrt[6]{x}$

Q.No:6 The substitution $u=\tan\left(\frac{x}{2}\right)$ transforms the integral $\int\frac{1}{1+\sin x+\cos x}dx$, into

(a) $\int\frac{1}{1+u}du$ (b) $\frac{1}{2}\int\frac{1}{(1+u)}du$ (c) $\int\frac{1}{u^2+u+1}du$ (d) $\int\frac{2}{(1+u)}du$

Q.No:7 If $f(x)=(x)^{x+1}$ then $f'(x)$ is equal to:

(a) $\left(1+\frac{1}{x}+\ln x\right)x^{x+1}$ (b) $\left(\ln x+\frac{1}{x}\right)x^{x+1}$ (c) $(1+\ln x)x^{x+1}$ (d) $\left(1+\frac{1}{x}+\ln x\right)x^{x}$

Q.No:8 The partial fraction decomposition of $\frac{x^3+1}{x^2(x-1)}$ takes the form:

(a) $\frac{A}{x^2}+\frac{B}{x}+\frac{C}{x-1}$ (b) $1+\frac{A}{x^2}+\frac{B}{x}+\frac{C}{x-1}$ (c) $1+\frac{A}{x^2}+\frac{B}{x-1}$ (d) $\frac{A}{x^2}+\frac{B}{x-1}$

Q.No:9 To evaluate the integral $\int\frac{\sqrt{x^2+4}}{x}dx$, we use the substitution

(a) $x=2\tan u$ (b) $x=\tan u$ (c) $x=2\sec u$ (d) $x=2\sin u$

Q.No:10 The integral $\int\cos^3(x)(\sin x)^{\frac{-1}{2}}dx$ is equal to:

(a) $2\cos^{\frac{1}{2}}(x)-\frac{2}{5}\cos^{\frac{5}{2}}(x)+c,$ (b) $2\sin^{\frac{1}{2}}(x)-\frac{2}{5}\sin^{\frac{5}{2}}(x)+c,$ (c) $2\sin^{\frac{1}{2}}(x)+\frac{2}{5}\sin^{\frac{5}{2}}(x)+c$

(d) $\sin^{\frac{1}{2}}(x)-\frac{2}{5}\sin^{\frac{5}{2}}(x)+c$

Q.No:11 The integral $\int \frac{1}{\sqrt{8-2x-x^2}}dx,$ with suitable substitution is equal to:

(a) $\int \frac{1}{\sqrt{u^2-7}}du$ (b) $\int \frac{1}{\sqrt{u^2-9}}du$ (c) $\int \frac{1}{\sqrt{9-u^2}}du$ (d) $\int \frac{1}{\sqrt{7-u^2}}du$

Q.No:12 The improper integral $\int_2^{\infty} \frac{1}{(x-1)^2}dx,$

(a) converges to -1 (b) converges to 0 (c) converges to 1 (d) diverges

Q.No:13 The area of the region bounded by the graphs of $y=\frac{2}{x}, \; x=1, \; x=3 \; and \; y=0$ is equal to:

(a) $\ln 3$ (b) $2\ln(3)-2$ (c) $2\ln(3)$ (d) $\ln(3)-2$

Q.No:14 The arc length of the graph of the curve $y=\cosh x, \quad 0\le x\le 4$ is equal to:

(a) $\sinh(4)-1$ (b) $\cosh(4)-1$ (c) $\sinh(4)$ (d) $\cosh(4)$

Q.No:15 The surface area generated by revolving the curve of the function $f(x)=\sqrt{4-x^2}, \; -2\le x\le 2$ around the **x-axis** is equal to:

(a) 16π (b) 4π (c) 8π (d) 6π

Q.No:16 If $(x,y)-coordinates$ of a point are $(0,-2)$ then its $(r,\theta)-coordinates$ are:

(a) $(2,\pi)$ (b) $(2,-\pi)$ (c) $\left(2,\frac{3\pi}{2}\right)$ (d) $\left(2,\frac{-3\pi}{2}\right)$

Q.No:17 The length of the curve $r=2\cos\theta, \; 0\le\theta\le\frac{\pi}{2}$ is equal to:

(a) $\frac{\pi}{2}$ (b) π (c) 2π (d) $\frac{2\pi}{3}$

Q.No:18 The polar equation that has the same graph as the Cartesian equation $x^2+y^2-x=2\sqrt{x^2+y^2}$ is :

(a) $r=2+\sin\theta$ (b) $r=2+\cos\theta$ (c) $r=2-\cos\theta$ (d) $r=2-\sin\theta$

Q.No:19 The polar equation $r=2+2\sin\theta$ represents:

(a) Cardioid (b) Circle (c) Ellipse (d) Straight line

Q.No:20 The slope of the tangent line to the curve $x=4t+1, y=t^2-2$ at $t=1$ is equal to:

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) $-\frac{1}{4}$

<u>**Full Questions**</u>

Question No: 21 Approximate $\int_0^{\sqrt{\pi}} \sin\left(4x^2\right)\, dx$, using **Simpson's rule** with n=4. **[6]**

Solution: Let $f(x) = \sin\left(4x^2\right)$.

$$x_0 = 0,\ \ x_1 = \frac{\sqrt{\pi}}{4},\ \ x_2 = \frac{\sqrt{\pi}}{2},\ \ x_3 = \frac{3\sqrt{\pi}}{4}\ \ and\ \ x_4 = \sqrt{\pi}$$

(1)

$$\int_0^{\sqrt{\pi}} \sin\left(4x^2\right)\, dx \approx \frac{\sqrt{\pi}}{3\times 4}\{f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)\}$$

(2)

$$\approx \frac{\sqrt{\pi}}{12}\{(0) + 4\times(0.70711) + 2\times(0) + 4\times(0.70711) + (0)\}$$

$$\approx \frac{\sqrt{\pi}}{12}\{(0) + 2.8284 + 2\times(0) + 2.8284 + (0)\}$$

(2)

$$\approx \frac{\sqrt{\pi}}{12}\{5.6568\}$$

$$\approx 0.83553$$

(1)

Question No: 22 Evaluate the integral $\int \sin^{-1}(x)\, dx$. **[4]**

Solution: By Integrations by parts (with $u = \sin^{-1} x\ and\ dv = 1$) we obtain:

$$\int \sin^{-1}(x)\, dx = x\sin^{-1}(x) - \int \frac{x}{\sqrt{1-x^2}}dx$$ (2)

$$= x\sin^{-1}(x) + \sqrt{1-x^2} + c$$ (2)

Question No: 23 Evaluate the integral $\int \frac{2x+3}{x^2+2x+3}dx$ **[5]**

Solution:

$$\int \frac{2x+3}{x^2+2x+3}dx = \int \frac{2x+2}{x^2+2x+3}dx + \int \frac{1}{x^2+2x+3}dx \qquad (2)$$

$$= \int \frac{2x+2}{x^2+2x+3}dx + \int \frac{1}{(x+1)^2+2}dx \qquad (1)$$

$$= \ln\left|x^2+2x+3\right| + \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right) + c \qquad (2$$

Question No: 24 Determine whether the integral $\int_0^{\infty} \frac{e^x}{1+e^{2x}}dx$ converges or diverges **[5]**

Solution:

$$\int_0^{\infty} \frac{e^x}{1+e^{2x}}dx = \lim_{t\to\infty}\int_0^{t} \frac{e^x}{1+e^{2x}}dx \qquad (1)$$

Let $u = e^x$, then

$$\int_0^{t} \frac{e^x}{1+e^{2x}}dx = \int_1^{e^t} \frac{1}{1+u^2}du = \tan^{-1}\left(e^t\right) - \tan^{-1}(1) = \tan^{-1}\left(e^t\right) - \frac{\pi}{4}. \qquad (2)$$

Hence

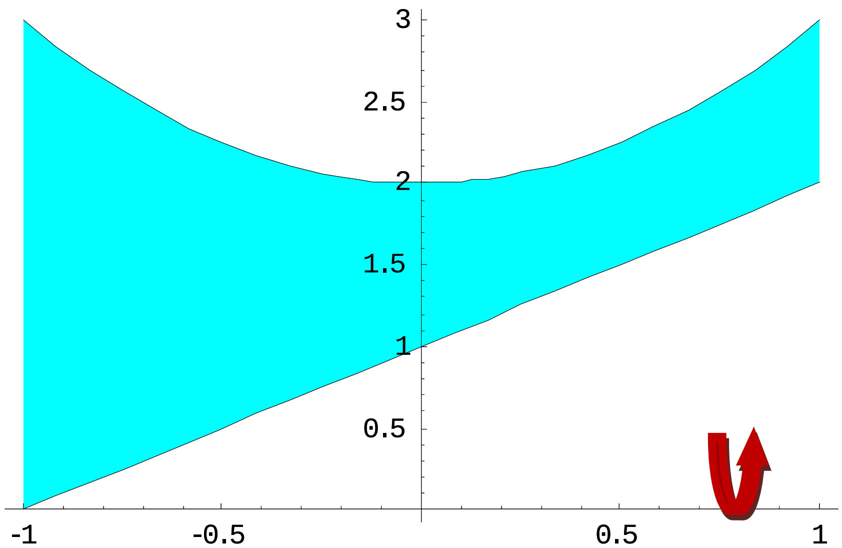
$$\int_0^{\infty} \frac{e^x}{1+e^{2x}}dx = \lim_{t\to\infty}\int_0^{t} \frac{e^x}{1+e^{2x}}dx = \lim_{t\to\infty}\left[\tan^{-1}\left(e^t\right) - \frac{\pi}{4}\right] = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}.$$

That is

$$\int_0^{\infty} \frac{e^x}{1+e^{2x}}dx \text{ converges and } \int_0^{\infty} \frac{e^x}{1+e^{2x}}dx = \frac{\pi}{4}. \qquad (2)$$

Question No: 25 Let **R** be the region bounded by the graphs of the functions $y = x^2 + 2$, $y = x + 1$, $x = -1$ and $x = 1$: Sketch the region **R** and **set up** an integral to find the **volume** of the solid generated by revolving the region **R** around the **x-axis. (Use Washer Method)** **[5]**

Solution:



(2)

$$V = \pi \int_{-1}^{1} \left[\left(x^2 + 2\right)^2 - \left(x + 1\right)^2 \right] dx$$ (3)

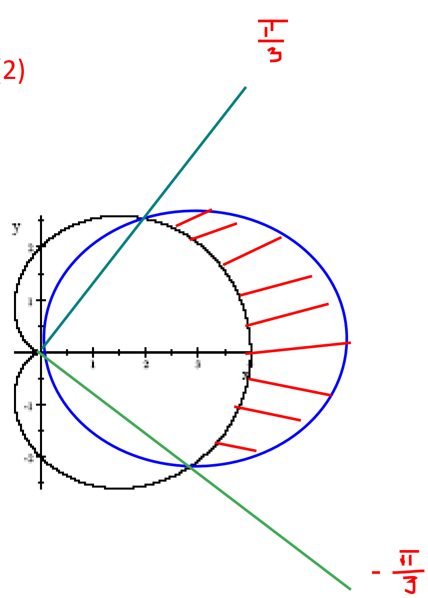
Question No: 26 Let **R** be the region which is **outside** the graph of $r = 2 + 2\cos\theta$ and **inside** the graph of the polar equation $r = 6\cos\theta$:

Sketch the region R and set up an integral that can be used to find the area of the region R. [5]

Solution

$$If\ r = 2 + 2\cos\theta = 6\cos\theta \Rightarrow \theta = \pm\frac{\pi}{3}$$ (1)

$$A = \frac{1}{2} \int_{\frac{-\pi}{3}}^{\frac{\pi}{3}} \left[\left(6\cos\theta\right)^2 - \left(2 + 2\cos\theta\right)^2 \right] d\theta$$ (2)



(2)