Answer the following questions:

Q1: [3+6]

a) A Markov chain $X_0, X_1, X_2, \ldots$ has the transition probability matrix

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \left\| \begin{array}{ccc} 0.6 & 0.3 & 0.1 \\ 0.3 & 0.3 & 0.4 \\ 0.4 & 0.1 & 0.5 \end{array} \right\| \end{array}$$

If the process starts in state $X_0 = 1$, determine the probability $\Pr\{X_0 = 1, X_1 = 0, X_2 = 2\}$

b) Consider a spare parts inventory model in which either 0, 1, or 2 repair parts are demanded in any period, with $\Pr\{\xi_n = 0\} = 0.4$, $\Pr\{\xi_n = 1\} = 0.3$, $\Pr\{\xi_n = 2\} = 0.3$ and suppose $s=0$ and $S=3$ Determine the transition probability matrix for the Markov chain $\{X_n\}$, where X_n is defined to be the quantity on hand at the end of period n.

Q2: [3+6]

a) A particle moves among the states 0, 1, 2 according to a Markov process whose transition probability matrix is

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \left\| \begin{array}{ccc} 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \end{array} \right\| \end{array}$$

Let X_n denote the position of the particle at the nth move. Calculate $\Pr\{X_n = 0 | X_0 = 0\}$ for $n = 0, 1, 2$

b) Determine whether the transition matrix

$$
P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \left\| \begin{array}{ccc} 1 & 0 & 0 \\ 0.1 & 0.6 & 0.3 \\ 0 & 0 & 1 \end{array} \right\| \end{array}
$$

represents an absorbing Markov chain or not, sketch Markov chain diagram and then find each of the following:

i) Starting in state 1, determine the probability that the Markov chain ends in state 0.

ii) Determine the mean time to absorption.

Q3: [7]

Let $\{X_n\}$, n=1,2,... be a Markov chain with transition probability matrix

$$
P = \begin{array}{c} \\ O \\ D \\ R \end{array} \begin{array}{c} \begin{array}{ccc} O & D & R \end{array} \\ \left\| \begin{array}{ccc} 0.9 & 0.1 & 0 \\ 0 & 0.9 & 0.1 \\ 1 & 0 & 0 \end{array} \right\| \end{array}
$$

Where X_n denote the condition of a machine of nth period with $X_n = 1$ means "operating", $X_n = 2$ means "deterioration" and $X_n = 3$ means "repairing". Find each of the following:

a) $\Pr\{X_1 = 1\}$, knowing that the process starts in state $X_0 = 1$

b) The limiting distribution

c) The long run rate of repairs per unit time.

1

9

Model Answer for Mid II 52/38/39

Q1

a) $pr\{X_0=1, X_1=0, X_2=2\}$

$= P_{i_0} P_{i_0 i_1} P_{i_1 i_2}$

(3)

$= P_1 P_{10} P_{02}$

$= 1\,(0.3)(0.1) = 0.03$

when the process starts in state $X_0 = 1$ i.e $P_1 = P_1(0) = 1$

b)

$$P = \begin{array}{c} \\ -1 \\ 0 \\ 1 \\ 2 \\ 3 \end{array}\begin{bmatrix} -1 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0.3 & 0.3 & 0.4 \\ 0 & 0 & 0.3 & 0.3 & 0.4 \\ 0 & 0 & 0.3 & 0.4 & 0 \\ 0.3 & 0.3 & 0.4 & 0.4 & 0 \\ 0 & 0.3 & 0.3 & 0.3 & 0.4 \\ 0 & 0 & 0.3 & 0.3 & 0.4 \end{bmatrix}$$

(6)

تجديد (−1, 0); تجديد (2); تجديد (3)

where $P_{ij} = \begin{cases} Pr(\xi_n = 3-j), & i \le 0 \rightarrow \text{تجديد} \\ Pr(\xi_n = i-j), & 0 < i \le 3 \rightarrow \text{عدم تجديد} \end{cases}$

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array}\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 0 \\ 0.1 & 0.6 & 0.3 \\ 0 & 0 & 1 \end{bmatrix}$$

$u = pr\{X_T = 0 \mid X_0 = 1\}$

$u = P_{10} + P_{11} u$

$u = 0.1 + 0.6 u$

$\Rightarrow u = 1/4$

$\therefore \boxed{u_{10} = \frac{1}{4}}$ prob. that M. chain ends in state 0

$v = E[T \mid X_0 = 1]$

$v = 1 + P_{11} v$

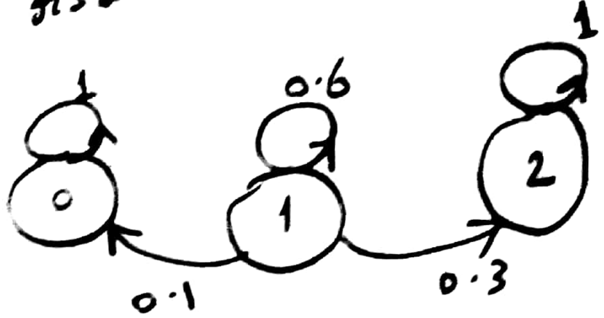
$v = 1 + 0.6 v$

$\Rightarrow v = \frac{1}{0.4}$

$\therefore v = 5/2$ (MTTA)

(3)

It's an absorbing Markov chain



(3)

Q2

a) $P^0_{00} = pr\{X_0 = 0 \mid X_0 = 0\} = 1$

$P^1_{00} = pr\{X_1 = 0 \mid X_0 = 0\} = 0$

9

$P^2_{00} = [0 \;\; 0.5 \;\; 0.5] \begin{bmatrix} 0 \\ 0.5 \\ 0.5 \end{bmatrix}$

$= 0.5$

(3)

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2

7

Q3

$$P = \begin{array}{c|ccc} & O & D & R \\ \hline O & 0.9 & 0.1 & 0 \\ D & 0 & 0.9 & 0.1 \\ R & 1 & 0 & 0 \end{array}$$

$\downarrow \pi_1 \quad \downarrow \pi_2 \quad \downarrow \pi_3$

a) $pr\{X_3 = 1\} = pr\{X_3 = 1 / X_0 = 1\}$

$= P^3_{11}$

$$P^2 = \begin{bmatrix} 0.9 & 0.1 & 0 \\ 0 & 0.9 & 0.1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.9 & 0.1 & 0 \\ 0 & 0.9 & 0.1 \\ 1 & 0 & 0 \end{bmatrix}$$

(2)

$$P^2 = \begin{bmatrix} 0.81 & 0.18 & 0.01 \\ 0.1 & 0.81 & 0.09 \\ 0.9 & 0.1 & 0 \end{bmatrix}$$

$$P^3_{11} = [0.81 \quad 0.18 \quad 0.01] \begin{bmatrix} 0.9 \\ 0 \\ 1 \end{bmatrix} = \boxed{0.739}$$

b) The limitting distn is $\pi = (\pi_1, \pi_2, \pi_3)$

$\pi_1 = 0.9\pi_1 + \pi_3 \Rightarrow 0.1\pi_1 = \pi_3 \Rightarrow \boxed{\pi_3 = 0.1\pi_1}$

(3) $\pi_2 = 0.1\pi_1 + 0.9\pi_2 \Rightarrow \boxed{\pi_1 = \pi_2}$

$\because \pi_1 + \pi_2 + \pi_3 = 1 \Rightarrow \pi_1 = 10/21, \pi_2 = 10/21, \pi_3 = 1/21$

$\therefore \pi = (10/21, 10/21, 1/21)$

c) The long run rate of repairs per unit time is

(2) $\pi_3 = \frac{1}{21}$ #