Continuous Probability Distributions

Chapter 7

Learning Objectives

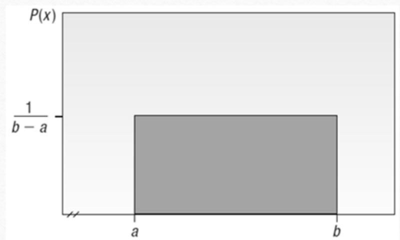
- List the characteristics of the uniform distribution.
- Compute probabilities using the uniform distribution
- List the characteristics of the normal probability distribution.
- Define and calculate z values.
- Determine the probability an observation is between two points on a normal probability distribution.
- Determine the probability an observation is above (or below) a point on normal probability distribution.

Uniform Probability Distribution

1. The distribution shape is rectangular.
2. Has a maximum value (b) and a minimum value (a).
3. the height of the distribution is constant or uniform for all values between *a* and *b*
4. The area of the rectangular is 1
5. The equation for the uniform probability distribution is:

$$P(x) = \frac{1}{b-a} \quad if\ a \leq x \leq b\ \ and\ 0\ elswhere$$

Uniform Probability Distribution



Uniform Probability Distribution

MEAN OF THE UNIFORM DISTRIBUTION	$\mu = \frac{a + b}{2}$	[7–1]
STANDARD DEVIATION OF THE UNIFORM DISTRIBUTION	$\sigma = \sqrt{\frac{(b - a)^2}{12}}$	[7–2]

Uniform Probability Distribution

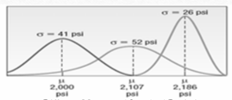
Southwest Arizona State University provides bus service to students while they are on campus. A bus arrives at the North Main Street and College Drive stop every 30 minutes between 6 A.M. and 11 P.M. during weekdays. Students arrive at the bus stop at random times. The time that a student waits is uniformly distributed from 0 to 30 minutes.

1. Draw a graph of this distribution.
2. Show that the area of this uniform distribution is 1.00.
3. How long will a student "typically" have to wait for a bus? In other words, what is the mean waiting time? What is the standard deviation of the waiting times?
4. What is the probability a student will wait more than 25 minutes?
5. What is the probability a student will wait between 10 and 20 minutes?

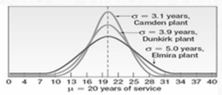
Normal Probability distribution

1. It is bell-shaped and has a single peak at the center of the distribution.
2. It is symmetrical about the mean
3. It is asymptotic: The curve gets closer and closer to the *X*-axis but never actually touches it.
4. The location of a normal distribution is determined by the mean, μ, the dispersion or spread of the distribution is determined by the standard deviation, σ .
5. The arithmetic mean, median, and mode are equal
6. The total area under the curve is 1.00; half the area under the normal curve is to the right of this center point and the other half to the left of it

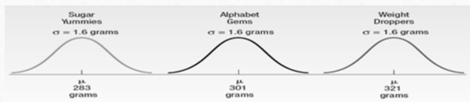
Normal Probability distribution



Different Means and Standard Deviations

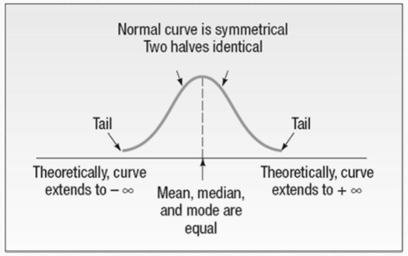


Equal Means and Different Standard Deviations



Different Means and Equal Standard Deviations

Normal Probability Distribution



The Empirical Rule

As part of its quality assurance program, the Autolite Battery Company conducts tests on battery life. For a particular D-cell alkaline battery, the mean life is 19 hours. The useful life of the battery follows a normal distribution with a standard deviation of 1.2 hours.

Answer the following questions.

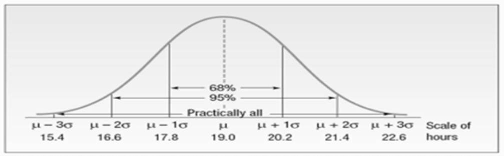
1. About 68 percent of the batteries failed between what two values?
2. About 95 percent of the batteries failed between what two values?
3. Virtually all of the batteries failed between what two values?

The Empirical Rule

We can use the results of the Empirical Rule to answer these questions.

1. About 68 percent of the batteries will fail between 17.8 and 20.2 hours by 19.0 ± 1(1.2) hours.
2. About 95 percent of the batteries will fail between 16.6 and 21.4 hours by 19.0 ± 2(1.2) hours.
3. Virtually all failed between 15.4 and 22.6 hours, found by 19.0 ± 3(1.2)

This information is summarized on the following chart.



The Standard Normal Probability Distribution

- The standard normal distribution is a normal distribution with a mean of 0 and a standard deviation of 1.
- It is also called the z distribution.
- A z-value is the signed distance between a selected value, designated *X*, and the population mean μ, divided by the population standard deviation, σ.
- The formula is:

$$Z = \frac{X - \mu}{\sigma}$$

Z-value

The weekly incomes of shift foremen in the glass industry follow the normal probability distribution with a mean of $1,000 and a standard deviation of $100.

What is the z-value for the income, let's call it X, of a foreman who earns $1,100 per week? For a foreman who earns $900 per week?

Example 1

- The mean weekly income of a shift foreman in the glass industry is normally distributed with a mean of $1,000 and a standard deviation of $100. What is the likelihood of selecting a foreman whose weekly income is between $1,000 and $1,100?
- We want $P(1000 < X < 1100)$
- First we have to get the z values.
- $P(0 < Z < 1) = ??$

Example 2

- The mean weekly income of a shift foreman in the glass industry is normally distributed with a mean of $1,000 and a standard deviation of $100. What is the likelihood of selecting a foreman whose weekly income is between $773 and $1,000?
- We want $P(773 < X < 1000)$
- First we have to get the z values.
- $P(-2.27 < Z < 0) = ??$

Example 3

- The mean weekly income of a shift foreman in the glass industry is normally distributed with a mean of $1,000 and a standard deviation of $100. What is the likelihood of selecting a foreman whose weekly income is between $1,100 and $1,200?
- We want $P(1100 < X < 1200)$
- First we have to get the z values.
- $P(1 < Z < 2) = ??$

Example 4

- The mean weekly income of a shift foreman in the glass industry is normally distributed with a mean of $1,000 and a standard deviation of $100. What is the likelihood of selecting a foreman whose weekly income is between $800 and $950?
- We want $P(800 < X < 1200)$
- First we have to get the z values.
- $P(-2 < Z < -0.5) = ??$

Example 5

- The mean weekly income of a shift foreman in the glass industry is normally distributed with a mean of $1,000 and a standard deviation of $100. What is the likelihood of selecting a foreman whose weekly income is between $865 and $1,157?
- We want $P(865 < X < 1157)$
- First we have to get the z values.
- $P(-1.35 < Z < 1.57) = ??$

Example 6

- The mean weekly income of a shift foreman in the glass industry is normally distributed with a mean of $1,000 and a standard deviation of $100. What is the likelihood of selecting a foreman whose weekly income is greater than $1220?
- We want $P(X > 1220)$
- First we have to get the z values.
- $P(Z > 2.2) = ??$

Example 7

- The mean weekly income of a shift foreman in the glass industry is normally distributed with a mean of $1,000 and a standard deviation of $100. What is the likelihood of selecting a foreman whose weekly income is less than $1050?
- We want $P(X < 1050)$
- First we have to get the z values.
- $P(Z < 0.5) = ??$

Example 8

- The mean weekly income of a shift foreman in the glass industry is normally distributed with a mean of $1,000 and a standard deviation of $100. What is the likelihood of selecting a foreman whose weekly income is grater than $788?
- We want $P(X > 788)$
- First we have to get the z values.
- $P(Z > -2.12) = ??$

Example 9

- The mean weekly income of a shift foreman in the glass industry is normally distributed with a mean of $1,000 and a standard deviation of $100. What is the likelihood of selecting a foreman whose weekly income is less than $940?
- We want $P(X < 940)$
- First we have to get the z values.
- $P(Z < -0.6) = ??$

Example 10

- The mean weekly income of a shift foreman in the glass industry is normally distributed with a mean of $1,000 and a standard deviation of $100. What is the minimum weekly income for the top 2.5%?
- We have $P(X > x) = 0.025$ and we want to know the value of x.
- First we have to get the z values.
- $P(Z > 1.96) = 0.025$
- Now, we got the z value, we know μ and σ. How can we get x??

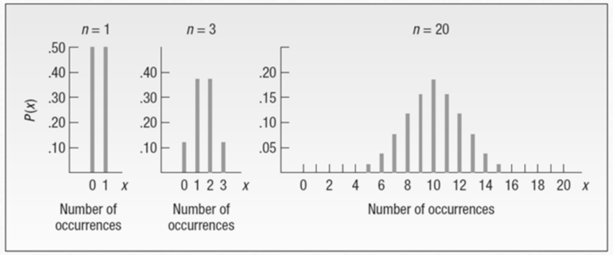
Example 11

- The mean weekly income of a shift foreman in the glass industry is normally distributed with a mean of $1,000 and a standard deviation of $100. What is the likelihood of selecting a foreman whose weekly income is between $865 and $1,157?
- We want $P(865 < X < 1157)$
- First we have to get the z values.
- $P(-1.35 < Z < 1.57) = ??$

The Normal Approximation to the Binomial

We can use the normal distribution (a continuous distribution) as a substitute for a binomial distribution (a discrete distribution) for large values of *n* because, as *n* increases, a binomial distribution gets closer and closer to a normal distribution. Depicts the change in the shape of a binomial distribution with $\pi = .50$ from an *n* of 1, to an *n* of 3, to an *n* of 20. Notice how the case where $n = 20$ approximates the shape of the normal distribution. That is, compare the case where $n = 20$ to the normal curve.

The Normal Approximation to the Binomial



The Normal Approximation to the Binomial

- When can we use the normal approximation to the binomial?

The normal probability distribution is a good approximation to the binomial probability distribution when $n\pi$ and $n(1-\pi)$ are both at least 5. However, before we apply the normal approximation, we must make sure that our distribution of interest is in fact a binomial distribution.

The Normal Approximation to the Binomial

- Example: Suppose the management of the Santoni Pizza Restaurant found that 70 percent of its new customers return for another meal. For a week in which 80 new (first-time) customers dined at Santoni's, what is the probability that 60 or more will return for another meal?

The Normal Approximation to the Binomial

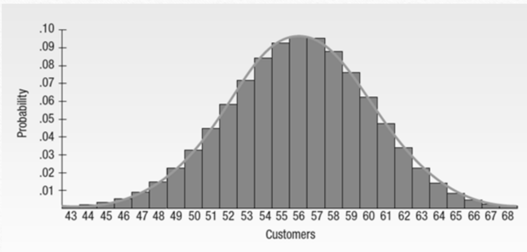
Number Returning	Probability	Number Returning	Probability
43	.001	56	.097
44	.002	57	.095
45	.003	58	.088
46	.006	59	.077
47	.009	60	.063
48	.015	61	.048
49	.023	62	.034
50	.033	63	.023
51	.045	64	.014
52	.059	65	.008
53	.072	66	.004
54	.084	67	.002
55	.093	68	.001

The Normal Approximation to the Binomial

We can find the probability of 60 or more returning by summing $.063 + .048 + \cdots + .001$, which is $.197$. However, a look at the plot on the following slide shows the similarity of this distribution to a normal distribution. All we need do is "smooth out" the discrete probabilities into a continuous distribution. Furthermore, working with a normal distribution will involve far fewer calculations than working with the binomial.

The trick is to let the discrete probability for 56 customers be represented by an area under the continuous curve between 55.5 and 56.5. Then let the probability for 57 customers be represented by an area between 56.5 and 57.5, and so on. This is just the opposite of rounding off the numbers to a whole number.

The Normal Approximation to the Binomial



The Normal Approximation to the Binomial

- Because we use the normal distribution to determine the binomial probability of 60 or more successes, we must subtract, in this case, .5 from 60. The value .5 is called the **continuity correction factor.** This small adjustment must be made because a continuous distribution (the normal distribution) is being used to approximate a discrete distribution (the binomial distribution). Subtracting, $60 - .5 = 59.5$.

> **CONTINUITY CORRECTION FACTOR** The value .5 subtracted or added, depending on the question, to a selected value when a discrete probability distribution is approximated by a continuous probability distribution.

The Normal Approximation to the Binomial

- How to Apply the Correction Factor?

Only four cases may arise. These cases are:

1. For the probability *at least* X occurs, use the area *above* $(X - .5)$.
2. For the probability that *more than* X occurs, use the area *above* $(X + .5)$.
3. For the probability that X *or fewer* occurs, use the area *below* $(X + .5)$.
4. For the probability that *fewer than* X occurs, use the area *below* $(X - .5)$.

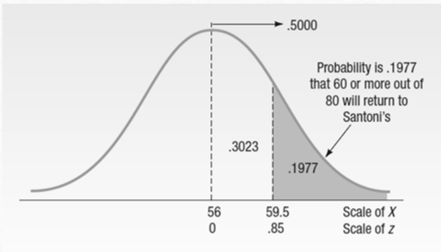
The Normal Approximation to the Binomial

To use the normal distribution to approximate the probability that 60 or more first-time Santoni customers out of 80 will return, follow the procedure shown below.

Step 1: Find the Z corresponding to an X of 59.5.

Step 2: Calculate the probability.

The Normal Approximation to the Binomial



The Normal Approximation to the Binomial

No doubt you will agree that using the normal approximation to the binomial is a more efficient method of estimating the probability of 60 or more first-time customers returning. The result compares favorably with that computed on page 243, using the binomial distribution. The probability using the binomial distribution is .197, whereas the probability using the normal approximation is .1977.