Chapter 31

Faraday's Law

CHAPTER OUTLINE

31.1 Faraday's Law of Induction

31.2 Motional emf

▪ Introduction

❖ We knew that the electric fields produced by stationary charges and the magnetic fields produced by moving charges.

❖ In this chapter explores the effects produced by magnetic fields that vary in time.

❖ Experiments conducted by Michael Faraday in England in 1831 and independently by Joseph Henry in the United States that same year **showed that an emf can be induced in a circuit by a changing magnetic field.**

Michael Faraday
British Physicist and Chemist
(1791–1867)

❖ The results of these experiments led to a very basic and important law known as *Faraday's law of induction*.

❖ **An emf (and therefore a current as well) can be induced in various processes that involve a change in a magnetic flux.**

31.1 Faraday's Law of Induction

- How an emf can be induced by a changing magnetic field.

- Consider a loop of wire connected to a sensitive ammeter.

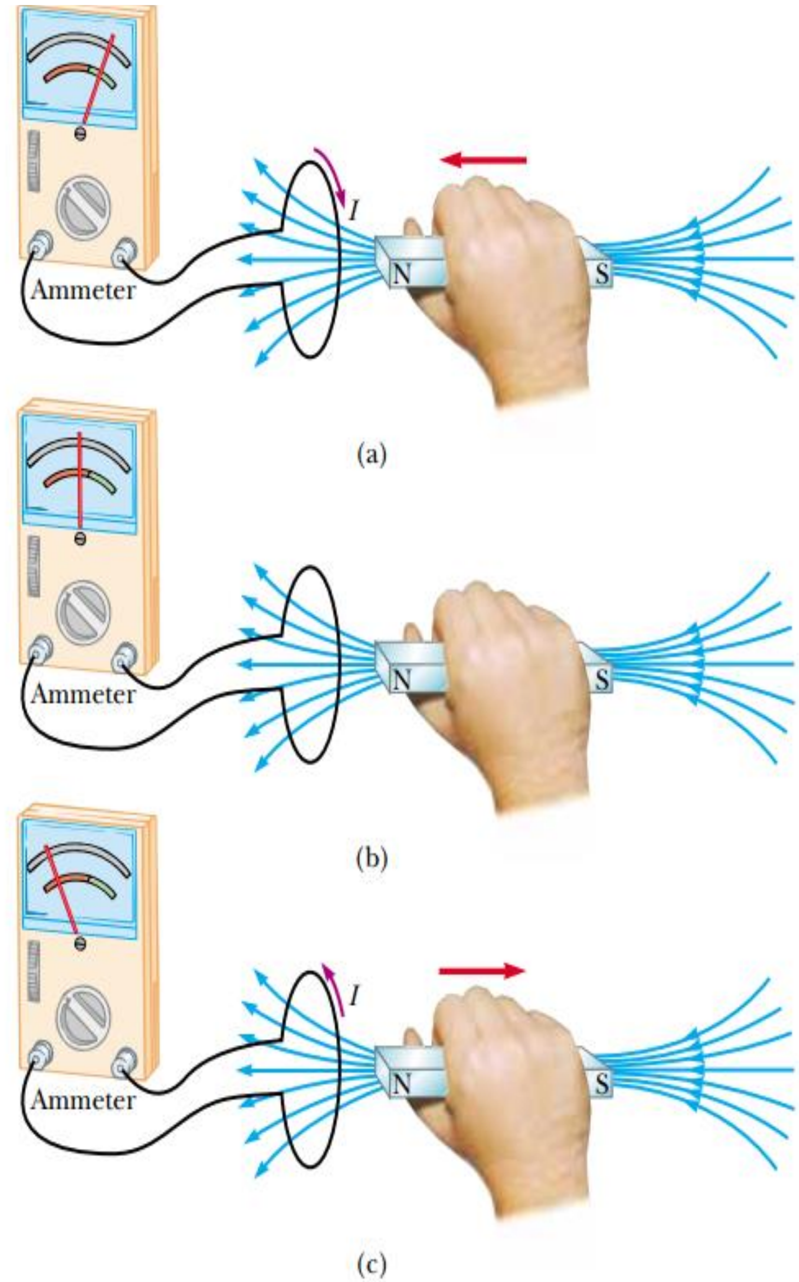
A. When a magnet is moved toward the loop, the galvanometer needle deflects in one direction, arbitrarily shown to the right.

B. When the magnet is brought to rest and held stationary relative to the loop , no deflection is observed.

C. When the magnet is moved away from the loop, the needle deflects in the opposite direction. Finally, if the magnet is held stationary and the loop is moved either toward or away from it, the needle deflects.

D. we conclude that the loop detects that the magnet is moving relative to it and we relate ***this detection to a change in magnetic field.*** Thus, it seems that a relationship exists between current and changing magnetic field.

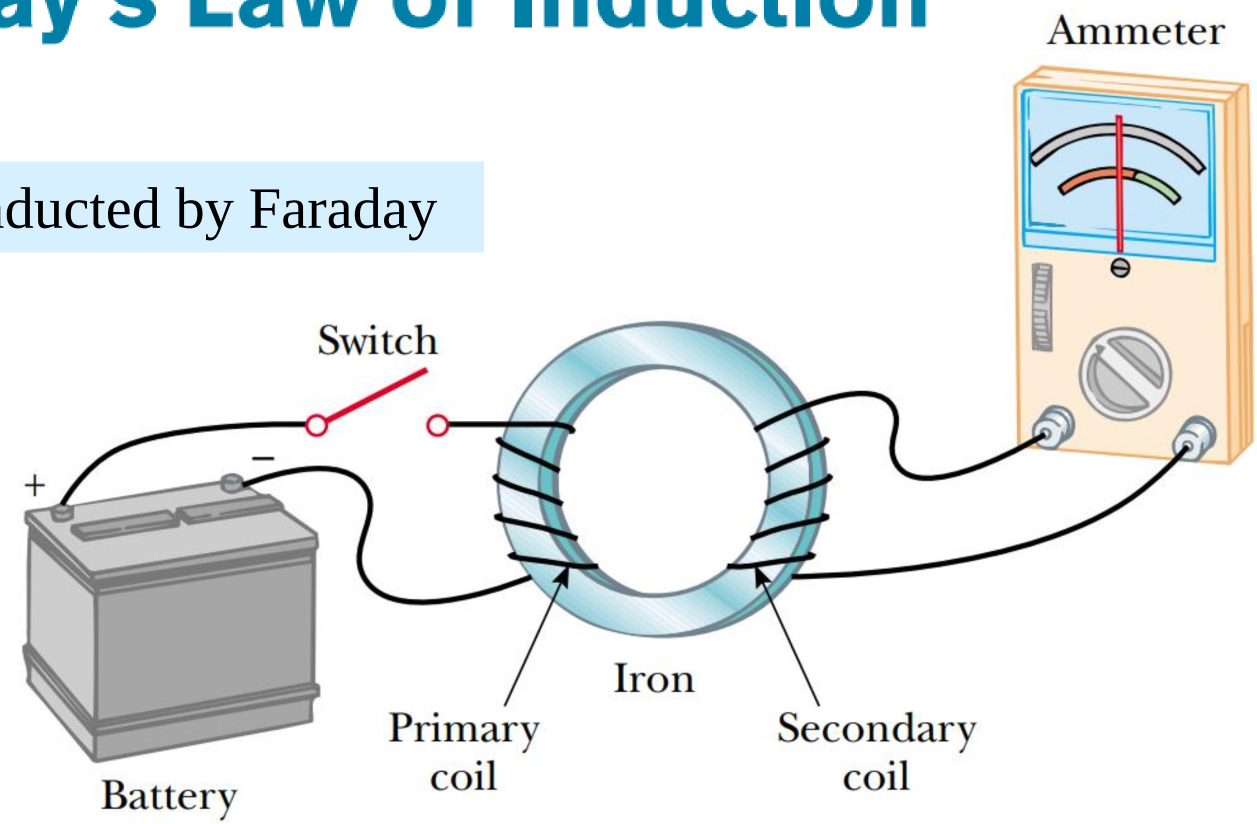
a current is set up even though no batteries are present in the circuit! We call such a current an *induced current* and say that it is produced by an *induced emf* .



An emf is induced in the circuit when the magnetic flux through the circuit changes with time.

31.1 Faraday's Law of Induction

- The experiment conducted by Faraday



A primary coil is connected to a switch and a battery. The coil is wrapped around an iron ring, and a current in the coil produces a magnetic field when the switch is closed. A secondary coil also is wrapped around the ring and is connected to a sensitive ammeter. No battery is present in the secondary circuit, and the secondary coil is not electrically connected to the primary coil. Any current detected in the secondary circuit must be induced by some external agent.

At the instant the switch is closed, the galvanometer needle deflects in one direction and then returns to zero. The current in the primary circuit produces a magnetic field that penetrates the secondary circuit which changes from zero to some value over some finite time, and this changing field induces a current in the secondary circuit.
When switch is opened, the needle deflects in the opposite direction and again returns to zero.

31.1 Faraday's Law of Induction

- Faraday concluded that **an electric current can be induced in a circuit (the secondary circuit in our setup) by a changing magnetic field.**

- The induced current exists for only a short time while the magnetic field through the secondary coil is changing, so that **an induced emf is produced in the secondary circuit by the changing magnetic field.**

- An emf is induced in the circuit when the magnetic flux through the circuit changes with time.

The emf induced in a circuit is directly proportional to the time rate of change of the magnetic flux through the circuit.

31.1 Faraday's Law of Induction

Faraday's law of induction

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

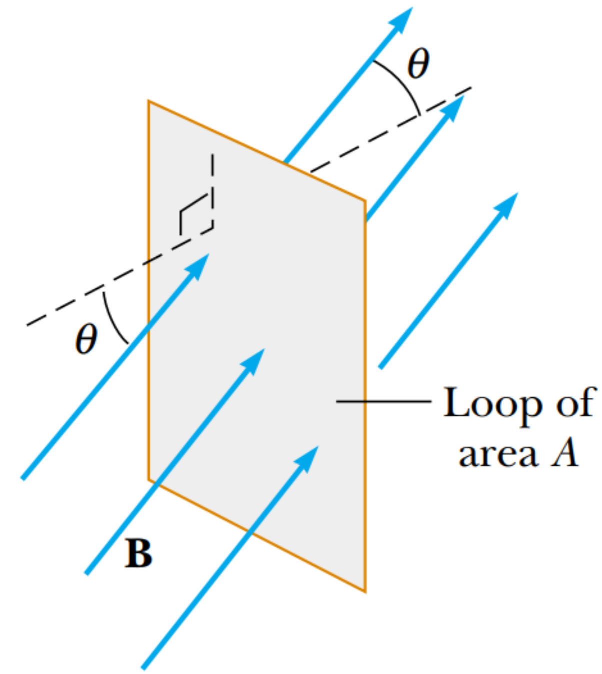
where $\Phi_B = \int \mathbf{B} \cdot d\mathbf{A}$ is the magnetic flux through the circuit.

If the circuit is a coil consisting of N loops all of the same area and if Φ_B is the magnetic flux through one loop, an emf is induced in every loop. emfs add;

$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$

31.1 Faraday's Law of Induction

A conducting loop that encloses an area A in the presence of a uniform magnetic field $\vec{B}$ The angle between $\vec{B}$ and the normal to the loop is θ



induced emf can be expressed as

$$\mathcal{E} = -\frac{d}{dt}(BA\cos\theta)$$

- The magnitude of **B** can change with time.
- The area enclosed by the loop can change with time.
- The angle θ between **B** and the normal to the loop can change with time.
- Any combination of the above can occur.

31.1 Faraday's Law of Induction

Quick Quiz 31.1 A circular loop of wire is held in a uniform magnetic field, with the plane of the loop perpendicular to the field lines. Which of the following will *not* cause a current to be induced in the loop? (a) crushing the loop; (b) rotating the loop about an axis perpendicular to the field lines; (c) keeping the orientation of the loop fixed and moving it along the field lines; (d) pulling the loop out of the field.

31.1 Faraday's Law of Induction

Quick Quiz 31.2 Figure 31.4 shows a graphical representation of the field magnitude versus time for a magnetic field that passes through a fixed loop and is oriented perpendicular to the plane of the loop. The magnitude of the magnetic field at any time is uniform over the area of the loop. Rank the magnitudes of the emf generated in the loop at the five instants indicated, from largest to smallest.

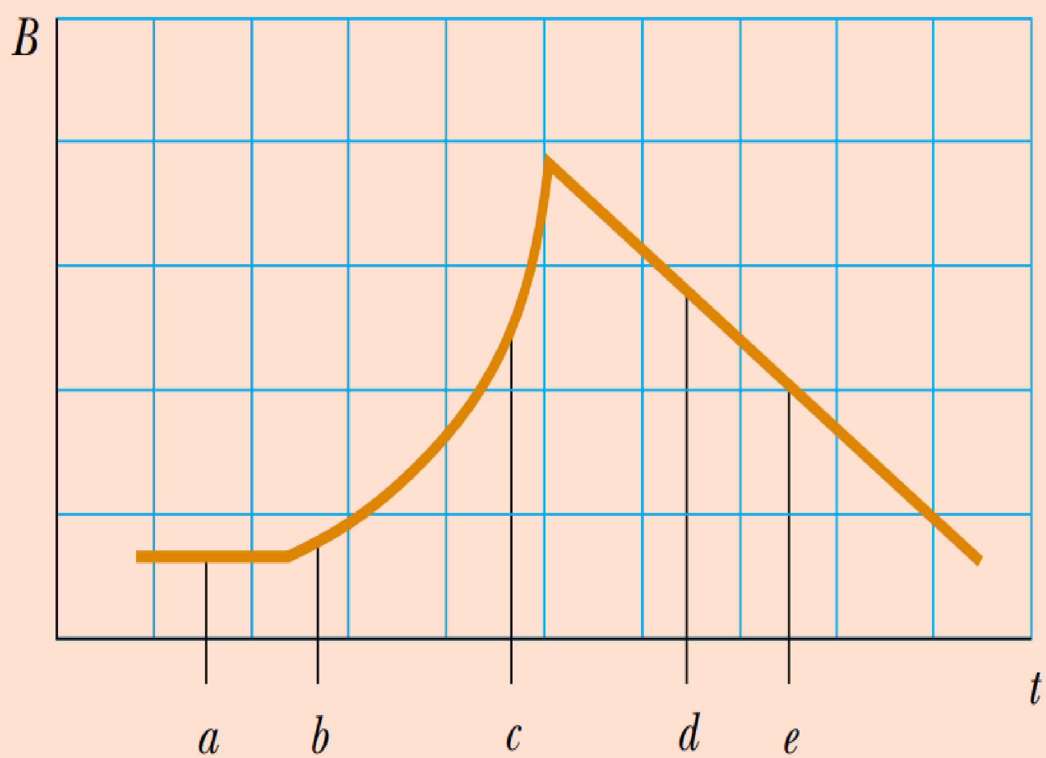


Figure 31.4 (Quick Quiz 31.2) The time behavior of a magnetic field through a loop.

31.1 Faraday's Law of Induction

Example 31.1 One Way to Induce an emf in a Coil

A coil consists of 200 turns of wire. Each turn is a square of side 18 cm, and a uniform magnetic field directed perpendicular to the plane of the coil is turned on. If the field changes linearly from 0 to 0.50 T in 0.80 s, what is the magnitude of the induced emf in the coil while the field is changing?

Solution The area of one turn of the coil is $(0.18\text{ m})^2 = 0.0324\text{ m}^2$. The magnetic flux through the coil at $t = 0$ is zero because $B = 0$ at that time. At $t = 0.80$ s, the magnetic flux through one turn is $\Phi_B = BA = (0.50\text{ T})(0.0324\text{ m}^2) = 0.0162\text{ T}\cdot\text{m}^2$. Therefore, the magnitude of the induced emf is, from Equation 31.2,

$$|\mathcal{E}| = N\frac{\Delta\Phi_B}{\Delta t} = 200\,\frac{(0.0162\text{ T}\cdot\text{m}^2 - 0)}{0.80\text{ s}}$$

$$= 4.1\text{ T}\cdot\text{m}^2/\text{s} = 4.1\text{ V}$$

You should be able to show that $1\text{ T}\cdot\text{m}^2/\text{s} = 1\text{ V}$.

What If? What if you were asked to find the magnitude of the induced current in the coil while the field is changing? Can you answer this question?

Answer If the ends of the coil are not connected to a circuit, the answer to this question is easy—the current is zero! (Charges will move within the wire of the coil, but they cannot move into or out of the ends of the coil.) In order for a steady current to exist, the ends of the coil must be connected to an external circuit. Let us assume that the coil is connected to a circuit and that the total resistance of the coil and the circuit is 2.0 Ω. Then, the current in the coil is

$$I = \frac{\mathcal{E}}{R} = \frac{4.1\text{ V}}{2.0\,\Omega} = 2.0\text{ A}$$

Section 31.1 Faraday's Law of Induction

Selected Solved Problems (Chapter # 31)

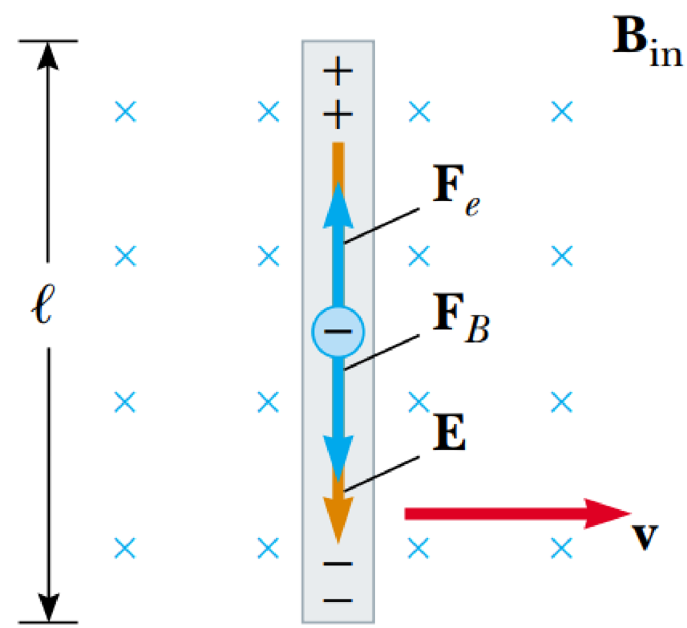
2. A flat loop of wire consisting of a single turn of cross-sectional area 8.00 cm^2 is perpendicular to a magnetic field that increases uniformly in magnitude from 0.500 T to 2.50 T in 1.00 s. What is the resulting induced current if the loop has a resistance of 2.00 Ω?

5. A strong electromagnet produces a uniform magnetic field of 1.60 T over a cross-sectional area of 0.200 m^2. We place a coil having 200 turns and a total resistance of 20.0 Ω around the electromagnet. We then smoothly reduce the current in the electromagnet until it reaches zero in 20.0 ms. What is the current induced in the coil?

31.2 Motional emf

❖ In previous section, an emf is induced in a stationary circuit placed in a magnetic field when the field changes with time.

❖ Here, **motional emf** is the emf induced in a conductor moving through a constant magnetic field.



- Straight conductor of length l is moving through a uniform magnetic field directed into the page.
- The conductor is moving in a direction perpendicular to the field with constant velocity under the influence of some external agent.
- Electrons in the conductor experience a magnetic force that is directed along the length l, perpendicular to both $\vec{V}$ and $\vec{B}$.
- Electrons move to the lower end of the conductor and accumulate there, leaving a net positive charge at the upper end.
- Electric field $\vec{E}$ is produced inside the conductor.
- Magnetic force is downward qVB on charges remaining in the conductor is balanced by the upward electric force $q\vec{E}$.

31.2 Motional emf

The condition for equilibrium requires that

$$qE = qvB \qquad \text{or} \qquad E = vB$$

$$\Delta V = E\ell$$

Thus, for the equilibrium condition,

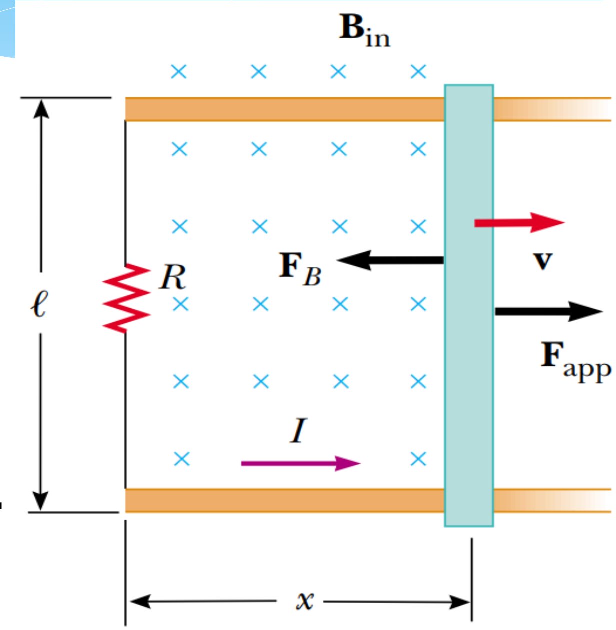
$$\Delta V = E\ell = B\ell v$$

Thus, **a potential difference is maintained between the ends of the conductor as long as the conductor continues to move through the uniform magnetic field.**

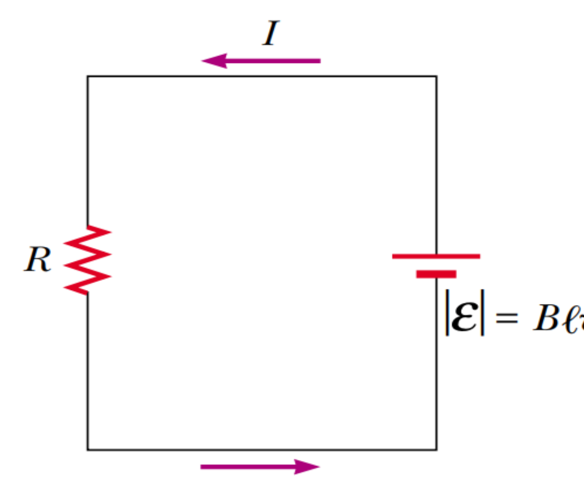
31.2 Motional emf

❑ How a changing magnetic flux causes an induced current in a closed circuit.

- Consider a circuit consisting of a conducting bar of length l sliding along two fixed parallel conducting rails
- A uniform and constant magnetic field $\vec{B}$ is applied perpendicular to the plane of the circuit.
- As the bar is pulled to the right with a velocity $\vec{V}$ under the influence of an applied force $\vec{F}_{app}$,
- Free charges in the bar experience a magnetic force directed along the length of the bar.
- This force sets up an induced current because the charges are free to move in the closed conducting path.
- In this case, the rate of change of magnetic flux through the loop and the corresponding induced motional emf across the moving bar are proportional to the change in area of the loop.
- If the bar is pulled to the right with a constant velocity, the work done by the applied force appears as internal energy in the resistor R.



(a)



(b)

31.2 Motional emf

- The area enclosed by the circuit at any instant is lx, where x is the position of the bar.

- Magnetic flux through that area is

$$\Phi_B = B\ell x$$

x changes with time at a rate $\frac{dx}{dt} = v$, we find that the induced motional emf is

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(B\ell x) = -B\ell\frac{dx}{dt}$$

Motional emf

$$\mathcal{E} = -B\ell v$$

- The resistance of the circuit is R, the magnitude of the induced current is

$$I = \frac{|\mathcal{E}|}{R} = \frac{B\ell v}{R}$$

31.2 Motional emf

- The applied force does work on the conducting bar, thereby moving charges through a magnetic field. Their movement through the field causes the charges to move along the bar with some average drift velocity, and hence a current is established.

- The bar moves with constant velocity, the applied force must be equal in magnitude and opposite in direction to the magnetic force.

❑ The power delivered by the applied force is

$$\mathcal{P} = F_{\text{app}}v = (I\ell B)v = \frac{B^2\ell^2v^2}{R} = \frac{\mathcal{E}^2}{R}$$

Quick Quiz 31.5 In Figure 31.10, a given applied force of magnitude F_{app} results in a constant speed v and a power input $\mathcal{P}$. Imagine that the force is increased so that the constant speed of the bar is doubled to $2v$. Under these conditions, the new force and the new power input are (a) $2F$ and $2\mathcal{P}$ (b) $4F$ and $2\mathcal{P}$ (c) $2F$ and $4\mathcal{P}$ (d) $4F$ and $4\mathcal{P}$.

Quick Quiz 31.6 You wish to move a rectangular loop of wire into a region of uniform magnetic field at a given speed so as to induce an emf in the loop. The plane of the loop remains perpendicular to the magnetic field lines. In which orientation should you hold the loop while you move it into the region of magnetic field in order to generate the largest emf? (a) with the long dimension of the loop parallel to the velocity vector (b) with the short dimension of the loop parallel to the velocity vector (c) either way—the emf is the same regardless of orientation.

Section 31.2 Motional emf

Selected Solved Problems (Chapter # 31)

13. A long solenoid has $n = 400$ turns per meter and carries a current given by $I = (30.0\text{ A})(1 - e^{-1.60t})$. Inside the solenoid and coaxial with it is a coil that has a radius of 6.00 cm and consists of a total of $N = 250$ turns of fine wire (Fig. P31.13). What emf is induced in the coil by the changing current?

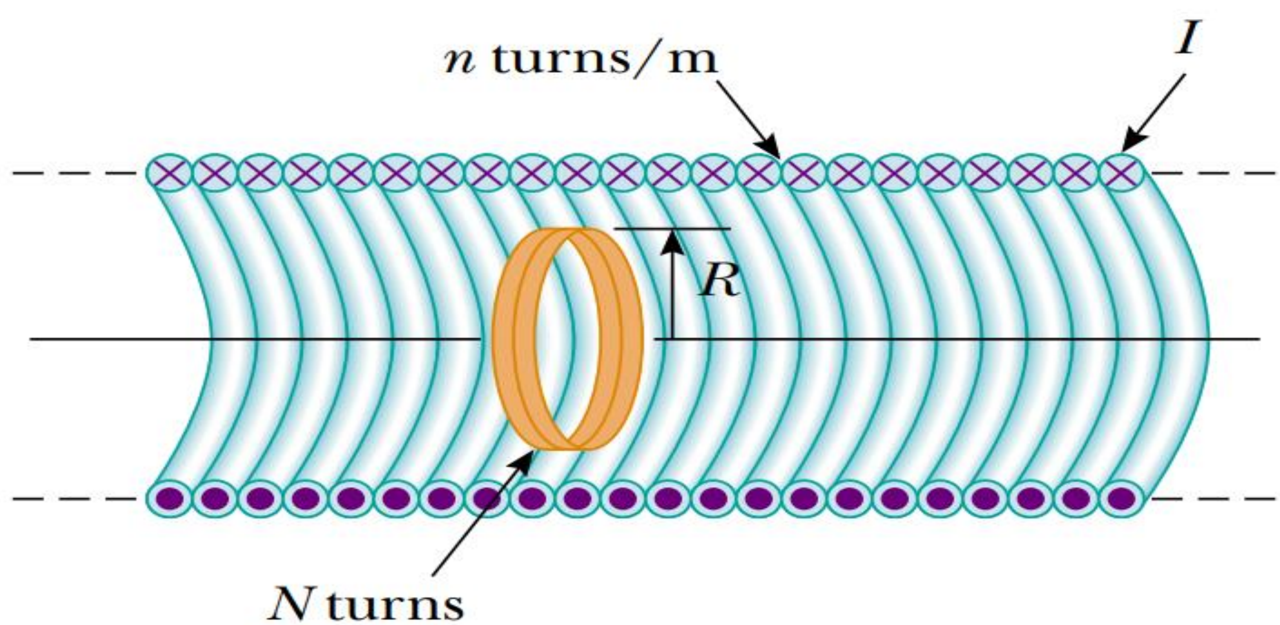


Figure P31.13

Section 31.2 Motional emf

Selected Solved Problems (Chapter # 31)

20. Consider the arrangement shown in Figure P31.20. Assume that $R = 6.00\ \Omega$, $\ell = 1.20$ m, and a uniform 2.50-T magnetic field is directed into the page. At what speed should the bar be moved to produce a current of 0.500 A in the resistor?

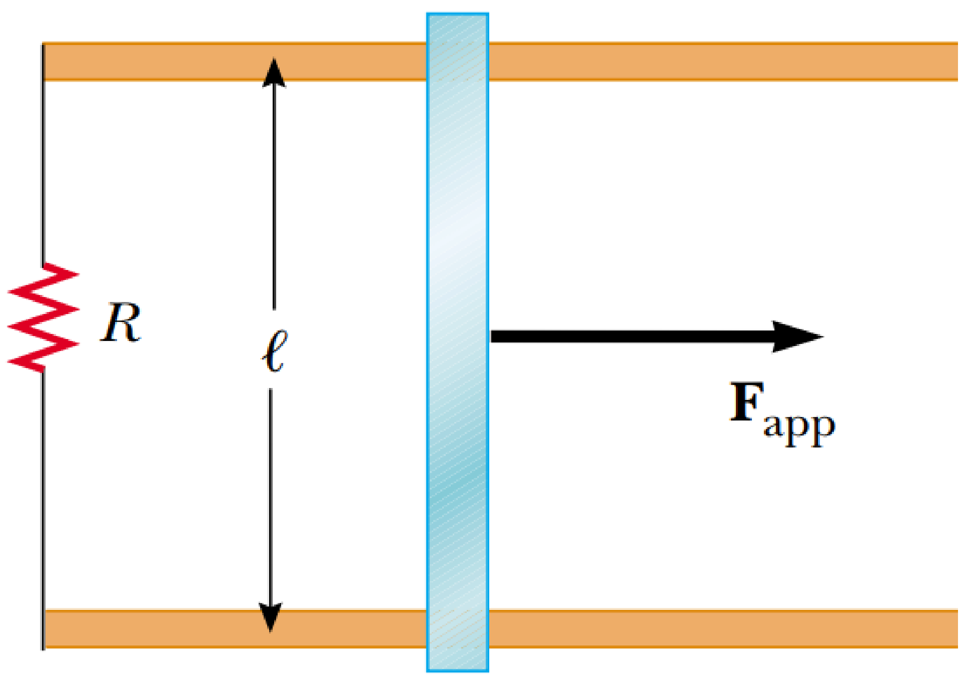


Figure P31.20 Problems 20, 21, and 22.

Chapter # 31 Problems

Selected Old exam questions

Q24. A straight conductor of a length 60cm is moving with a velocity of 2.5 m/s through a uniform magnetic field 1.5 T vertically into the page. The induced ε is:

24. يتحرك موصل معدني مستقيم طوله 40 cm وبسرعة 1.5 m/s وذلك خلال مجال مغناطيسي منتظم يتجه عمودياً على مستوى الورقة. إن قيمة القوة المحركة الحثية للملف ε هي:

A) -2.05 V B) -2.25V C) -7.13 V D) -1.26 V E) -5.10 V

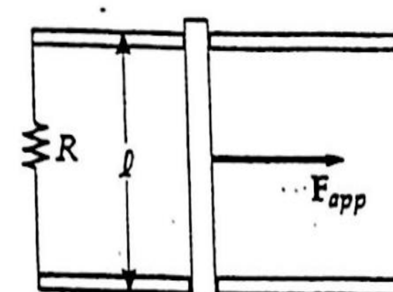
Q25. Based on the previous question 24, and if a resistance of 10 Ω is connected in series. Then the induced current will be:

25. استناداً إلى السؤال السابق رقم 24 ، وإذا ربطت مقاومة أومية Ω 10 على التوالي. فإن التيار الحثي الناتج هو:

A) 245 mA B) 125 mA C) 345 mA D)710 mA E)225 mA

س٢٥- يتحرك موصل طوله $1m$ علي موصلين أفقيين بدون إحتكاك في مجال مغناطيسي $4T$ عمودي على الحركة إلى داخل الورقة. إذا كانت مقدار المقاومة 16Ω فان مقدار القوة اللازمة لتحريك القضيب الى اليمين بسرعة $4\ m/s$ يساوي:

Q25- A bar of length 1 m moves on two horizontal frictionless rails as shown in the figure. If R= 16 Ω and a 4 T magnetic field is directed perpendicularly into the paper, the applied force required to move the bar to the right at a constant speed of 4 m/s equals to:



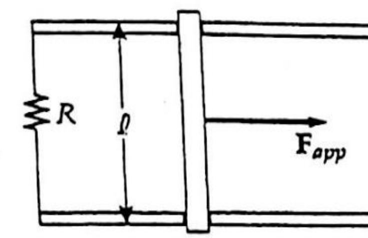
A) 2 B) 4 C) 6 D) 8

Chapter # 31 Problems

Selected Old exam questions

س23- يتحرك قضيب طوله $1m$ علي قضيبين أفقيين بدون إحتكاك في مجال مغناطيسي $3T$ عمودي على الحركة كما هو موضح بالشكل. إذا كانت مقدار المقاومة 6Ω فان مقدار القوّة اللازمة لتحريك القضيب الى اليمين بسرعة $10\ m/s$ يساوي:

Q23- A bar of length 1 *m* moves on two horizontal frictionless rails as shown in the figure. If R= 6 Ω and a 3 *T* magnetic field is directed perpendicularly into the paper, the applied force required to move the bar to the right at a constant speed of 10 *m/s* equals to:



A) 3 *N* B) 15 *N* C) 125 *N* D) 200 *N*

س24- ملف حلزوني طويل $(n = 1200\ turns/m)$ يمر به تيار $30\ A$ مقدار المجال المغناطيسي بمركز الملف يساوي:

Q24- A long solenoid (n = 1200 *turns/m*) has a current of a 30 *A* in its winding. The magnitude of the resulting magnetic field at the center point on the axis of the solenoid is:

A) 45.2 *mT* B) 36.2 *mT* C) 52 *μT* D) 0.60 *mT*

س25- من الممكن إنتاج تيار مستحث في ملف بواسطة:

Q25- Induced current can be produced in a coil by:

A) moving a magnet bar towards the coil B) applying a constant magnetic field in the coil
C) fixing the magnitude of the magnetic flux D) making the magnetic field equal zero

Chapter # 31 Problems

Selected Old exam questions

س26- قانون فاراداي في الحث (التحريض) والذي يربط بين القوة الدافعة الكهربائية $\mathscr{E}$ والتدفق المغناطيسي Φ والزمن t هو:

Q26- Faraday's law of induction relating the electromotive force $\mathscr{E}$, magnetic flux Φ, and time t is:

A) $\mathscr{E} = -dt/d\Phi$ B) $t = -d\mathscr{E}/d\Phi$ C) $\mathscr{E} = -d\Phi/dt$ D) $\Phi = -d\mathscr{E}/dt$

س27- إذا كان المجال المغناطيسي يساوي $15\ mT$ فما كثافة الطاقة المغناطيسية لوحدة الحجم؟

Q27- If the magnetic field is $15\ mT$, what is the magnetic energy density?

A) 179.1 B) 89.5 C) 11.9 D) 5.96

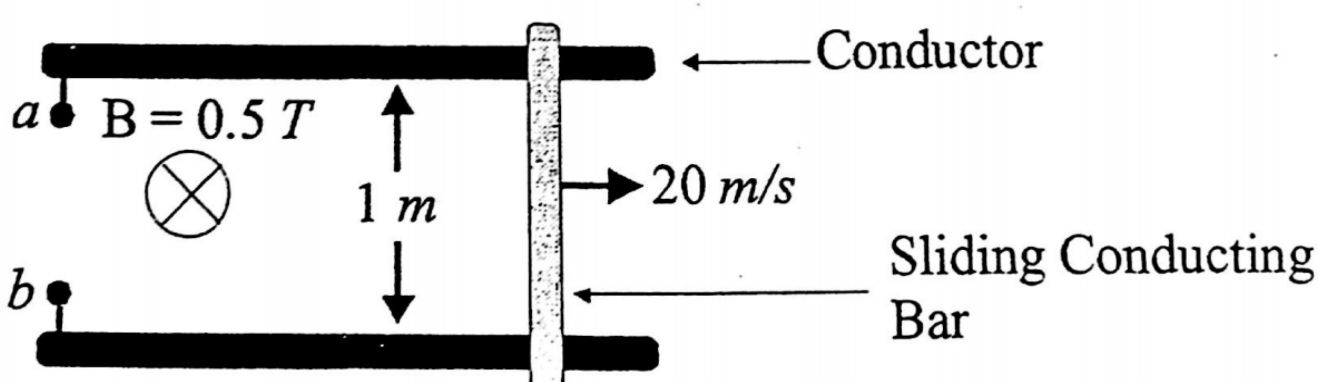
س28- يتناسب معامل الحث الذاتي لملف سولينويد تناسبا طرديا مع مربع:

Q28- The inductance of a solenoid is proportional to the square of its:

A) length l B) area A C) current I D) turns N

س29- القوة الدافعة المستحثة بين a و b هي:

Q29- The induced electromotive force between a and b equals:



A) 10 B) 40 C) 2.5 D) 1.25