

QUESTIONS


- In an analogy between electric current and automobile traffic flow, what would correspond to charge? What would correspond to current?
- Newspaper articles often contain a statement such as “10 000 volts of electricity surged through the victim’s body.” What is wrong with this statement?
- What factors affect the resistance of a conductor?
- What is the difference between resistance and resistivity?
- Two wires A and B of circular cross section are made of the same metal and have equal lengths, but the resistance of wire A is three times greater than that of wire B. What is the ratio of their cross-sectional areas? How do their radii compare?
- Do all conductors obey Ohm’s law? Give examples to justify your answer.
- We have seen that an electric field must exist inside a conductor that carries a current. How is it possible in view of the fact that in electrostatics we concluded that the electric field must be zero inside a conductor?
- A very large potential difference is not necessarily required to produce long sparks in air. With a device called *Jacob’s ladder*, a potential difference of about 10 kV produces an electric arc a few millimeters long between the bottom ends of two curved rods that project upward from the power supply. (The device is seen in classic mad-scientist horror movies and in Figure Q27.8.) The arc rises, climbing the rods and getting longer and longer. It disappears when it reaches the top; then a new spark immediately forms at the bottom and the process repeats. Explain these phenomena. Why does the arc rise? Why does a new arc appear only after the previous one is gone?
- When the voltage across a certain conductor is doubled, the current is observed to increase by a factor of three. What can you conclude about the conductor?
- In the water analogy of an electric circuit, what corresponds to the power supply, resistor, charge, and potential difference?
- Use the atomic theory of matter to explain why the resistance of a material should increase as its temperature increases.
- Why might a “good” electrical conductor also be a “good” thermal conductor?
- How does the resistance for copper and for silicon change with temperature? Why are the behaviors of these two materials different?
- Explain how a current can persist in a superconductor without any applied voltage.
- What single experimental requirement makes superconducting devices expensive to operate? In principle, can this limitation be overcome?
- What would happen to the drift velocity of the electrons in a wire and to the current in the wire if the electrons could move freely without resistance through the wire?
- If charges flow very slowly through a metal, why does it not require several hours for a light to come on when you throw a switch?
- In a conductor, changes in the electric field that drives the electrons through the conductor propagate with a speed close to the speed of light, although the drift velocity of the electrons is very small. Explain how these statements can both be true. Does one particular electron move from one end of the conductor to the other?
- Two conductors of the same length and radius are connected across the same potential difference. One conductor has twice the resistance of the other. To which conductor is more power delivered?
- Two lightbulbs both operate from 120 V. One has a power of 25 W and the other 100 W. Which bulb has higher resistance? Which bulb carries more current?
- Car batteries are often rated in ampere-hours. Does this designate the amount of current, power, energy, or charge that can be drawn from the battery?
- If you were to design an electric heater using Nichrome wire as the heating element, what parameters of the wire could you vary to meet a specific power output, such as 1 000 W?



Figure Q27.8


PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging □ = full solution available in the *Student Solutions Manual and Study Guide*

 = coached solution with hints available at <http://www.pse6.com>  = computer useful in solving problem

 = paired numerical and symbolic problems

Section 27.1 Electric Current

- In a particular cathode ray tube, the measured beam current is $30.0 \mu\text{A}$. How many electrons strike the tube screen every 40.0 s ?
- A teapot with a surface area of 700 cm^2 is to be silver plated. It is attached to the negative electrode of an electrolytic cell containing silver nitrate ($\text{Ag}^+ \text{NO}_3^-$). If the cell is powered by a 12.0-V battery and has a resistance of 1.80Ω , how long does it take for a 0.133-mm layer of silver to build up on the teapot? (The density of silver is $10.5 \times 10^3 \text{ kg/m}^3$.)
-  Suppose that the current through a conductor decreases exponentially with time according to the equation $I(t) = I_0 e^{-t/\tau}$ where I_0 is the initial current (at $t = 0$), and τ is a constant having dimensions of time. Consider a fixed observation point within the conductor. (a) How much charge passes this point between $t = 0$ and $t = \tau$? (b) How much charge passes this point between $t = 0$ and $t = 10\tau$? (c) **What If?** How much charge passes this point between $t = 0$ and $t = \infty$?
- In the Bohr model of the hydrogen atom, an electron in the lowest energy state follows a circular path $5.29 \times 10^{-11} \text{ m}$ from the proton. (a) Show that the speed of the electron is $2.19 \times 10^6 \text{ m/s}$. (b) What is the effective current associated with this orbiting electron?
- A small sphere that carries a charge q is whirled in a circle at the end of an insulating string. The angular frequency of rotation is ω . What average current does this rotating charge represent?
- The quantity of charge q (in coulombs) that has passed through a surface of area 2.00 cm^2 varies with time according to the equation $q = 4t^3 + 5t + 6$, where t is in seconds. (a) What is the instantaneous current through the surface at $t = 1.00 \text{ s}$? (b) What is the value of the current density?
- An electric current is given by the expression $I(t) = 100 \sin(120\pi t)$, where I is in amperes and t is in seconds. What is the total charge carried by the current from $t = 0$ to $t = (1/240) \text{ s}$?
- Figure P27.8 represents a section of a circular conductor of nonuniform diameter carrying a current of 5.00 A . The

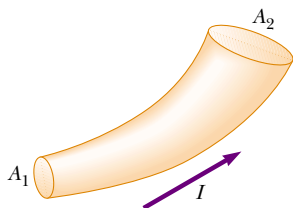



Figure P27.8

radius of cross section A_1 is 0.400 cm . (a) What is the magnitude of the current density across A_1 ? (b) If the current density across A_2 is one-fourth the value across A_1 , what is the radius of the conductor at A_2 ?

- The electron beam emerging from a certain high-energy electron accelerator has a circular cross section of radius 1.00 mm . (a) The beam current is $8.00 \mu\text{A}$. Find the current density in the beam, assuming that it is uniform throughout. (b) The speed of the electrons is so close to the speed of light that their speed can be taken as $c = 3.00 \times 10^8 \text{ m/s}$ with negligible error. Find the electron density in the beam. (c) How long does it take for Avogadro's number of electrons to emerge from the accelerator?
- A Van de Graaff generator produces a beam of 2.00-MeV deuterons, which are heavy hydrogen nuclei containing a proton and a neutron. (a) If the beam current is $10.0 \mu\text{A}$, how far apart are the deuterons? (b) Is the electric force of repulsion among them a significant factor in beam stability? Explain.
- An aluminum wire having a cross-sectional area of $4.00 \times 10^{-6} \text{ m}^2$ carries a current of 5.00 A . Find the drift speed of the electrons in the wire. The density of aluminum is 2.70 g/cm^3 . Assume that one conduction electron is supplied by each atom.

Section 27.2 Resistance

- Calculate the current density in a gold wire at 20°C , if an electric field of 0.740 V/m exists in the wire.
- A lightbulb has a resistance of 240Ω when operating with a potential difference of 120 V across it. What is the current in the lightbulb?
- A resistor is constructed of a carbon rod that has a uniform cross-sectional area of 5.00 mm^2 . When a potential difference of 15.0 V is applied across the ends of the rod, the rod carries a current of $4.00 \times 10^{-3} \text{ A}$. Find (a) the resistance of the rod and (b) the rod's length.
-  A 0.900-V potential difference is maintained across a 1.50-m length of tungsten wire that has a cross-sectional area of 0.600 mm^2 . What is the current in the wire?
- A conductor of uniform radius 1.20 cm carries a current of 3.00 A produced by an electric field of 120 V/m . What is the resistivity of the material?
- Suppose that you wish to fabricate a uniform wire out of 1.00 g of copper. If the wire is to have a resistance of $R = 0.500 \Omega$, and if all of the copper is to be used, what will be (a) the length and (b) the diameter of this wire?
- Gold is the most ductile of all metals. For example, one gram of gold can be drawn into a wire 2.40 km long. What is the resistance of such a wire at 20°C ? You can find the necessary reference information in this textbook.

19. (a) Make an order-of-magnitude estimate of the resistance between the ends of a rubber band. (b) Make an order-of-magnitude estimate of the resistance between the 'heads' and 'tails' sides of a penny. In each case state what quantities you take as data and the values you measure or estimate for them. (c) WARNING! Do not try this at home! What is the order of magnitude of the current that each would carry if it were connected across a 120-V power supply?
20. A solid cube of silver (density = 10.5 g/cm^3) has a mass of 90.0 g. (a) What is the resistance between opposite faces of the cube? (b) Assume each silver atom contributes one conduction electron. Find the average drift speed of electrons when a potential difference of $1.00 \times 10^{-5} \text{ V}$ is applied to opposite faces. The atomic number of silver is 47, and its molar mass is 107.87 g/mol.
21. A metal wire of resistance R is cut into three equal pieces that are then connected side by side to form a new wire the length of which is equal to one-third the original length. What is the resistance of this new wire?
22. Aluminum and copper wires of equal length are found to have the same resistance. What is the ratio of their radii?
23. A current density of $6.00 \times 10^{-13} \text{ A/m}^2$ exists in the atmosphere at a location where the electric field is 100 V/m. Calculate the electrical conductivity of the Earth's atmosphere in this region.
24. The rod in Figure P27.24 is made of two materials. The figure is not drawn to scale. Each conductor has a square cross section 3.00 mm on a side. The first material has a resistivity of $4.00 \times 10^{-3} \Omega \cdot \text{m}$ and is 25.0 cm long, while the second material has a resistivity of $6.00 \times 10^{-3} \Omega \cdot \text{m}$ and is 40.0 cm long. What is the resistance between the ends of the rod?

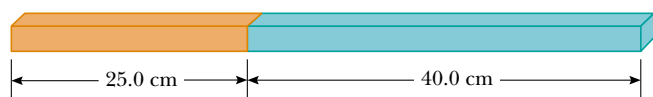


Figure P27.24

Section 27.3 A Model for Electrical Conduction

25. If the magnitude of the drift velocity of free electrons in a copper wire is $7.84 \times 10^{-4} \text{ m/s}$, what is the electric field in the conductor?
26. If the current carried by a conductor is doubled, what happens to the (a) charge carrier density? (b) current density? (c) electron drift velocity? (d) average time interval between collisions?
27. Use data from Example 27.1 to calculate the collision mean free path of electrons in copper. Assume the average thermal speed of conduction electrons is $8.60 \times 10^5 \text{ m/s}$.


Section 27.4 Resistance and Temperature

28. While taking photographs in Death Valley on a day when the temperature is 58.0°C , Bill Hiker finds that a certain voltage applied to a copper wire produces a current of 1.000 A. Bill then travels to Antarctica and applies the

- same voltage to the same wire. What current does he register there if the temperature is -88.0°C ? Assume that no change occurs in the wire's shape and size.
29. A certain lightbulb has a tungsten filament with a resistance of 19.0Ω when cold and 140Ω when hot. Assume that the resistivity of tungsten varies linearly with temperature even over the large temperature range involved here, and find the temperature of the hot filament. Assume the initial temperature is 20.0°C .
30. A carbon wire and a Nichrome wire are connected in series, so that the same current exists in both wires. If the combination has a resistance of $10.0 \text{ k}\Omega$ at 0°C , what is the resistance of each wire at 0°C so that the resistance of the combination does not change with temperature? The total or equivalent resistance of resistors in series is the sum of their individual resistances.
31. An aluminum wire with a diameter of 0.100 mm has a uniform electric field of 0.200 V/m imposed along its entire length. The temperature of the wire is 50.0°C . Assume one free electron per atom. (a) Use the information in Table 27.1 and determine the resistivity. (b) What is the current density in the wire? (c) What is the total current in the wire? (d) What is the drift speed of the conduction electrons? (e) What potential difference must exist between the ends of a 2.00-m length of the wire to produce the stated electric field?
32. **Review problem.** An aluminum rod has a resistance of 1.234Ω at 20.0°C . Calculate the resistance of the rod at 120°C by accounting for the changes in both the resistivity and the dimensions of the rod.
33. What is the fractional change in the resistance of an iron filament when its temperature changes from 25.0°C to 50.0°C ?
34. The resistance of a platinum wire is to be calibrated for low-temperature measurements. A platinum wire with resistance 1.00Ω at 20.0°C is immersed in liquid nitrogen at 77 K (-196°C). If the temperature response of the platinum wire is linear, what is the expected resistance of the platinum wire at -196°C ? ($\alpha_{\text{platinum}} = 3.92 \times 10^{-3}/^\circ\text{C}$)
35. The temperature of a sample of tungsten is raised while a sample of copper is maintained at 20.0°C . At what temperature will the resistivity of the tungsten be four times that of the copper?

Section 27.6 Electrical Power

36. A toaster is rated at 600 W when connected to a 120-V source. What current does the toaster carry, and what is its resistance?
37. A Van de Graaff generator (see Figure 25.29) is operating so that the potential difference between the high-voltage electrode B and the charging needles at A is 15.0 kV. Calculate the power required to drive the belt against electrical forces at an instant when the effective current delivered to the high-voltage electrode is $500 \mu\text{A}$.
38. In a hydroelectric installation, a turbine delivers 1 500 hp to a generator, which in turn transfers 80.0% of the mechanical energy out by electrical transmission. Under

- these conditions, what current does the generator deliver at a terminal potential difference of 2 000 V?
- 39.**  What is the required resistance of an immersion heater that increases the temperature of 1.50 kg of water from 10.0°C to 50.0°C in 10.0 min while operating at 110 V?
- 40.** One rechargeable battery of mass 15.0 g delivers to a CD player an average current of 18.0 mA at 1.60 V for 2.40 h before the battery needs to be recharged. The recharger maintains a potential difference of 2.30 V across the battery and delivers a charging current of 13.5 mA for 4.20 h. (a) What is the efficiency of the battery as an energy storage device? (b) How much internal energy is produced in the battery during one charge–discharge cycle? (c) If the battery is surrounded by ideal thermal insulation and has an overall effective specific heat of 975 J/kg°C, by how much will its temperature increase during the cycle?
- 41.** Suppose that a voltage surge produces 140 V for a moment. By what percentage does the power output of a 120-V, 100-W lightbulb increase? Assume that its resistance does not change.
- 42.** A 500-W heating coil designed to operate from 110 V is made of Nichrome wire 0.500 mm in diameter. (a) Assuming that the resistivity of the Nichrome remains constant at its 20.0°C value, find the length of wire used. (b) **What If?** Now consider the variation of resistivity with temperature. What power will the coil of part (a) actually deliver when it is heated to 1 200°C?
- 43.** A coil of Nichrome wire is 25.0 m long. The wire has a diameter of 0.400 mm and is at 20.0°C. If it carries a current of 0.500 A, what are (a) the magnitude of the electric field in the wire, and (b) the power delivered to it? (c) **What If?** If the temperature is increased to 340°C and the voltage across the wire remains constant, what is the power delivered?
- 44.** Batteries are rated in terms of ampere-hours (A·h). For example, a battery that can produce a current of 2.00 A for 3.00 h is rated at 6.00 A·h. (a) What is the total energy, in kilowatt-hours, stored in a 12.0-V battery rated at 55.0 A·h? (b) At \$0.060 0 per kilowatt-hour, what is the value of the electricity produced by this battery?
- 45.** A 10.0-V battery is connected to a 120-Ω resistor. Ignoring the internal resistance of the battery, calculate the power delivered to the resistor.
- 46.** Residential building codes typically require the use of 12-gauge copper wire (diameter 0.205 3 cm) for wiring receptacles. Such circuits carry currents as large as 20 A. A wire of smaller diameter (with a higher gauge number) could carry this much current, but the wire could rise to a high temperature and cause a fire. (a) Calculate the rate at which internal energy is produced in 1.00 m of 12-gauge copper wire carrying a current of 20.0 A. (b) **What If?** Repeat the calculation for an aluminum wire. Would a 12-gauge aluminum wire be as safe as a copper wire?
- 47.** An 11.0-W energy-efficient fluorescent lamp is designed to produce the same illumination as a conventional 40.0-W incandescent lightbulb. How much money does the user of the energy-efficient lamp save during 100 hours of use? Assume a cost of \$0.080 0/kWh for energy from the power company.
- 48.** We estimate that 270 million plug-in electric clocks are in the United States, approximately one clock for each person. The clocks convert energy at the average rate 2.50 W. To supply this energy, how many metric tons of coal are burned per hour in coal-fired electric generating plants that are, on average, 25.0% efficient? The heat of combustion for coal is 33.0 MJ/kg.
- 49.** Compute the cost per day of operating a lamp that draws a current of 1.70 A from a 110-V line. Assume the cost of energy from the power company is \$0.060 0/kWh.
- 50. Review problem.** The heating element of a coffee maker operates at 120 V and carries a current of 2.00 A. Assuming that the water absorbs all of the energy delivered to the resistor, calculate how long it takes to raise the temperature of 0.500 kg of water from room temperature (23.0°C) to the boiling point.
- 51.** A certain toaster has a heating element made of Nichrome wire. When the toaster is first connected to a 120-V source (and the wire is at a temperature of 20.0°C), the initial current is 1.80 A. However, the current begins to decrease as the heating element warms up. When the toaster reaches its final operating temperature, the current drops to 1.53 A. (a) Find the power delivered to the toaster when it is at its operating temperature. (b) What is the final temperature of the heating element?
- 52.** The cost of electricity varies widely through the United States; \$0.120/kWh is one typical value. At this unit price, calculate the cost of (a) leaving a 40.0-W porch light on for two weeks while you are on vacation, (b) making a piece of dark toast in 3.00 min with a 970-W toaster, and (c) drying a load of clothes in 40.0 min in a 5 200-W dryer.
- 53.** Make an order-of-magnitude estimate of the cost of one person's routine use of a hair dryer for 1 yr. If you do not use a blow dryer yourself, observe or interview someone who does. State the quantities you estimate and their values.

Additional Problems

- 54.** One lightbulb is marked '25 W 120 V,' and another '100 W 120 V'; this means that each bulb has its respective power delivered to it when plugged into a constant 120-V potential difference. (a) Find the resistance of each bulb. (b) How long does it take for 1.00 C to pass through the dim bulb? Is the charge different in any way upon its exit from the bulb versus its entry? (c) How long does it take for 1.00 J to pass through the dim bulb? By what mechanisms does this energy enter and exit the bulb? (d) Find how much it costs to run the dim bulb continuously for 30.0 days if the electric company sells its product at \$0.070 0 per kWh. What product *does* the electric company sell? What is its price for one SI unit of this quantity?
- 55.** A charge Q is placed on a capacitor of capacitance C . The capacitor is connected into the circuit shown in Figure P27.55, with an open switch, a resistor, and an initially uncharged capacitor of capacitance $3C$. The switch is then closed and the circuit comes to equilibrium. In terms of Q and C , find (a) the final potential difference between the plates of each capacitor, (b) the charge on each capacitor,

and (c) the final energy stored in each capacitor. (d) Find the internal energy appearing in the resistor.

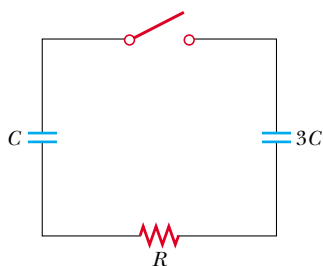


Figure P27.55

56. A high-voltage transmission line with a diameter of 2.00 cm and a length of 200 km carries a steady current of 1 000 A. If the conductor is copper wire with a free charge density of 8.49×10^{28} electrons/m³, how long does it take one electron to travel the full length of the line?

57. A more general definition of the temperature coefficient of resistivity is


$$\alpha = \frac{1}{\rho} \frac{d\rho}{dT}$$

where ρ is the resistivity at temperature T . (a) Assuming that α is constant, show that

$$\rho = \rho_0 e^{\alpha(T - T_0)}$$

where ρ_0 is the resistivity at temperature T_0 . (b) Using the series expansion $e^x \approx 1 + x$ for $x \ll 1$, show that the resistivity is given approximately by the expression $\rho = \rho_0 [1 + \alpha(T - T_0)]$ for $\alpha(T - T_0) \ll 1$.

58. A high-voltage transmission line carries 1 000 A starting at 700 kV for a distance of 100 mi. If the resistance in the wire is 0.500 Ω /mi, what is the power loss due to resistive losses?

59.  An experiment is conducted to measure the electrical resistivity of Nichrome in the form of wires with different lengths and cross-sectional areas. For one set of measurements, a student uses 30-gauge wire, which has a cross-sectional area of 7.30×10^{-8} m². The student measures the potential difference across the wire and the current in the wire with a voltmeter and an ammeter, respectively. For each of the measurements given in the table taken on wires of three different lengths, calculate the resistance of the wires and the corresponding values of the resistivity. What is the average value of the resistivity, and how does this value compare with the value given in Table 27.1?

L (m)	ΔV (V)	I (A)	R (Ω)	ρ ($\Omega \cdot \text{m}$)
0.540	5.22	0.500		
1.028	5.82	0.276		
1.543	5.94	0.187		

60. An electric utility company supplies a customer's house from the main power lines (120 V) with two copper wires, each of which is 50.0 m long and has a resistance of 0.108 Ω per 300 m. (a) Find the voltage at the customer's


house for a load current of 110 A. For this load current, find (b) the power the customer is receiving and (c) the electric power lost in the copper wires.

61. A straight cylindrical wire lying along the x axis has a length of 0.500 m and a diameter of 0.200 mm. It is made of a material that obeys Ohm's law with a resistivity of $\rho = 4.00 \times 10^{-8} \Omega \cdot \text{m}$. Assume that a potential of 4.00 V is maintained at $x = 0$, and that $V = 0$ at $x = 0.500$ m. Find (a) the electric field \mathbf{E} in the wire, (b) the resistance of the wire, (c) the electric current in the wire, and (d) the current density \mathbf{J} in the wire. Express vectors in vector notation. (e) Show that $\mathbf{E} = \rho \mathbf{J}$.

62. A straight cylindrical wire lying along the x axis has a length L and a diameter d . It is made of a material that obeys Ohm's law with a resistivity ρ . Assume that potential V is maintained at $x = 0$, and that the potential is zero at $x = L$. In terms of L , d , V , ρ , and physical constants, derive expressions for (a) the electric field in the wire, (b) the resistance of the wire, (c) the electric current in the wire, and (d) the current density in the wire. Express vectors in vector notation. (e) Prove that $\mathbf{E} = \rho \mathbf{J}$.

63. The potential difference across the filament of a lamp is maintained at a constant level while equilibrium temperature is being reached. It is observed that the steady-state current in the lamp is only one tenth of the current drawn by the lamp when it is first turned on. If the temperature coefficient of resistivity for the lamp at 20.0°C is 0.00450 ($^{\circ}\text{C}$)⁻¹, and if the resistance increases linearly with increasing temperature, what is the final operating temperature of the filament?

64. The current in a resistor decreases by 3.00 A when the voltage applied across the resistor decreases from 12.0 V to 6.00 V. Find the resistance of the resistor.

65.  An electric car is designed to run off a bank of 12.0-V batteries with total energy storage of 2.00×10^7 J. (a) If the electric motor draws 8.00 kW, what is the current delivered to the motor? (b) If the electric motor draws 8.00 kW as the car moves at a steady speed of 20.0 m/s, how far will the car travel before it is "out of juice"?

66. **Review problem.** When a straight wire is heated, its resistance is given by $R = R_0 [1 + \alpha(T - T_0)]$ according to Equation 27.21, where α is the temperature coefficient of resistivity. (a) Show that a more precise result, one that includes the fact that the length and area of the wire change when heated, is

$$R = \frac{R_0 [1 + \alpha(T - T_0)] [1 + \alpha'(T - T_0)]}{[1 + 2\alpha'(T - T_0)]}$$

where α' is the coefficient of linear expansion (see Chapter 19). (b) Compare these two results for a 2.00-m-long copper wire of radius 0.100 mm, first at 20.0°C and then heated to 100.0°C.

67. The temperature coefficients of resistivity in Table 27.1 were determined at a temperature of 20°C. What would they be at 0°C? Note that the temperature coefficient of resistivity at 20°C satisfies $\rho = \rho_0 [1 + \alpha(T - T_0)]$, where ρ_0 is the resistivity of the material at $T_0 = 20^\circ\text{C}$. The temperature

coefficient of resistivity α' at 0°C must satisfy the expression $\rho = \rho_0[1 + \alpha'T]$, where ρ_0 is the resistivity of the material at 0°C .

68. An oceanographer is studying how the ion concentration in sea water depends on depth. She does this by lowering into the water a pair of concentric metallic cylinders (Fig. P27.68) at the end of a cable and taking data to determine the resistance between these electrodes as a function of depth. The water between the two cylinders forms a cylindrical shell of inner radius r_a , outer radius r_b , and length L much larger than r_b . The scientist applies a potential difference ΔV between the inner and outer surfaces, producing an outward radial current I . Let ρ represent the resistivity of the water. (a) Find the resistance of the water between the cylinders in terms of L , ρ , r_a , and r_b . (b) Express the resistivity of the water in terms of the measured quantities L , r_a , r_b , ΔV , and I .

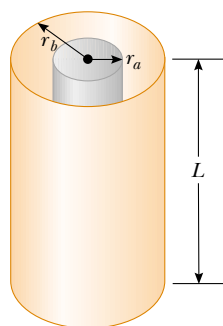


Figure P27.68

69. In a certain stereo system, each speaker has a resistance of $4.00\ \Omega$. The system is rated at $60.0\ \text{W}$ in each channel, and each speaker circuit includes a fuse rated $4.00\ \text{A}$. Is this system adequately protected against overload? Explain your reasoning.
70. A close analogy exists between the flow of energy by heat because of a temperature difference (see Section 20.7) and the flow of electric charge because of a potential difference. The energy dQ and the electric charge dq can both be transported by free electrons in the conducting material. Consequently, a good electrical conductor is usually a good thermal conductor as well. Consider a thin conducting slab of thickness dx , area A , and electrical conductivity σ , with a potential difference dV between opposite faces. Show that the current $I = dq/dt$ is given by the equation on the left below:

	Thermal Conduction
Charge Conduction	(Eq. 20.14)
$\frac{dq}{dt} = \sigma A \left \frac{dV}{dx} \right $	$\frac{dQ}{dt} = kA \left \frac{dT}{dx} \right $

In the analogous thermal conduction equation on the right, the rate of energy flow dQ/dt (in SI units of joules per second) is due to a temperature gradient dT/dx , in a material of thermal conductivity k . State analogous rules relating the direction of the electric current to the change in potential, and relating the direction of energy flow to the change in temperature.

71. Material with uniform resistivity ρ is formed into a wedge as shown in Figure P27.71. Show that the resistance between face A and face B of this wedge is

$$R = \rho \frac{L}{w(y_2 - y_1)} \ln \left(\frac{y_2}{y_1} \right)$$

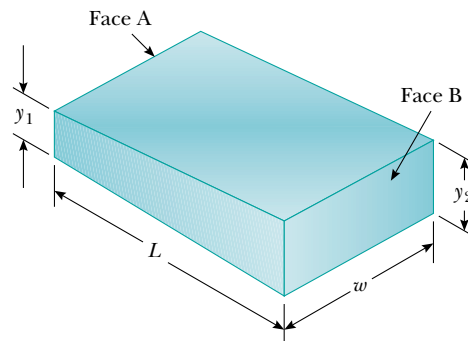


Figure P27.71

72. A material of resistivity ρ is formed into the shape of a truncated cone of altitude h as shown in Figure P27.72. The bottom end has radius b , and the top end has radius a . Assume that the current is distributed uniformly over any circular cross section of the cone, so that the current density does not depend on radial position. (The current density does vary with position along the axis of the cone.) Show that the resistance between the two ends is described by the expression

$$R = \frac{\rho}{\pi} \left(\frac{h}{ab} \right)$$

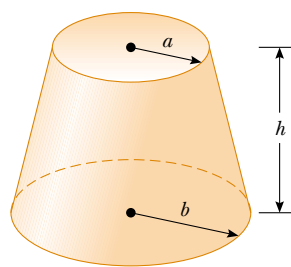


Figure P27.72

73. The dielectric material between the plates of a parallel-plate capacitor always has some nonzero conductivity σ . Let A represent the area of each plate and d the distance between them. Let κ represent the dielectric constant of the material. (a) Show that the resistance R and the capacitance C of the capacitor are related by

$$RC = \frac{\kappa\epsilon_0}{\sigma}$$

- (b) Find the resistance between the plates of a 14.0-nF capacitor with a fused quartz dielectric.
74. The current-voltage characteristic curve for a semiconductor diode as a function of temperature T is given by the equation

$$I = I_0(e^{e\Delta V/k_B T} - 1)$$

Here the first symbol e represents Euler's number, the base of natural logarithms. The second e is the charge on the electron. The k_B stands for Boltzmann's constant, and T is the absolute temperature. Set up a spreadsheet to calculate I and $R = \Delta V/I$ for $\Delta V = 0.400$ V to 0.600 V in increments of 0.005 V. Assume $I_0 = 1.00$ nA. Plot R versus ΔV for $T = 280$ K, 300 K, and 320 K.

- 75. Review problem.** A parallel-plate capacitor consists of square plates of edge length ℓ that are separated by a distance d , where $d \ll \ell$. A potential difference ΔV is maintained between the plates. A material of dielectric constant κ fills half of the space between the plates. The dielectric slab is now withdrawn from the capacitor, as shown in Figure P27.75. (a) Find the capacitance when the left edge of the dielectric is at a distance x from the center of the capacitor. (b) If the dielectric is removed at a constant speed v , what is the current in the circuit as the dielectric is being withdrawn?

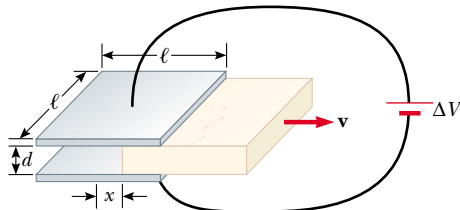


Figure P27.75

Answers to Quick Quizzes

- 27.1** d, b = c, a. The current in part (d) is equivalent to two positive charges moving to the left. Parts (b) and (c) each represent four positive charges moving in the same direction because negative charges moving to the left are equivalent to positive charges moving to the right. The current in part (a) is equivalent to five positive charges moving to the right.
- 27.2** (b). The currents in the two paths add numerically to equal the current coming into the junction, without regard for the directions of the two wires coming out of the junction. This is indicative of scalar addition. Even though we can assign a direction to a current, it is not a vector. This suggests a deeper meaning for vectors besides that of a quantity with magnitude and direction.
- 27.3** (a). The current in each section of the wire is the same even though the wire constricts. As the cross-sectional area A decreases, the drift velocity must increase in order for the constant current to be maintained, in accordance with Equation 27.4. As A decreases, Equation 27.11 tells us that R increases.
- 27.4** (b). The doubling of the radius causes the area A to be four times as large, so Equation 27.11 tells us that the resistance decreases.
- 27.5** (b). The slope of the tangent to the graph line at a point is the reciprocal of the resistance at that point. Because the slope is increasing, the resistance is decreasing.
- 27.6** (a). When the filament is at room temperature, its resistance is low, and hence the current is relatively large. As the filament warms up, its resistance increases, and the current decreases. Older lightbulbs often fail just as they are turned on because this large initial current “spike” produces rapid temperature increase and mechanical stress on the filament, causing it to break.
- 27.7** (c). Because the potential difference ΔV is the same across the two bulbs and because the power delivered to a conductor is $\mathcal{P} = I\Delta V$, the 60-W bulb, with its higher power rating, must carry the greater current. The 30-W bulb has the higher resistance because it draws less current at the same potential difference.
- 27.8** $I_a = I_b > I_c = I_d > I_e = I_f$. The current I_a leaves the positive terminal of the battery and then splits to flow through the two bulbs; thus, $I_a = I_c + I_e$. From Quick Quiz 27.7, we know that the current in the 60-W bulb is greater than that in the 30-W bulb. Because charge does not build up in the bulbs, we know that the same amount of charge flowing into a bulb from the left must flow out on the right; consequently, $I_c = I_d$ and $I_e = I_f$. The two currents leaving the bulbs recombine to form the current back into the battery, $I_f + I_d = I_b$.