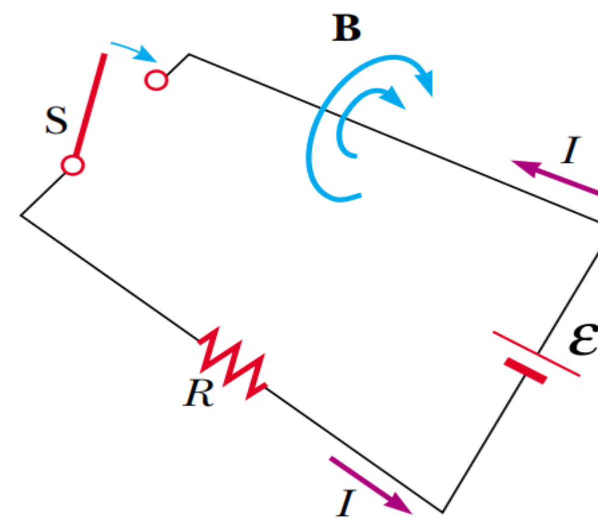
Chapter 32

Inductance

CHAPTER OUTLINE

32.1 Self-Inductance

- In chapter 31, an emf and a current are induced in a circuit when the magnetic flux through the area enclosed by the circuit changes with time.



1. When the switch is closed, the current does not immediately jump from zero to its maximum value ε/R
2. As the current increases with time, the magnetic flux through the circuit loop due to this current also increases with time. *This increasing flux creates an induced emf in the circuit.*

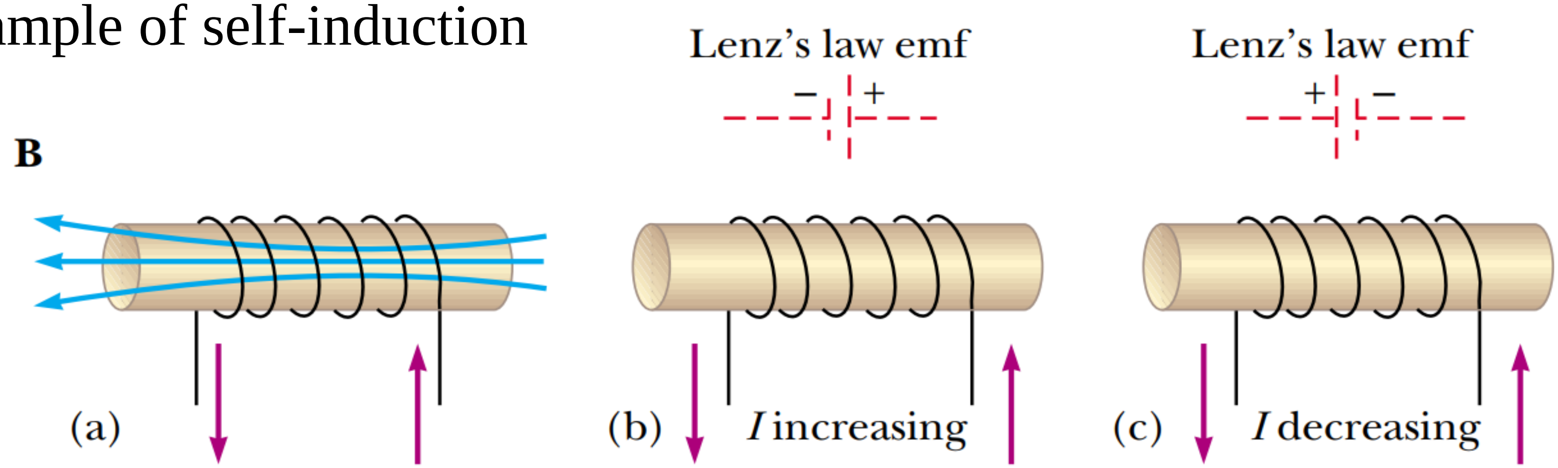
3. The direction of the induced emf is such that it would cause an induced current in the loop (if the loop did not already carry a current), which would establish a magnetic field opposing the change in the original magnetic field. Thus, the direction of the induced emf is opposite the direction of the emf of the battery.

❖ This effect is called **self-induction** because the changing flux through the circuit and the resultant induced emf arise from the circuit itself. The emf set up in this case is called a **self-induced emf.**

➢ The direction of the induced emf, it is also called *a back emf*

32.1 Self-Inductance

A second example of self-induction



- A coil wound on a cylindrical core and the current in the coil is either increases or decreases with time.
- The current is in the direction as shown in (a), a magnetic field directed from right to left is set up inside the coil.
- As the current changes with time, the magnetic flux through the coil also changes and induces an emf in the coil.
- The polarity of this induced emf must be such that it opposes the change in the magnetic field from the current.
- If the current is increasing, the polarity of the induced emf is as shown in (b)
- If the current is decreasing, the polarity of the induced emf is as shown in (c)

32.1 Self-Inductance

- From Faraday's law that the induced emf is equal to the negative of the time rate of change of the magnetic flux.
- The magnetic flux is proportional to the magnetic field due to the current, which in turn is proportional to the current in the circuit.
- Therefore, **a self-induced emf is always proportional to the time rate of change of the current.**

For any coil, we find that

$$\varepsilon_L = -L\frac{dI}{dt}$$

where ***L*** is a proportionality constant—called the **inductance** of the coil

- From Faraday's law, the inductance of a closely spaced coil of *N* turns carrying a current *I* and containing *N* turns is

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -N\frac{d(\mu_o nIA)}{dt} = -N\mu_o nA\frac{dI}{dt} = -\frac{NBA}{I}\frac{dI}{dt}$$

$$\varepsilon = -\frac{N\Phi_B}{I}\frac{dI}{dt} = -L\frac{dI}{dt}$$

$$L = \frac{N\Phi_B}{I}$$

32.1 Self-Inductance

Inductance of an *N*-turn coil

$$L = \frac{N\Phi_B}{I}$$

where it is assumed that the same magnetic flux passes through each turn.

Inductance

$$L = -\frac{\mathcal{E}_L}{dI/dt}$$

- Inductance is a measure of the opposition to a *change* in current.

The SI unit of inductance is the **henry** (H) which is 1 volt-second per ampere

$$1\ \text{H} = 1\ \frac{\text{V}\cdot\text{s}}{\text{A}}$$

32.1 Self-Inductance

Example 32.1 Inductance of a Solenoid

Find the inductance of a uniformly wound solenoid having N turns and length ℓ. Assume that ℓ is much longer than the radius of the windings and that the core of the solenoid is air.

Solution We can assume that the interior magnetic field due to the current is uniform and given by Equation 30.17:

$$B = \mu_0 n I = \mu_0 \frac{N}{\ell} I$$

where $n = N/\ell$ is the number of turns per unit length. The magnetic flux through each turn is

$$\Phi_B = BA = \mu_0 \frac{NA}{\ell} I$$

where A is the cross-sectional area of the solenoid. Using this expression and Equation 32.2, we find that

$$L = \frac{N\Phi_B}{I} = \frac{\mu_0 N^2 A}{\ell} \qquad (32.4)$$

This result shows that L depends on geometry and is proportional to the square of the number of turns. Because $N = n\ell$, we can also express the result in the form

$$L = \mu_0 \frac{(n\ell)^2}{\ell} A = \mu_0 n^2 A\ell = \mu_0 n^2 V \qquad (32.5)$$

where $V = A\ell$ is the interior volume of the solenoid.

What If? What would happen to the inductance if you inserted a ferromagnetic material inside the solenoid?

Answer The inductance would increase. For a given current, the magnetic flux in the solenoid is much greater because of the increase in the magnetic field originating from the magnetization of the ferromagnetic material. For example, if the material has a magnetic permeability of $500\mu_0$, the inductance increases by a factor of 500.

32.1 Self-Inductance

Example 32.2 Calculating Inductance and emf

(A) Calculate the inductance of an air-core solenoid containing 300 turns if the length of the solenoid is 25.0 cm and its cross-sectional area is 4.00 cm^2.

Solution Using Equation 32.4, we obtain

$$L = \frac{\mu_0 N^2 A}{\ell}$$

$$= \frac{(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A})(300)^2(4.00 \times 10^{-4}\ \mathrm{m}^2)}{25.0 \times 10^{-2}\ \mathrm{m}}$$

$$= 1.81 \times 10^{-4}\ \mathrm{T \cdot m^2/A} = \boxed{0.181\ \mathrm{mH}}$$

(B) Calculate the self-induced emf in the solenoid if the current it carries is decreasing at the rate of 50.0 A/s.

Solution Using Equation 32.1 and given that $dI/dt = -50.0$ A/s, we obtain

$$\mathcal{E}_L = -L\frac{dI}{dt} = -(1.81 \times 10^{-4}\ \mathrm{H})(-50.0\ \mathrm{A/s})$$

$$= \boxed{9.05\ \mathrm{mV}}$$

Section 32.1 Self-Inductance

Selected Solved Problems (Chapter # 32)

6. An emf of 24.0 mV is induced in a 500-turn coil at an instant when the current is 4.00 A and is changing at the rate of 10.0 A/s. What is the magnetic flux through each turn of the coil?

7. An inductor in the form of a solenoid contains 420 turns, is 16.0 cm in length, and has a cross-sectional area of 3.00 cm^2. What uniform rate of decrease of current through the inductor induces an emf of 175 μV?

9. A 40.0-mA current is carried by a uniformly wound air-core solenoid with 450 turns, a 15.0-mm diameter, and 12.0-cm length. Compute (a) the magnetic field inside the solenoid, (b) the magnetic flux through each turn, and (c) the inductance of the solenoid. (d) **What If?** If the current were different, which of these quantities would change?

32.3 Energy in a Magnetic Field

- The emf induced in an inductor prevents a battery from establishing an instantaneous current, the battery must provide more energy than in a circuit without the inductor.

- The energy supplied by the battery appears as internal energy in the resistor, while the remaining energy is stored in the magnetic field of the inductor.

$$I\mathcal{E} = I^2R + LI\frac{dI}{dt}$$

Recognizing

1. εI as the rate at which energy is supplied by the battery.
2. I^2R as the rate at which energy is delivered to the resistor.
3. we see that $LI\ (dI/dt)$ must represent the rate at which energy is being stored in the inductor.

32.3 Energy in a Magnetic Field

➢ The energy stored in the inductor at any time

$$\frac{dU}{dt} = LI\,\frac{dI}{dt}$$

➢ To find the total energy stored in the inductor when the current is *I*

Energy stored in an inductor

$$U = \int dU = \int_0^I LI\,dI = L\int_0^I I\,dI$$

$$U = \frac{1}{2}LI^2$$

32.3 Energy in a Magnetic Field

- Energy density of a magnetic field
  - Consider a solenoid whose inductance $L = \mu_0 n^2 A\ell$
- The magnetic field of a solenoid $B = \mu_0 n I$

$$U = \tfrac{1}{2} L I^2 = \tfrac{1}{2} \mu_0 n^2 A\ell \left(\frac{B}{\mu_0 n}\right)^2 = \frac{B^2}{2\mu_0} A\ell$$

Because ***Al*** is the volume of the solenoid, the magnetic energy density, or *the energy stored per unit volume in the magnetic field of the inductor* is

Magnetic energy density

$$u_B = \frac{U}{A\ell} = \frac{B^2}{2\mu_0}$$

- The energy density is proportional to the square of the field magnitude.

32.3 Energy in a Magnetic Field

Quick Quiz 32.5 You are performing an experiment that requires the highest possible energy density in the interior of a very long solenoid. Which of the following increases the energy density? (More than one choice may be correct.) (a) increasing the number of turns per unit length on the solenoid (b) increasing the cross-sectional area of the solenoid (c) increasing only the length of the solenoid while keeping the number of turns per unit length fixed (d) increasing the current in the solenoid.

Section 32.3 Energy in a Magnetic Field

Selected Solved Problems (Chapter # 32)

29. Calculate the energy associated with the magnetic field of a 200-turn solenoid in which a current of 1.75 A produces a flux of 3.70×10^{-4} Wb in each turn.

30. The magnetic field inside a superconducting solenoid is 4.50 T. The solenoid has an inner diameter of 6.20 cm and a length of 26.0 cm. Determine (a) the magnetic energy density in the field and (b) the energy stored in the magnetic field within the solenoid.

31. An air-core solenoid with 68 turns is 8.00 cm long and has a diameter of 1.20 cm. How much energy is stored in its magnetic field when it carries a current of 0.770 A?

37. A uniform electric field of magnitude 680 kV/m throughout a cylindrical volume results in a total energy of 3.40 μJ. What magnetic field over this same region stores the same total energy?

Chapter # 32 Problems

Selected Old exam questions

Q26. The unit of the self-inductance coefficient is:

26. إن وحدة قياس معامل اللف (الحث) الذاتي:

A) $\frac{T}{s}$ B) $\frac{V s^2}{T}$ C) $\frac{V s}{A}$ D) $\frac{H}{A}$ E) T^2/s

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Q27. An air-core solenoid contains 500 turns with a length 40cm and a cross-sectional area $6cm^2$. Then the self-inductance is:

27. يحتوي ملف حلزوني هوائي على 500 لفة، طوله الكلي 40 cm ومساحة مقطعه $6\ cm^2$، فإن معامل اللف الذاتي لهذا الملف هو:

A) 0.471 mH B) 0.146 mH C) 0.834mH D) 0.656 mH E) 0.780 mH

Q28. If u_E is the energy density resulted from an applied electric field E and u_B is the energy density from an applied magnetic field B. Then the ratio u_E/u_B will be:

28. لتكن u_E كثافة الطاقة الناجمة عن المجال الكهربائي E، وكذلك u_B تمثل كثافة الطاقة الناجمة عن المجال المغناطيسي B. فإن النسبة بين الكثافتين u_E/u_B هي:

A) $\frac{\varepsilon_0 E}{\mu_0 . B}$ B) $\varepsilon_0 \mu_0 (\frac{E}{B})^2$ C) $\varepsilon_0 \mu_0 (\frac{E}{B})$ D) $\frac{1}{\varepsilon_0 \mu_0} (\frac{E}{B})^2$ E) $\frac{1}{\varepsilon_0 \mu^2_0} (\frac{E}{B})^2$

Chapter # 32 Problems

Selected Old exam questions

س٢٦- ملف سولينويد طويل يمر به تيار $10\ A$ ومعامل الحث الذاتي له $10\ \mu H$ ، قيمة الطاقة المخزنة بالملف تساوي:

Q26- A long solenoid has a self inductance of $10\ \mu H$. The energy stored in its mag. field when it carries a current of $10\ A$ is:

A) $50\ \mu J$ B) $250\ \mu J$ C) $500\ \mu J$ D) $5000\ \mu J$

س26- أي القيم التالية لاتعتمد على التيار I ؟

Q26- Which is of the following independent on the electric current I?

A) Self Inductance L B) Mag. Flux Φ C) Mag. force on a conductor F D) Mag. Field B

س27- ملف حلزوني طويل يمر به تيار $10\ A$ ومعامل الحث الذاتي له $10\ \mu H$ ، قيمة الطاقة المخزنة تساوي:

Q27- A long solenoid has a self inductance of $10\ \mu H$. The energy stored in its mag. field when it carries a current of $10\ A$ is:

A) $50\ \mu J$ B) $50\ mJ$ C) $250\ \mu J$ D) $500\ \mu J$

س27- إذا كان المجال المغناطيسي يساوي $15\ mT$ فما كثافة الطاقة المغناطيسية لوحدة الحجم؟

Q27- If the magnetic field is $15\ mT$, what is the magnetic energy density?

A) 179.1 B) 89.5 C) 11.9 D) 5.96

س29- يحمل ملف حثه الذاتي $200\ mH$ تياراً مستقراً أو ثابتاً قيمته $0.5\ A$ ، وعند فتح قاطع الدائرة الكهربائية فإن القيمة الفعالة لهذا التيار تتناقص بشكل خطي حتى تبلغ الصفراً وذلك خلال زمن قدره $10\ mS$، القوة الدافعة المستحثة في الملف تساوي:

Q29- A $200\ mH$ inductor carries a steady current of $0.5\ A$. When the switch in the circuit is opened, the current is effectively zero after $10\ mS$. What is the induced electromotive force (emf) in the inductor during this time?

A) 10000 B) 1000 C) 100 D) 10