

Magnetic Fields

CHAPTER OUTLINE

29.1 Magnetic Fields and Forces

29.2 Magnetic Force Acting on a Current-Carrying Conductor

29.4 Motion of a Charged Particle in a Uniform Magnetic Field

29.5 Applications Involving Charged Particles Moving in a Magnetic Field

Examines the forces that act on moving charges and on current carrying wires in the presence of a magnetic field.

Brief on the History of Magnetism

- Greeks discovered that the stone magnetite (Fe_3O_4) attracts pieces of iron.
- Every magnet, regardless of its shape, has two poles, called *north* (N) and
- ✤ south (S) poles,
- Poles (N–N or S–S) repel each other, and opposite poles (N–S) attract each other.
- electric charges can be isolated (witness the electron and proton) whereas a single magnetic pole has never been isolated. That is, magnetic poles are always found in pairs.

No matter how many times a permanent magnet is cut in two, each piece always has a north and a south pole.

Brief on the History of Magnetism

- The relationship between magnetism and electricity was discovered in 1819
- Oersted found that an electric current in a wire deflected a nearby compass needle.
- Faraday and Joseph Henry (1797–1878) showed that an electric current can be produced in a circuit either by moving a magnet near the circuit or by changing the current in a nearby circuit. These observations demonstrate that a changing magnetic field creates an electric field.
 - Theoretical work by Maxwell showed that a changing electric field creates a magnetic field.



Hans Christian Oersted

Danish Physicist and Chemist (1777–1851)

Oersted is best known for observing that a compass needle deflects when placed near a wire carrying a current. This important discovery was the first evidence of the connection between electric and magnetic phenomena. Oersted was also the first to prepare pure aluminum. (North Wind Picture Archives)

- The region of space surrounding any *moving* electric charge also contains a magnetic field.
- A magnetic field also surrounds a magnetic substance making up a permanent magnet.
- The symbol B has been used to represent a magnetic field.
- The direction of the magnetic field B at any location is the direction in which a compass needle points at that location. As with the electric field, we can represent the magnetic field by means of drawings with *magnetic field lines*.
- The magnetic field lines outside the magnet point away from north poles and toward south poles.



Compass needles can be used to trace the magnetic field lines in the region outside a bar magnet.



(a) Magnetic field pattern surrounding a bar magnet as displayed with iron filings.

- (b) Magnetic field pattern between *opposite* poles (N–S) of two bar magnets.
- (c) Magnetic field pattern between *like* poles (N–N) of two bar magnets.

A magnetic field **B** at some point in space can be defined in terms of the magnetic force \mathbf{F}_B . The field exerts on a charged particle moving with a velocity **v**.

Properties of the magnetic force on a charge moving in a magnetic field B

- ✤ The magnitude F_B of the magnetic force exerted on the particle is proportional to the charge q and to the speed v of the particle.
- * The magnitude and direction of \mathbf{F}_B depend on the velocity of the particle and on the magnitude and direction of the magnetic field **B**.
- When a charged particle moves parallel to the magnetic field vector, the magnetic force acting on the particle is zero.

Properties of the magnetic force on a charge moving in a magnetic field B

- ★ When the particle's velocity vector makes any angle θ ≠ 0 with the magnetic field, the magnetic force acts in a direction perpendicular to both v and B; that is, F_B is perpendicular to the plane formed by v and B
 - The magnetic force exerted on a positive charge is in the direction opposite the direction of the magnetic force exerted on a negative charge moving in the same direction.
- * The magnitude of the magnetic force exerted on the moving particle is proportional to $\sin \theta$, where θ is the angle the particle's velocity vector makes with the direction of **B**.

We can summarize these observations by writing the magnetic force in the form

Vector expression for the magnetic force on a charged particle moving in a magnetic field

 $\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$



The direction of the magnetic force \mathbf{F}_B acting on a charged particle moving with a velocity \mathbf{v} in the presence of a magnetic field \mathbf{B} . (a) The magnetic force is perpendicular to both \mathbf{v} and \mathbf{B} . (b) Oppositely directed magnetic forces $\mathbf{F}B$ are exerted on two oppositely charged particles moving at the same velocity in a magnetic field.

 Two right-hand rules for determining the direction of the magnetic force acting on a particle with charge q moving with a velocity v in a magnetic field B

 $\mathbf{F}_{B} = q \mathbf{v} \times \mathbf{B}$

First Rule

- The fingers point in the direction of v, with B coming out of your palm.
- 2. So that you can curl your fingers in the direction of **B**.
- 3. The direction of $\mathbf{v} \times \mathbf{B}$, and the force on a positive charge, is the direction in which the thumb points .



Alternative Rule

- 1. The vector **v** is in the direction of your thumb.
- 2. **B** in the direction of your fingers.
- 3. The force \mathbf{F}_B on a positive charge is in the direction of your palm, as if you are pushing the particle with your hand.



(b)

□ The magnetic field is defined in terms of the force acting on a moving charged particle.

Magnitude of the magnetic force on a charged particle moving in a magnetic field

$$F_B = |q| vB \sin \theta$$

where θ is the smaller angle between **v** and **B**.

 F_B is zero when v is parallel or antiparallel to **B** ($\theta = 0$ or 180°) maximum when v is perpendicular to **B** ($\theta = 90^\circ$).

Important differences between electric and magnetic forces:

- The electric force acts along the direction of the electric field, whereas the magnetic force acts perpendicular to the magnetic field.
- The electric force acts on a charged particle regardless of whether the particle is moving,
- whereas the magnetic force acts on a charged particle only when the particle is in motion.
- > The electric force does work in displacing a charged particle,
- whereas the magnetic force associated with a steady magnetic field does no work when a particle is displaced because the force is perpendicular to the displacement.

The kinetic energy of a charged particle moving through a magnetic field cannot be altered by the magnetic field alone



The tesla

The SI unit of magnetic field: tesla (T)

$$1 T = 1 \frac{N}{C \cdot m/s}$$
$$1 T = 1 \frac{N}{A \cdot m}$$

A non-SI magnetic-field unit in common use, called the gauss (G)

$$1 \text{ T} = 10^4 \text{ G}.$$

Quick Quiz 29.1 The north-pole end of a bar magnet is held near a positively charged piece of plastic. Is the plastic (a) attracted, (b) repelled, or (c) unaffected by the magnet?

Quick Quiz 29.2 A charged particle moves with velocity **v** in a magnetic field **B**. The magnetic force on the particle is a maximum when **v** is (a) parallel to **B**, (b) perpendicular to **B**, (c) zero.

Quick Quiz 29.3 An electron moves in the plane of this paper toward the top of the page. A magnetic field is also in the plane of the page and directed toward the right. The direction of the magnetic force on the electron is (a) toward the top of the page, (b) toward the bottom of the page, (c) toward the left edge of the page, (d) toward the right edge of the page, (e) upward out of the page, (f) downward into the page.

Table 29.1

Some Approximate Magnetic Field Magnitudes

Source of Field	Field Magnitude (T)
Strong superconducting laboratory magnet	30
Strong conventional laboratory magnet	2
Medical MRI unit	1.5
Bar magnet	10^{-2}
Surface of the Sun	10^{-2}
Surface of the Earth	$0.5 imes10^{-4}$
Inside human brain (due to nerve impulses)	10^{-13}

Example 29.1 An Electron Moving in a Magnetic Field

An electron in a television picture tube moves toward the front of the tube with a speed of 8.0×10^6 m/s along the *x* axis (Fig. 29.5). Surrounding the neck of the tube are coils of wire that create a magnetic field of magnitude 0.025 T, directed at an angle of 60° to the *x* axis and lying in the *xy* plane.

(A) Calculate the magnetic force on the electron using Equation 29.2.

 $F_B = |q| vB \sin \theta$

= $(1.6 \times 10^{-19} \text{ C})(8.0 \times 10^6 \text{ m/s})(0.025 \text{ T})(\sin 60^\circ)$

 $= 2.8 \times 10^{-14} \,\mathrm{N}$





Selected Solved Problems (Chapter # 29)

7. A proton moving at 4.00 × 10⁶ m/s through a magnetic field of 1.70 T experiences a magnetic force of magnitude 8.20 × 10⁻¹³ N. What is the angle between the proton's velocity and the field?

9. A proton moves with a velocity of v = (2î - 4j + k) m/s in a region in which the magnetic field is B = (î + 2j - 3k) T. What is the magnitude of the magnetic force this charge experiences?

A current-carrying wire also experiences a force when placed in a magnetic field. Similar to magnetic force exerted on a single charged particle when the particle moves through a magnetic field.

Current is a collection of many charged particles in motion; hence, the resultant force exerted by the field on the wire is the vector sum of the individual forces exerted on all the charged particles making up the current.

Notation Notes for \vec{B}

➢ If \vec{B} lies in the plane of the page or is present in a perspective drawing, we use blue vectors or blue field lines with arrowheads.



Cont. Notation Notes for \overline{B}

□ In non-perspective illustrations, magnetic field is depicted perpendicular with directed in or out of the page.

a. A series of *blue dots* represent the tips of arrows coming toward you shows perpendicular and *directed out of the page* magnetic field (\overline{B}_{out}) .

b. Magnetic field (\overline{B}_{in}) is directed perpendicularly into the page, we use *blue crosses*, which represent the feathered tails of arrows fired away from you.



B into page:									
	×	×	×	×	×	×	×		
	×	×	×	×	×	×	×		
	×	×	×	×	×	×	×		
	×	×	×	×	×	×	×		
	×	×	×	×	\times	×	×		
	×	×	×	×	×	×	×		

Magnetic force acting on a current-carrying conductor



Figure 29.7 (a) A wire suspended vertically between the poles of a magnet. (b) The setup shown in part (a) as seen looking at the south pole of the magnet, so that the magnetic field (blue crosses) is directed into the page. When there is no current in the wire, it remains vertical. (c) When the current is upward, the wire deflects to the left. (d) When the current is downward, the wire deflects to the right.

Force on a segment of current-carrying wire in a uniform magnetic field

□ Considering a straight segment of wire of length *L* and cross-sectional area *A*, carrying a current *I* in a uniform magnetic field \vec{B}

☐ The magnetic force exerted on a charge q moving with a drift velocity \vec{V}_d

 $q\mathbf{v}_d \times \mathbf{B}$

□ The total force acting on the wire

$$\mathbf{F}_B = (q\mathbf{v}_d \times \mathbf{B}) nAL$$
$$\mathbf{F}_B = I \mathbf{L} \times \mathbf{B}$$



Figure 29.8 A segment of a current-carrying wire in a magnetic field **B**. The magnetic force exerted on each charge making up the current is $q\mathbf{v}_d \times \mathbf{B}$ and the net force on the segment of length L is $I\mathbf{L} \times \mathbf{B}$.



 \vec{L} is a vector that points in the direction of the current *I* and has a magnitude equal to the length *L* of the segment.

Quick Quiz 29.4 The four wires shown in Figure 29.11 all carry the same current from point *A* to point *B* through the same magnetic field. In all four parts of the figure, the points *A* and *B* are 10 cm apart. Rank the wires according to the magnitude of the magnetic force exerted on them, from greatest to least.



Figure 29.11 (Quick Quiz 29.4) Which wire experiences the greatest magnetic force?

Quick Quiz 29.5 A wire carries current in the plane of this paper toward the top of the page. The wire experiences a magnetic force toward the right edge of the page. The direction of the magnetic field causing this force is (a) in the plane of the page and toward the left edge, (b) in the plane of the page and toward the bottom edge, (c) upward out of the page, (d) downward into the page.

Selected Solved Problems (Chapter # 29)

- 12. A wire carries a steady current of 2.40 A. A straight section of the wire is 0.750 m long and lies along the x axis within a uniform magnetic field, $\mathbf{B} = 1.60\hat{\mathbf{k}}$ T. If the current is in the + x direction, what is the magnetic force on the section of wire?
- 14. A conductor suspended by two flexible wires as shown in Figure P29.14 has a mass per unit length of 0.040 0 kg/m. What current must exist in the conductor in order for the tension in the supporting wires to be zero when the magnetic field is 3.60 T into the page? What is the required direction for the current?



Figure P29.14

- □ Consider the special case of a positively charged particle moving in a uniform magnetic field with the initial velocity vector of the particle perpendicular to the field.
- Assume that the direction of the magnetic field is into the page.
- As the particle changes the direction of its velocity in response to the magnetic force, the magnetic force remains perpendicular to the velocity.
- □ The force is always perpendicular to the velocity, the path of the particle is a circle.
- □ The particle moving in a circle in a plane perpendicular to the magnetic field.
- □ The rotation is counterclockwise for a positive charge. If q were negative, the rotation would be clockwise.



□ Equate this magnetic force to the product of the particle mass and the centripetal acceleration

$$\sum F = ma_c$$

$$F_B = qvB = \frac{mv^2}{r}$$

The radius of the path is
$$r = \frac{mv}{qB}$$

> The angular speed
$$\Theta$$
 of the particle $\omega = \frac{v}{r} = \frac{qB}{m}$

It is called cyclotron frequency because charged particles circulate at this angular frequency in the type of accelerator called a *cyclotron*.

The period of the motion (the time interval the particle requires to complete one revolution).

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

- □ If a charged particle moves in a uniform magnetic field with its velocity at some arbitrary angle with respect to **B**, its path is a helix.
- □ if the field is directed in the *x* direction, there is no component of force in the *x* direction. $a_x = 0$ the *x* component of velocity remains constant.
- □ The magnetic force q v x B causes the components v_y and v_z to change in time, and the resulting motion is a helix whose axis is parallel to the magnetic field.



A charged particle having a velocity vector that has a component parallel to a uniform magnetic field moves in a helical path.

Above equations is still apply provided that *v* is replaced by

$$V_{\perp} = \sqrt{V_y^2 + V_z^2}$$

Quick Quiz 29.8 A charged particle is moving perpendicular to a magnetic field in a circle with a radius r. An identical particle enters the field, with **v** perpendicular to **B**, but with a higher speed v than the first particle. Compared to the radius of the circle for the first particle, the radius of the circle for the second particle is (a) smaller (b) larger (c) equal in size.

Quick Quiz 29.9 A charged particle is moving perpendicular to a magnetic field in a circle with a radius *r*. The magnitude of the magnetic field is increased. Compared to the initial radius of the circular path, the radius of the new path is (a) smaller (b) larger (c) equal in size.

Example 29.6 A Proton Moving Perpendicular to a Uniform Magnetic Field

A proton is moving in a circular orbit of radius 14 cm in a uniform 0.35-T magnetic field perpendicular to the velocity of the proton. Find the linear speed of the proton.

Solution From Equation 29.13, we have

$$v = \frac{qBr}{m_p} = \frac{(1.60 \times 10^{-19} \,\text{C}) \,(0.35 \,\text{T}) \,(0.14 \,\text{m})}{1.67 \times 10^{-27} \,\text{kg}}$$
$$= 4.7 \times 10^6 \,\text{m/s}$$

What If? What if an electron, rather than a proton, moves in a direction perpendicular to the same magnetic field with this same linear speed? Will the radius of its orbit be different?

Answer An electron has a much smaller mass than a proton, so the magnetic force should be able to change its velocity much easier than for the proton. Thus, we should expect the radius to be smaller. Looking at Equation 29.13, we see that r is proportional to m with q, B, and v the same for the electron as for the proton. Consequently, the radius will be smaller by the same factor as the ratio of masses m_e/m_p .

□ Charged particles move in a nonuniform magnetic field, the motion is complex.

□ For example, in a magnetic field that is strong at the ends and weak in the middle.



❑ The particles can oscillate back and forth between two positions. A charged particle starting at one end spirals along the field lines until it reaches the other end, where it reverses its path and spirals back. This configuration is known as a *magnetic bottle* because charged particles can be trapped within it.

- The Van Allen radiation belts consist of charged particles (mostly electrons and protons) surrounding the Earth in doughnut-shaped regions.
- The particles, trapped by the Earth's nonuniform magnetic field, spiral around the field lines from pole to pole.
- Most cosmic rays are deflected by the Earth's magnetic field and never reach the atmosphere. However, some of the particles become trapped; it is these particles that make up the Van Allen belts.
- When the particles are located over the poles, they sometimes collide with atoms in the atmosphere, causing the atoms to emit visible light. Such collisions are the origin of the beautiful Aurora Borealis, or Northern Lights, in the northern hemisphere and the Aurora Australis in the southern hemisphere.





Selected Solved Problems (Chapter # 29)

30. A singly charged positive ion has a mass of 3.20×10^{-26} kg. After being accelerated from rest through a potential difference of 833 V, the ion enters a magnetic field of 0.920 T along a direction perpendicular to the direction of the field. Calculate the radius of the path of the ion in the field.

37. A cosmic-ray proton in interstellar space has an energy of 10.0 MeV and executes a circular orbit having a radius equal to that of Mercury's orbit around the Sun (5.80 \times 10¹⁰ m). What is the magnetic field in that region of space?

□ A charge moving with a velocity \overline{V} in the presence of both an electric field \overline{E} and a magnetic field \overline{B} experiences both an electric force $q\overline{E}$ and a magnetic force $q \overline{V} \times \overline{B}$.

The total force (called the Lorentz force) acting on the charge is

$$\vec{F}_{Total} = \vec{F}_{elec.} + \vec{F}_B$$
$$= q\vec{E} + q\vec{V} \times \vec{B}$$
$$= q(\vec{E} + \vec{V} \times \vec{B})$$

Velocity Selector



When a positively charged particle is moving with velocity \mathbf{v} in the presence of a magnetic field directed into the page and an electric field directed downward, it experiences a downward electric force $q\mathbf{E}$ and an upward magnetic force $q\mathbf{v} \times \mathbf{B}$.

(b) When these forces balance, the particle moves in a horizontal line through the fields.

- ➤ All particles move with the same velocity is achieved by applying a combination of an electric field and a magnetic field.
- A uniform electric field is directed vertically downward (in the plane of the page)
- A uniform magnetic field is applied in the direction perpendicular to the electric field (into the page).
- ➤ If q is positive and the velocity \vec{V} is to the right, the magnetic force $(q \vec{V} \times \vec{B})$ is upward and the electric force $q\vec{E}$ is downward.
- > When the magnitudes of the two fields are chosen so that $q\vec{E} = q \vec{V} \times \vec{B}$, the particle moves in a straight horizontal line through the region of the fields.

From the expression qE = qvB, we find that

 $v = \frac{E}{B}$

- > Only those particles having speed \vec{V} pass undeflected through the perpendicular electric and magnetic fields.
- ➤ The magnetic force exerted on particles moving at speeds greater than this is stronger than the electric force, and the particles are deflected upward.
- > Those moving at speeds less than this are deflected downward.

Selected Solved Problems (Chapter # 29)

41. Singly charged uranium-238 ions are accelerated through a potential difference of 2.00 kV and enter a uniform magnetic field of 1.20 T directed perpendicular to their velocities. (a) Determine the radius of their circular path. (b) Repeat for uranium-235 ions. What If? How does the ratio of these path radii depend on the accelerating voltage and on the magnitude of the magnetic field?



