

# Chapter 28

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## Direct Current Circuits

### CHAPTER OUTLINE

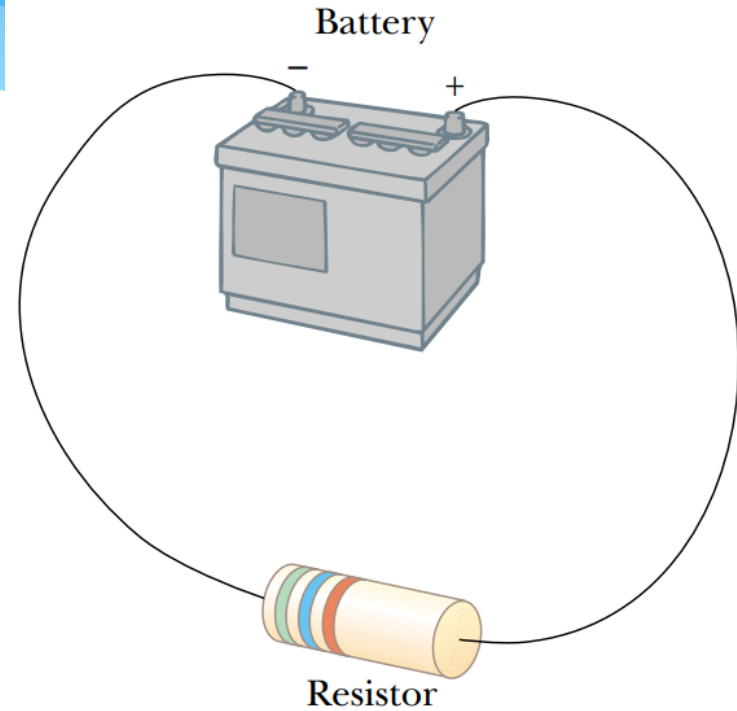
**28.1** Electromotive Force

**28.2** Resistors in Series and  
Parallel

**28.3** Kirchhoff's Rules

# 28.1 Electromotive Force

- Circuits are assumed to be in *steady state* (currents in the circuit are constant in magnitude and direction)
- Potential difference at the battery terminals is constant in a particular circuit, the current in the circuit is constant in magnitude and direction and is called **direct current (DC)**
- A battery is called a *source of electromotive force* or, **a source of emf**
- **The emf of a battery is the maximum possible voltage that the battery can provide between its terminals.**



A circuit consisting of a resistor connected to the terminals of a battery. Assume that the connecting wires have no resistance.

# 28.1 Electromotive Force

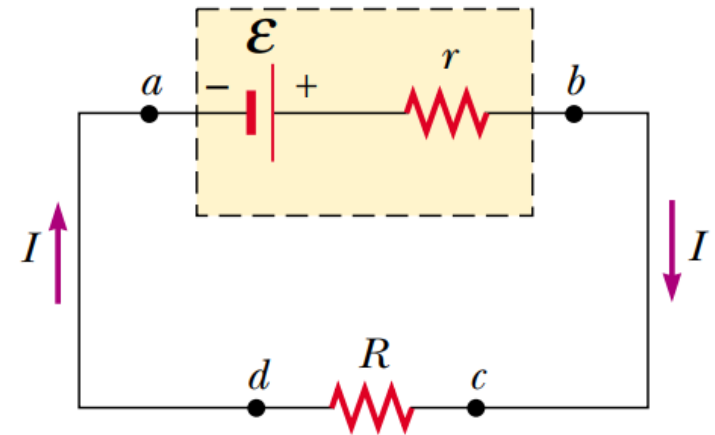
- Real battery is made of matter, there is resistance to the flow of charge within the battery. This resistance is called
- **internal resistance  $r$** .
- For an idealized battery with **zero internal resistance**, the **potential difference** across the battery (called its *terminal voltage*) **equals its emf**.
- For a real battery, **the terminal voltage** is **NOT** equal to the **emf** for a battery in a circuit in which there is a current.

➤ From  $a$  to  $b$

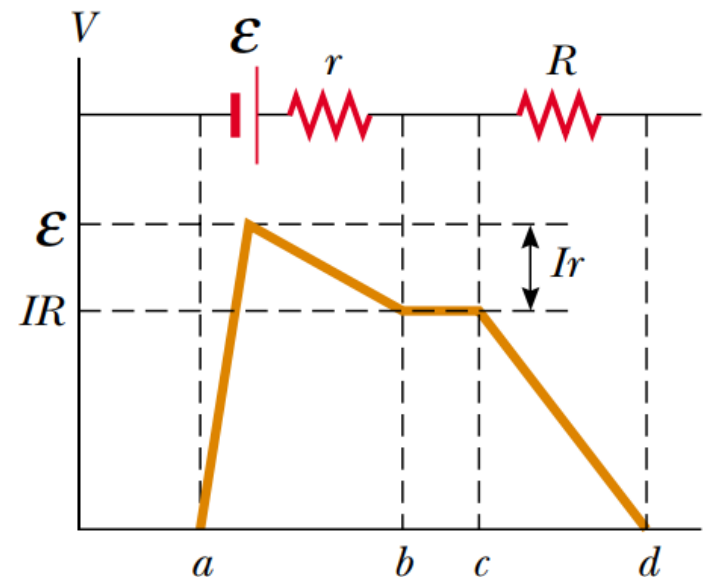
Negative terminal to the positive terminal, the potential *increases* by an amount  $\mathcal{E}$ .

However, as we move through the resistance  $r$ , the potential *decreases* by an amount  $Ir$ , where  $I$  is the current in the circuit.

Thus, the terminal voltage of the battery  $\Delta V = V_b - V_a$  is



(a)



(b)

$$\Delta V = \mathcal{E} - Ir$$

# 28.1 Electromotive Force

$$\Delta V = \mathcal{E} - Ir$$

- $\mathcal{E}$  note that is equivalent to the **open-circuit voltage**—that is, the *terminal voltage when the current is zero*.
- The terminal voltage  $\Delta V$  must equal the potential difference across the external resistance  $R$ , often called the **load resistance**.
- The emf is the voltage labeled on a battery
- The battery must supply energy to operate the device. The potential difference across the load resistance is  $\Delta V = IR$ .

$$\mathcal{E} = IR + Ir$$

$$I = \frac{\mathcal{E}}{R + r}$$

❖ The current in this simple circuit depends on both the load resistance  $R$  external to the battery and the internal resistance  $r$ . If  $R$  is much greater than  $r$ , as it is in many real-world circuits, we can neglect  $r$ .

## 28.1 Electromotive Force

➤ Total power output of the battery is

$$\mathcal{P} = I \Delta V$$

$$I\mathcal{E} = I^2R + I^2r$$

### Quick Quiz 28.1

In order to maximize the percentage of the power that is delivered from a battery to a device, the internal resistance of the battery should be

- (a) as low as possible
- (b) as high as possible
- (c) The percentage does not depend on the internal resistance

## 28.1 Electromotive Force

### Example 28.1 Terminal Voltage of a Battery

A battery has an emf of  $12.0\text{ V}$  and an internal resistance of  $0.05\ \Omega$ . Its terminals are connected to a load resistance of  $3.00\ \Omega$ .

**(A)** Find the current in the circuit and the terminal voltage of the battery.

**(B)** Calculate the power delivered to the load resistor, the power delivered to the internal resistance of the battery, and the power delivered by the battery.

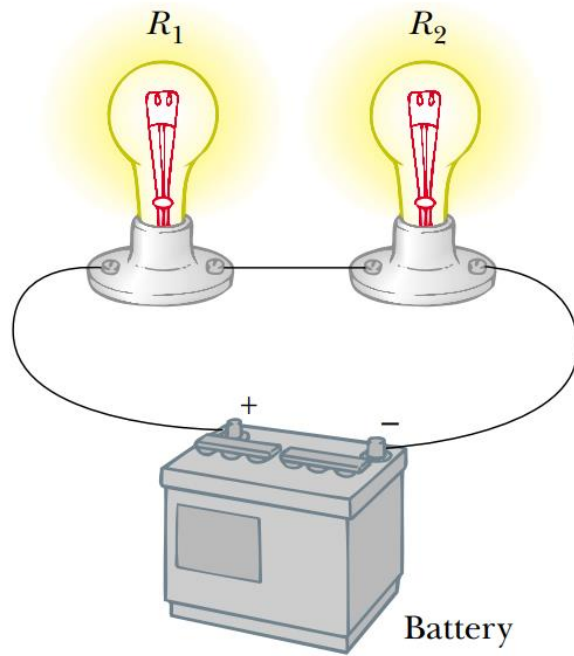
**What If?** As a battery ages, its internal resistance increases. Suppose the internal resistance of this battery rises to  $2.00\ \Omega$  toward the end of its useful life. How does this alter the ability of the battery to deliver energy?

## 28.1 Electromotive Force

### Electromotive Force Problem (Chapter # 28)

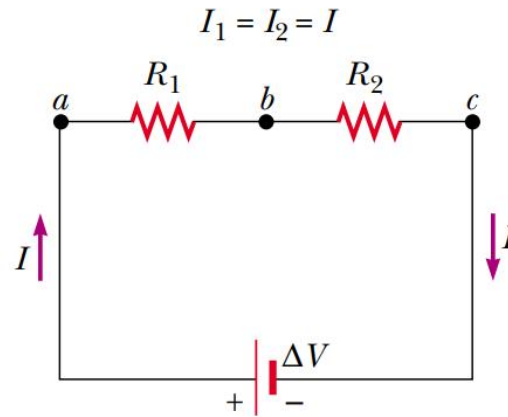
2. (a) What is the current in a  $5.60\text{-}\Omega$  resistor connected to a battery that has a  $0.200\text{-}\Omega$  internal resistance if the terminal voltage of the battery is  $10.0\text{ V}$ ? (b) What is the emf of the battery?

# 28.2 Resistors in Series and Parallel

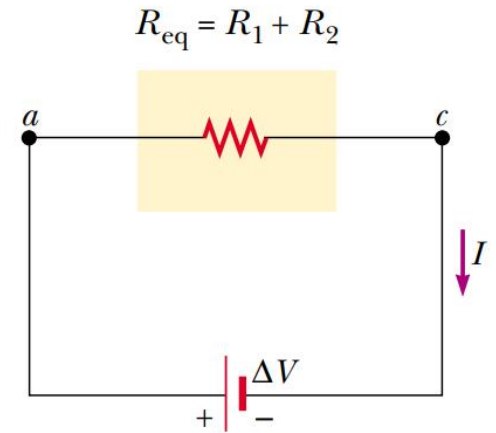


(a)

## ❖ Resistors in Series



(b)



(c)

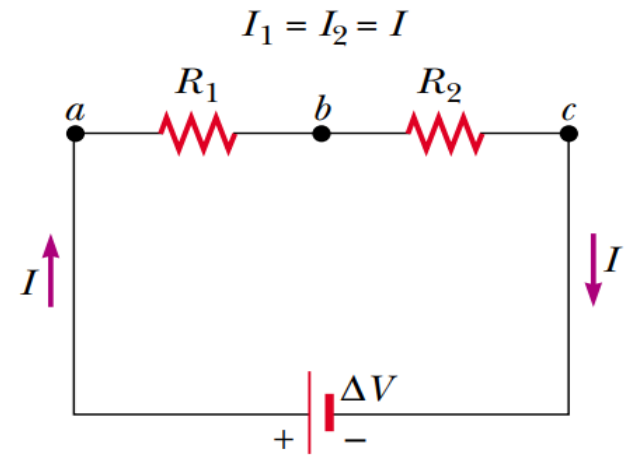
The resistors replaced with a single resistor having an equivalent resistance:  $R_{eq} = R_1 + R_2$ .

for a series combination of two resistors, the currents are the same in both resistors because the amount of charge that passes through  $R_1$  must also pass through  $R_2$  in the same time interval.



## 28.2 Resistors in Series and Parallel

### ❖ Resistors in Series



- ❖ The potential difference applied across the series combination of resistors will divide between the resistors. In Figure 28.4b, because the voltage drop from  $a$  to  $b$  equals  $IR_1$  and the voltage drop from  $b$  to  $c$  equals  $IR_2$ , the voltage drop from  $a$  to  $c$  is

$$\Delta V = IR_1 + IR_2 = I(R_1 + R_2) \qquad \Delta V = IR_{\text{eq}}$$

$$\Delta V = IR_{\text{eq}} = I(R_1 + R_2) \qquad \longrightarrow \qquad R_{\text{eq}} = R_1 + R_2$$

**The equivalent resistance of several resistors in series**

$$R_{\text{eq}} = R_1 + R_2 + R_3 + \cdots$$

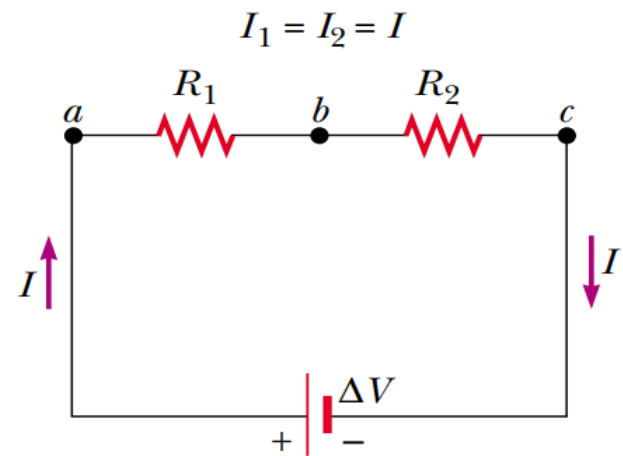
- ❖ The equivalent resistance of a series connection of resistors is the numerical sum of the individual resistances and is always greater than any individual resistance.

## 28.2 Resistors in Series and Parallel

### ❖ Resistors in Series

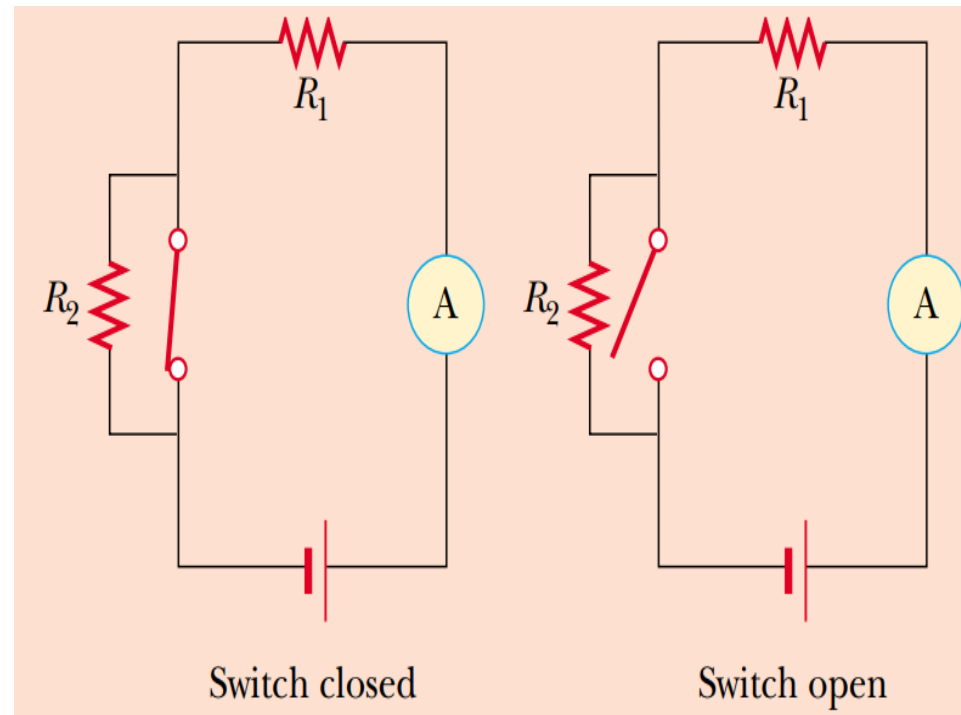
#### Quick Quiz 28.2

In Figure 28.4, imagine positive charges pass first through  $R_1$  and then through  $R_2$ . Compared to the current in  $R_1$ , the current in  $R_2$  is  
(a) smaller, (b) larger, or (c) the same



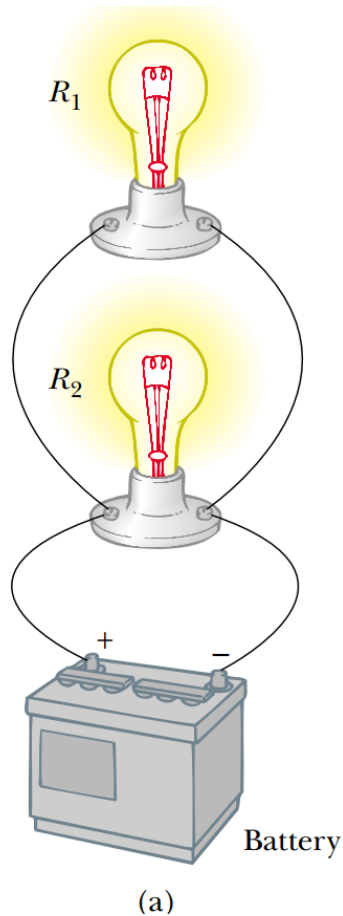
#### Quick Quiz 28.4

What happens to the reading on the ammeter when the switch is opened?  
(a) the reading goes up;  
(b) the reading goes down;  
(c) the reading does not change.

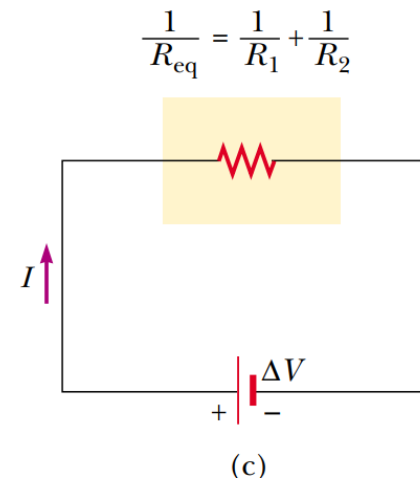
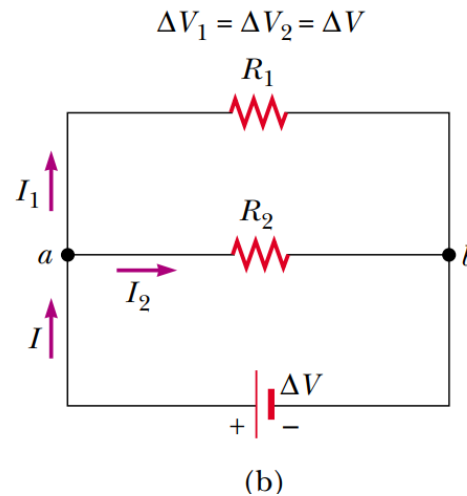


# 28.2 Resistors in Series and Parallel

## ❖ Resistors in Parallel



When charges reach point *a*, (called a *junction*), they split into two parts, with some going through  $R_1$  and the rest going through  $R_2$ . A **junction** is any point in a circuit where a current can split with result of less current in each individual resistor.



when resistors are connected in parallel, the potential differences across the resistors is the same.

# 28.2 Resistors in Series and Parallel

## ❖ Resistors in Parallel

Because the potential differences across the resistors are the same, the expression  $\Delta V = IR$  gives

$$I = I_1 + I_2 = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} = \Delta V \left( \frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{\Delta V}{R_{\text{eq}}}$$

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_{\text{eq}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$

**The equivalent resistance of several resistors in parallel**

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

- ❖ The inverse of the equivalent resistance of two or more resistors connected in parallel is equal to the sum of the inverses of the individual resistances. Furthermore, the equivalent resistance is always less than the smallest resistance in the group.

## 28.2 Resistors in Series and Parallel

### Quick Quiz 28.5

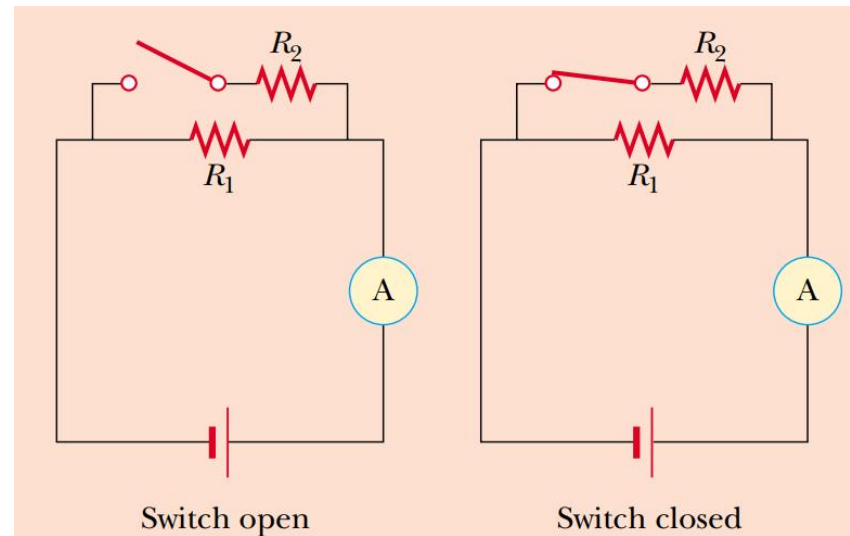
In Figure 28.4, imagine that we add a third resistor in series with the first two. Does the current in the battery (a) increase, (b) decrease, or (c) remain the same?

### Quick Quiz 28.6

In Figure 28.6, imagine that we add a third resistor in parallel with the first two. Does the current in the battery (a) increase, (b) decrease, or (c) remain the same?

### Quick Quiz 28.7

If the switch is closed (Fig. 28.7, right), there is current in  $R_2$ . What happens to the reading on the ammeter when the switch is closed? (a) the reading goes up; (b) the reading goes down; (c) the reading does not change.



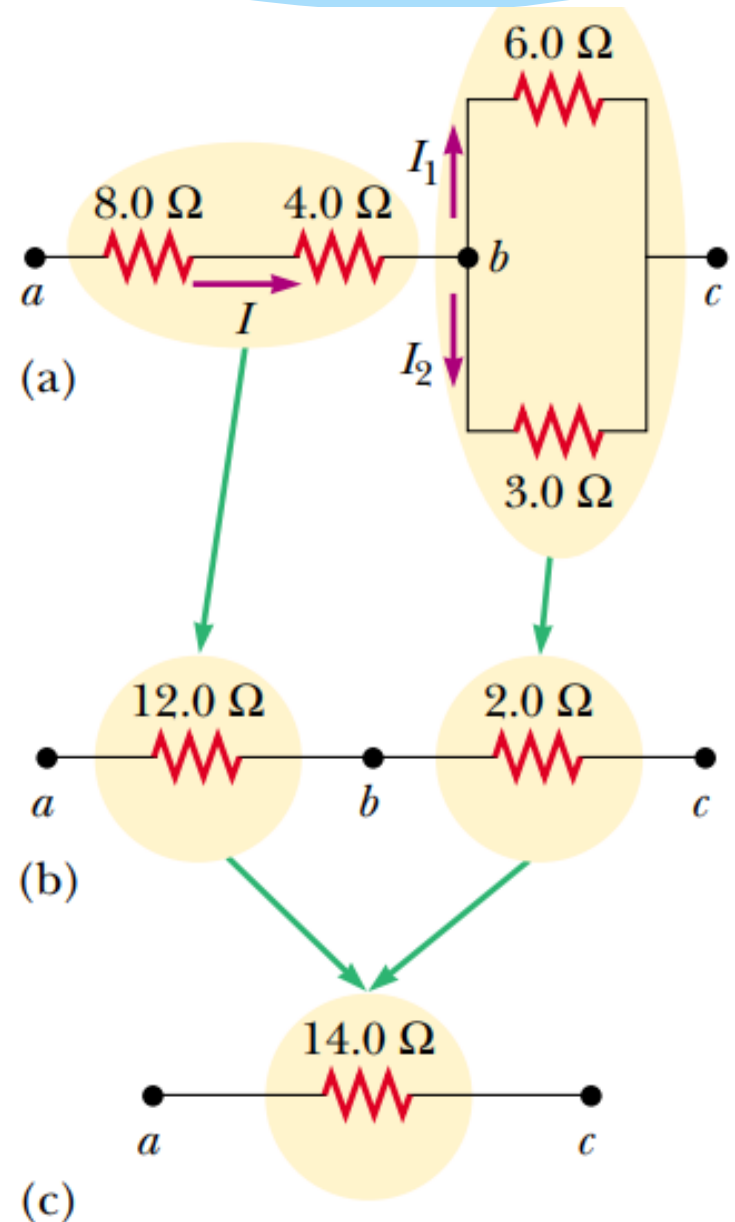
## 28.2 Resistors in Series and Parallel

### Example 28.4 Find the Equivalent Resistance

Four resistors are connected as shown in Figure 28.9a.

(A) Find the equivalent resistance between points  $a$  and  $c$ .

(B) What is the current in each resistor if a potential difference of 42 V is maintained between  $a$  and  $c$ ?



## 28.2 Resistors in Series and Parallel

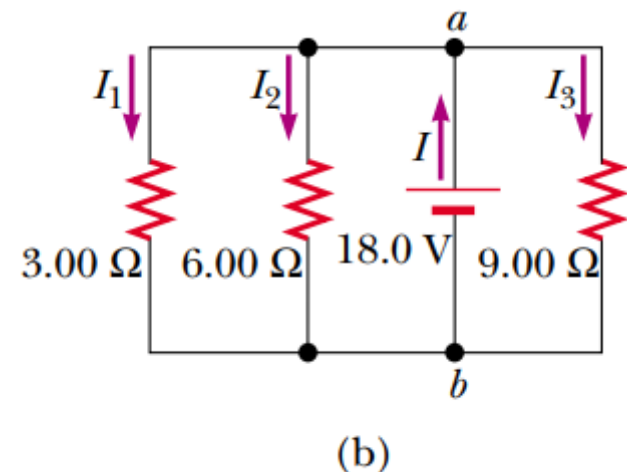
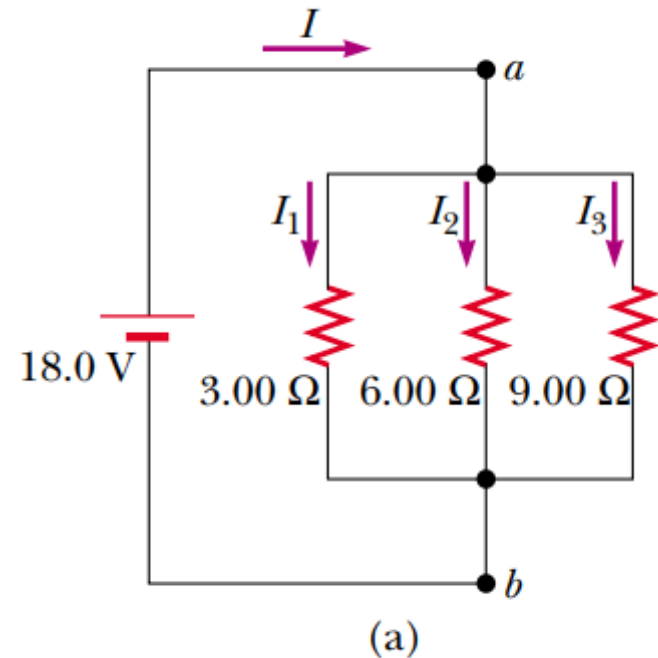
### Example 28.6 Three Resistors in Parallel

Three resistors are connected in parallel as shown in Figure 28.11a. A potential difference of  $18.0\text{ V}$  is maintained between points  $a$  and  $b$ .

**(A)** Find the current in each resistor.

**(B)** Calculate the power delivered to each resistor and the total power delivered to the combination of resistors.

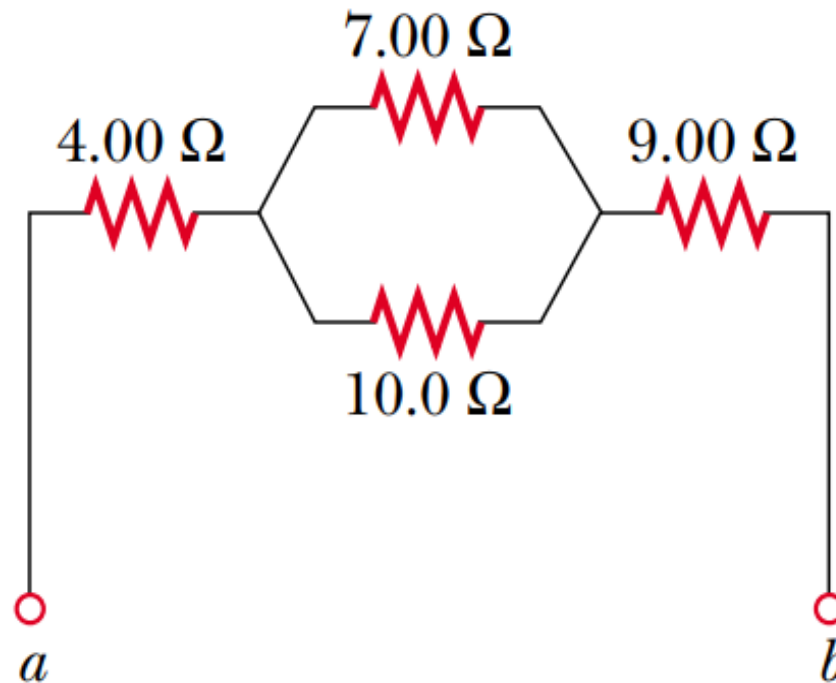
**(C)** Calculate the equivalent resistance of the circuit.



## 28.2 Resistors in Series and Parallel

### Selected Solved Problems (Chapter # 28)

6. (a) Find the equivalent resistance between points  $a$  and  $b$  in Figure P28.6. (b) A potential difference of  $34.0\text{ V}$  is applied between points  $a$  and  $b$ . Calculate the current in each resistor.



**Figure P28.6**



## 28.2 Resistors in Series and Parallel

### Selected Solved Problems (Chapter # 28)

8. Four copper wires of equal length are connected in series. Their cross-sectional areas are  $1.00 \text{ cm}^2$ ,  $2.00 \text{ cm}^2$ ,  $3.00 \text{ cm}^2$ , and  $5.00 \text{ cm}^2$ . A potential difference of  $120 \text{ V}$  is applied across the combination. Determine the voltage across the  $2.00\text{-cm}^2$  wire.

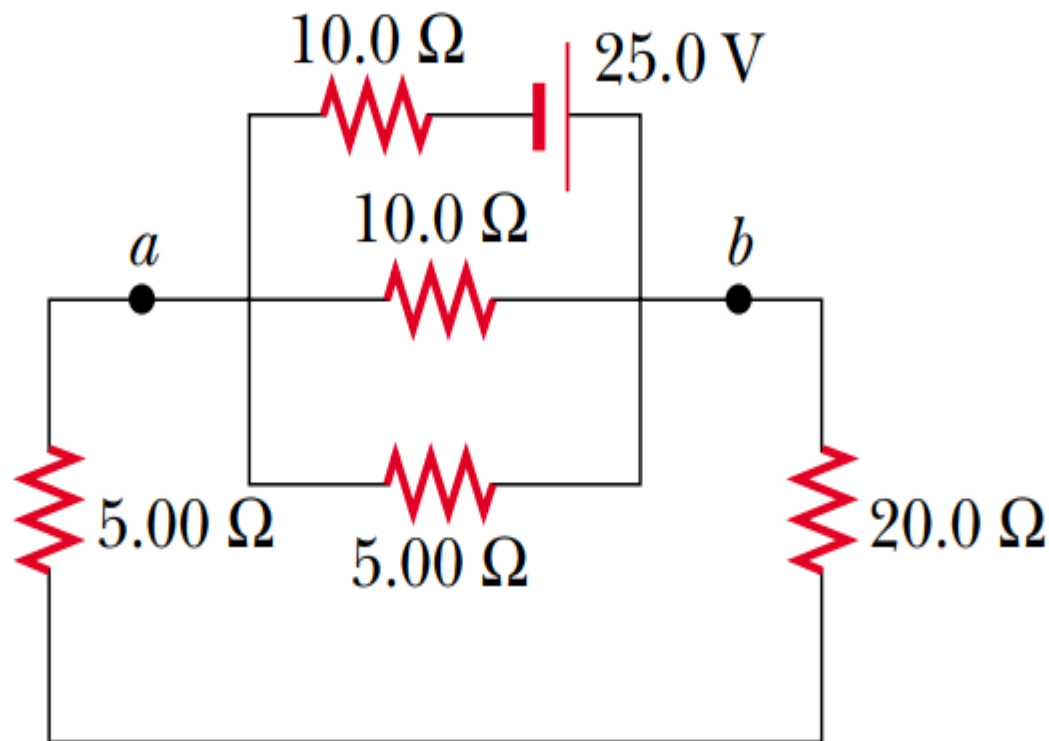
## 28.2 Resistors in Series and Parallel

### Selected Solved Problems (Chapter # 28)



9.

Consider the circuit shown in Figure P28.9. Find (a) the current in the  $20.0\text{-}\Omega$  resistor and (b) the potential difference between points  $a$  and  $b$ .

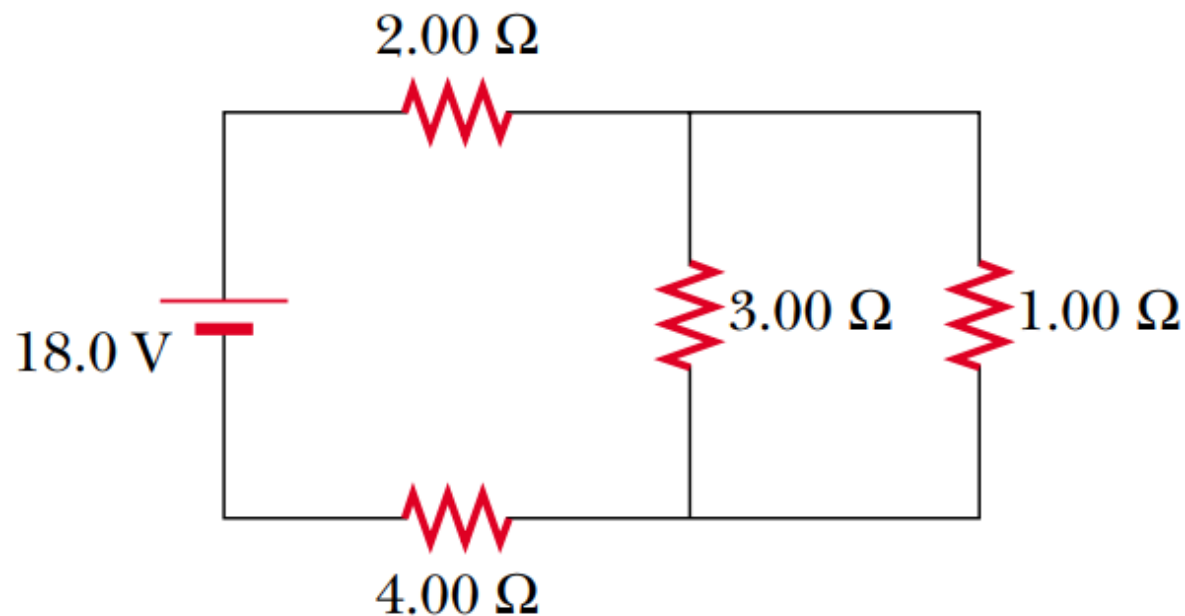


**Figure P28.9**

## 28.2 Resistors in Series and Parallel

### Selected Solved Problems (Chapter # 28)

- 15.** Calculate the power delivered to each resistor in the circuit shown in Figure P28.15.



**Figure P28.15**

## 28.3 Kirchhoff's Rules

❖ *Because it is not often possible to reduce a circuit to a single loop, we use **Kirchhoff's rules** for analyzing more complex circuits.*

1. **Junction rule.** The sum of the currents entering any junction in a circuit must equal the sum of the currents leaving that junction:

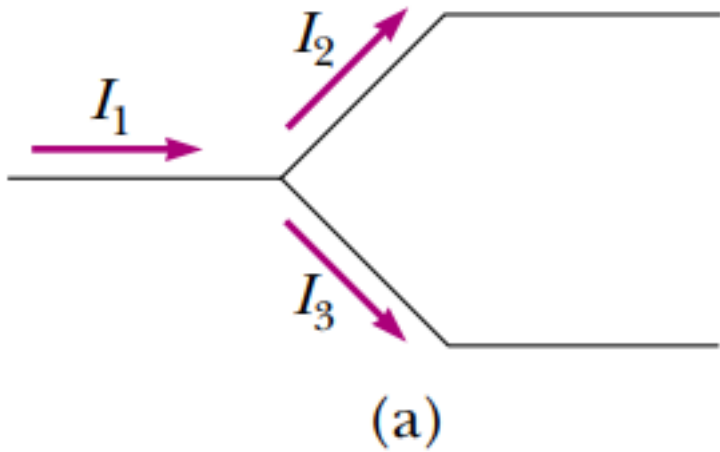
$$\sum I_{\text{in}} = \sum I_{\text{out}} \quad (28.9)$$

2. **Loop rule.** The sum of the potential differences across all elements around any closed circuit loop must be zero:

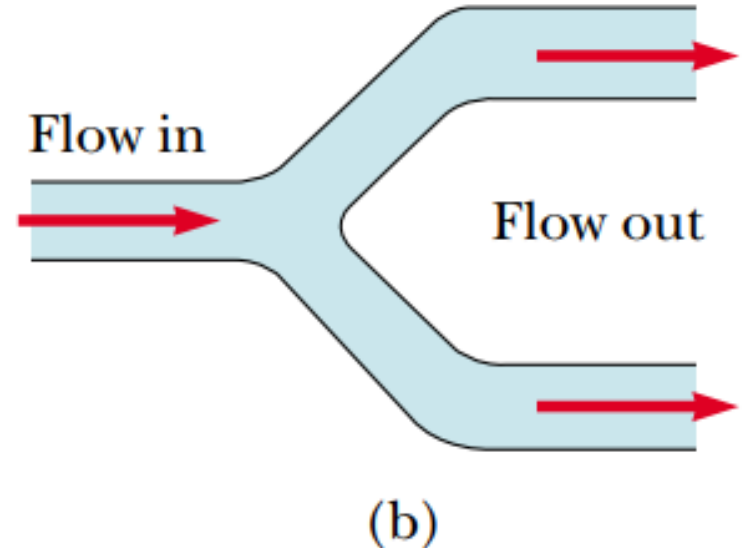
$$\sum_{\text{closed loop}} \Delta V = 0 \quad (28.10)$$

## 28.3 Kirchhoff's Rules

❑ **Junction Rule** (Kirchhoff's first rule is a statement of conservation of electric charge)



(a) Kirchhoff's junction rule. Conservation of charge requires that all charges entering a junction must leave that junction. Therefore,  $I_1 = I_2 + I_3$ .



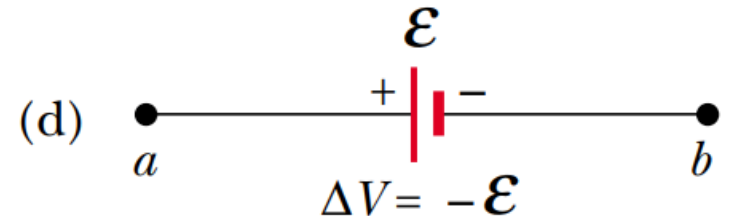
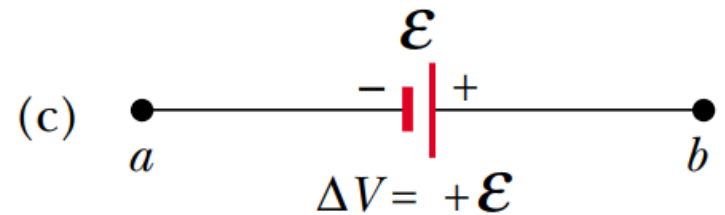
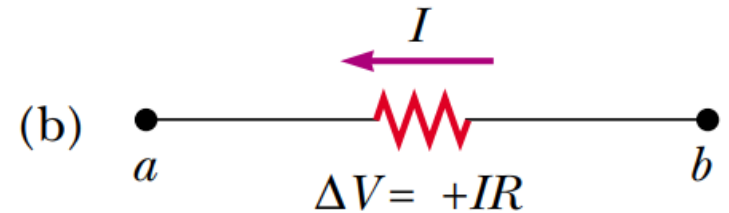
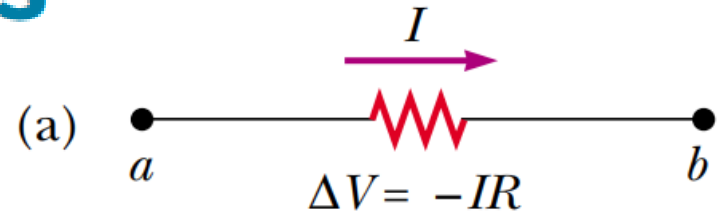
$$I_1 = I_2 + I_3$$

All charges that enter a given point in a circuit must leave that point because charge cannot build up at a point.

## 28.3 Kirchhoff's Rules

### ➤ **Loop Rule** (consider changes in electric potential)

- ❖ charges move from the high-potential end of a resistor toward the low potential end, if a resistor is traversed in the direction of the current, the potential difference  $\Delta V$  across the resistor is  **$-IR$**
- ❖ If a resistor is traversed in the direction *opposite* the current, the potential difference  $\Delta V$  across the resistor is  **$+IR$**
- ❖ If a source of emf (assumed to have zero internal resistance) is traversed in the direction of the emf (from **- to +**), the potential difference  **$\Delta V$  is  $+\mathcal{E}$**
- ❖ If a source of emf (assumed to have zero internal resistance) is traversed in the direction opposite the emf (from **+ to -**), the potential difference  **$\Delta V$  is  $-\mathcal{E}$**



Each circuit element is traversed from left to right.

# 28.3 Kirchhoff's Rules

## PROBLEM-SOLVING HINTS

### Kirchhoff's Rules

- Draw a circuit diagram, and label all the known and unknown quantities. You must assign a *direction* to the current in each branch of the circuit. Although the assignment of current directions is arbitrary, you must adhere rigorously to the assigned directions when applying Kirchhoff's rules.
- Apply the junction rule to any junctions in the circuit that provide new relationships among the various currents.
- Apply the loop rule to as many loops in the circuit as are needed to solve for the unknowns. To apply this rule, you must correctly identify the potential difference as you imagine crossing each element while traversing the closed loop (either clockwise or counterclockwise). Watch out for errors in sign!
- Solve the equations simultaneously for the unknown quantities. Do not be alarmed if a current turns out to be negative; *its magnitude will be correct and the direction is opposite to that which you assigned.*

## 28.3 Kirchhoff's Rules

### Quick Quiz 28.8

In using Kirchhoff's rules, you generally assign a separate unknown current to

- (a) each resistor in the circuit
- (b) each loop in the circuit
- (c) Each branch in the circuit
- (d) each battery in the circuit.

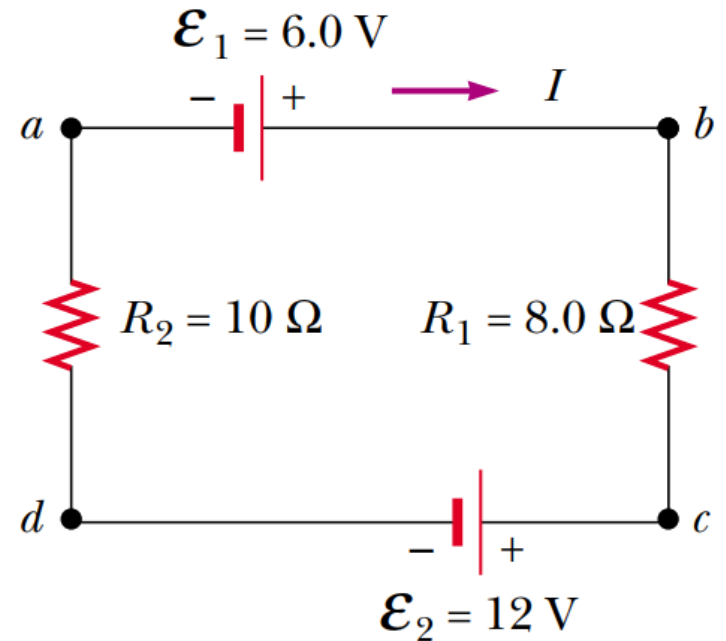


## 28.3 Kirchhoff's Rules

### Example 28.8 A Single-Loop Circuit

A single-loop circuit contains two resistors and two batteries, as shown in Figure 28.16. (Neglect the internal resistances of the batteries.)

- (A) Find the current in the circuit.
- (B) What power is delivered to each resistor? What power is delivered by the 12-V battery?

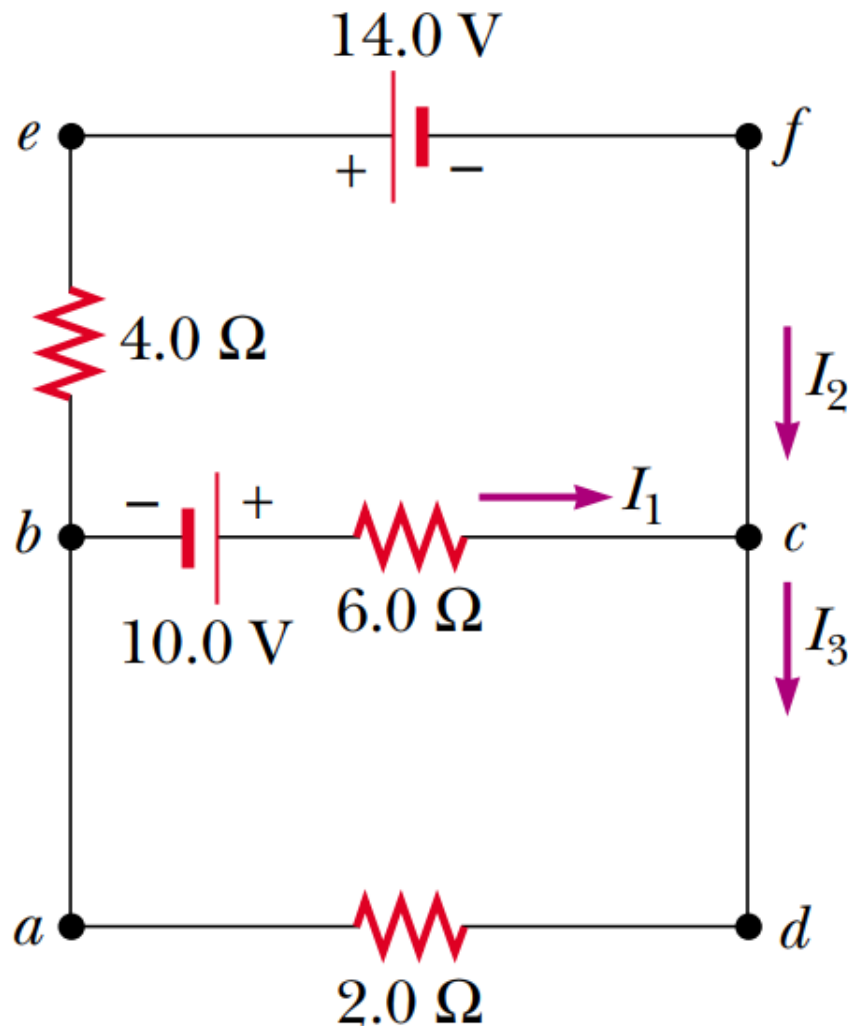


**What If?** What if the polarity of the 12.0-V battery were reversed? How would this affect the circuit?

## 28.3 Kirchhoff's Rules

### Example 28.9 Applying Kirchhoff's Rules

Find the currents  $I_1$ ,  $I_2$ , and  $I_3$  in the circuit shown in Figure 28.17.

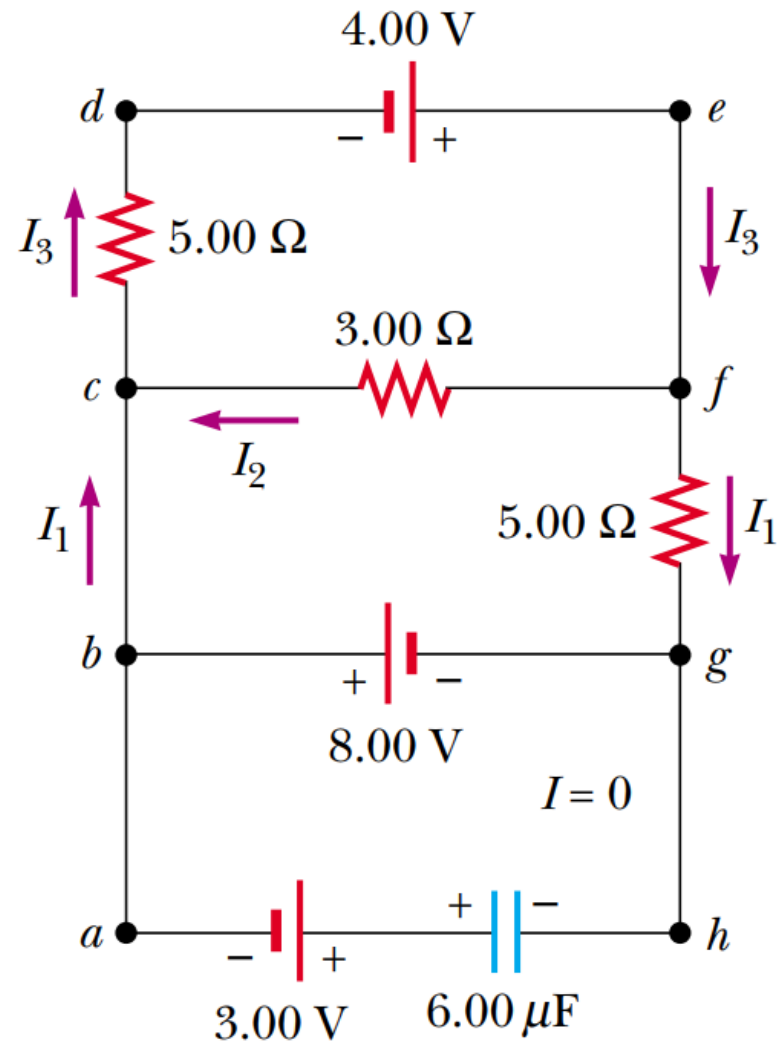


## 28.3 Kirchhoff's Rules

### Example 28.10 A Multiloop Circuit

(A) Under steady-state conditions, find the unknown current  $I_1$ ,  $I_2$ , and  $I_3$  in the multiloop circuit shown in Figure 28.18.

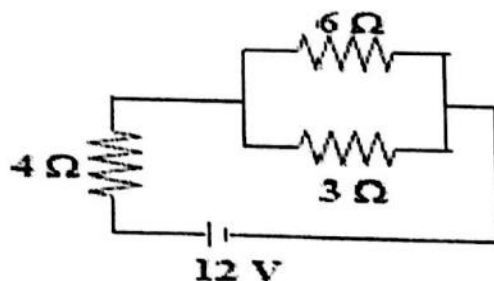
(B) What is the charge on the capacitor?



# Chapter # 28 Problems

## Selected Old exam questions

9. إن التيار المار من المقاومة  $4\ \Omega$  في الدائرة الكهربائية يساوي إلى:



A) 0.92 A

(B) 2 A

C) 3 A

D) 6 A

E) 1 A

س9) في الدائرة المرفقة إذا كان التيار المار بالمقاومة  $3\ \Omega$  يساوي 2A فإن التيار المار بالمقاومة  $5\ \Omega$  يساوي:

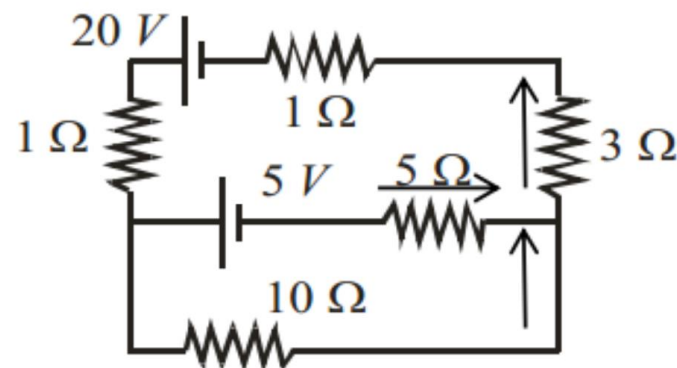
Q9) In the given circuit, if the electric current passes through the resistance  $3\ \Omega$  is 2A, the electric current passes through the resistance  $5\ \Omega$  equals:

A. 6 A

B. 3 A

C. 2 A

D. 1 A

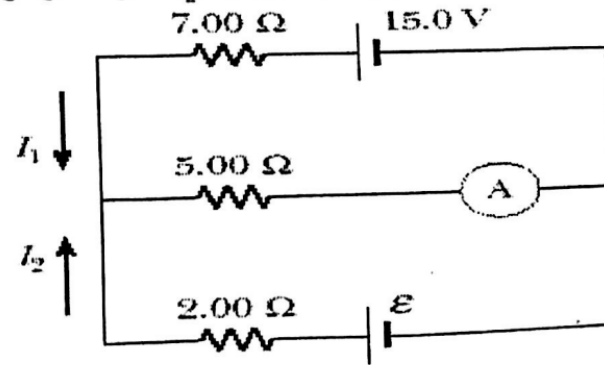


# Chapter # 28 Problems

## Selected Old exam questions

13. The ammeter shown in the circuit reads 2 A. The values of  $I_1$ ,  $I_2$ , and  $\mathcal{E}$ .

13. يقرأ مقياس التيار القيمة 2 A في الدارة المرسومة أدناه. فإن قيمة المقادير  $I_1$ ,  $I_2$ ,  $\mathcal{E}$  هي:



A) 1.286A, 0.612A, 10.9V

B) 0.286A, 0.322A, 12.6V

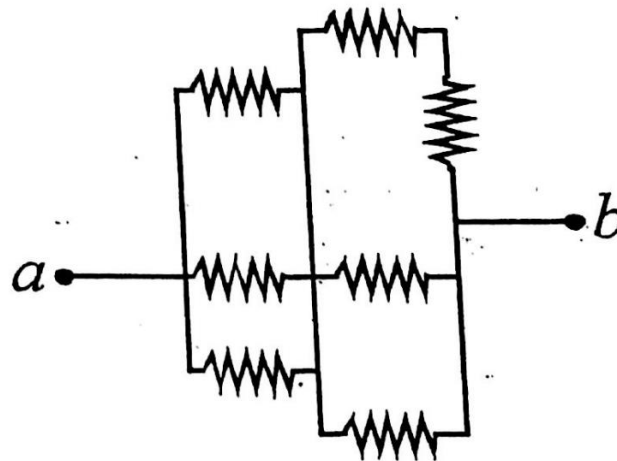
C) 1.006A, 1.004A, 14.6V

☒ D) 0.714A, 1.286A, 12.6V

E) 4.286A, 0.201A, 18.6V

س6- ما مقدار المقاومة المكافئة لمجموعة المقاومات في الرسم بين  $a$  و  $b$  إذا كان مقدار كل مقاومة يساوي  $R$ ؟

Q6- What is the equivalent resistance of the resistors, in the circuit, between  $a$  and  $b$  if each resistance is  $1 \Omega$ ?



A) 1.08

☒ B) 0.73

C) 7

D) 0.143