Chapter 27

Current and Resistance

CHAPTER OUTLINE

Electric charges that are *not* in equilibrium.

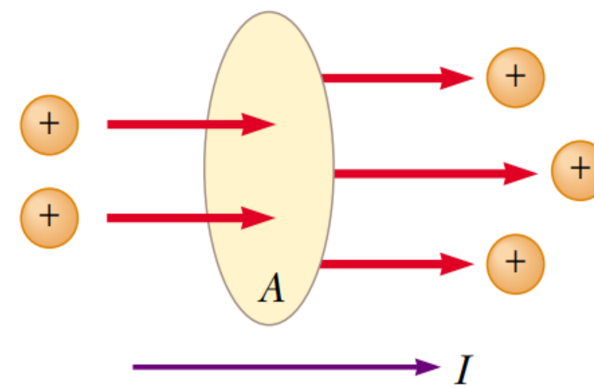
Not electrostatics

27.1 Electric Current

- Study the flow of electric charges through a piece of material depends on the material through which the charges are passing and the potential difference across the material.

Whenever there is a net flow of charge through some region, an electric current is said to exist.

- **The current (I) is the rate at which charge flows through this surface.**



$$I_{av} = \frac{\Delta Q}{\Delta t}$$

average current (I_{av}**)** is equal to the charge that passes through *A* per unit time

Charges in motion through an area *A*. The time rate at which charge flows through the area is defined as the current *I*. The direction of the current is the direction in which positive charges flow when free to do so.

$$I \equiv \frac{dQ}{dt}$$

instantaneous current *(I)* ***as the differential limit of average current.***

Charges passing through the surface can be positive or negative, or both.

27.1 Electric Current

The SI unit of current is the **ampere** (A):

$$1\ \text{A} = \frac{1\ \text{C}}{1\ \text{s}}$$

1 A of current is equivalent to 1 C of charge passing through the surface area in 1 s.

- **It is conventional to assign to the current the same direction as the flow of positive charge.**

- In electrical conductors, such as copper or aluminum, the current is due to the motion of negatively charged electrons.

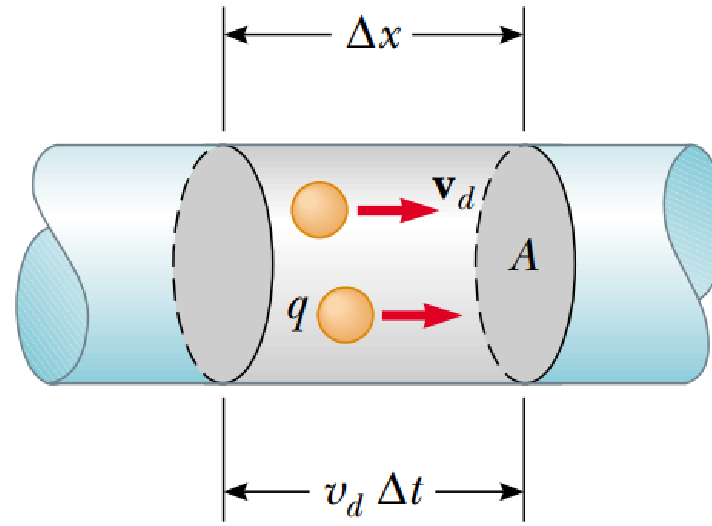
- In an ordinary conductor, **the direction of the current is opposite the direction of flow of electrons.**

- Moving charge (positive or negative) as a mobile **charge carrier.** For example, the mobile charge carriers in a metal are electrons.

27.1 Electric Current

Microscopic Model of Current

❖ The volume of a section of the conductor of length region is $A\,\Delta x$. n represents the number of mobile charge carriers per unit volume (the charge carrier density)



$$\Delta Q = \text{number of carriers in section} \times \text{charge per carrier} = (nA\,\Delta x)\,q$$

$$\Delta Q = (nAv_d\,\Delta t)\,q$$

Current in a conductor in terms of microscopic quantities

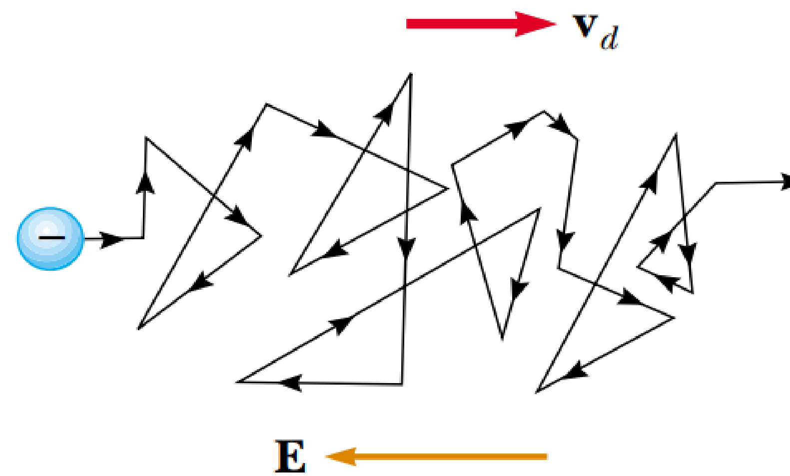
$$I_{\text{av}} = \frac{\Delta Q}{\Delta t} = nqv_dA$$

❖ The speed of the charge carriers v_d is an average speed called the **drift speed.**

27.1 Electric Current

Microscopic Model of Current

- When a potential difference is applied across the conductor (for example, by means of a battery), an electric field is set up in the conductor; this field exerts an electric force on the electrons, producing a current.



Note that the net motion of the electron is opposite the direction of the electric field

- The electrons do not move in straight lines along the conductor. Instead, they collide repeatedly with the metal atoms, and their resultant motion is complicated and zigzag.

- Energy transferred from the electrons to the metal atoms during collisions causes an increase in the vibrational energy of the atoms and a corresponding increase in the temperature of the conductor.

27.1 Electric Current

Quick Quiz 27.1 Consider positive and negative charges moving horizontally through the four regions shown in Figure 27.4. Rank the current in these four regions, from lowest to highest.

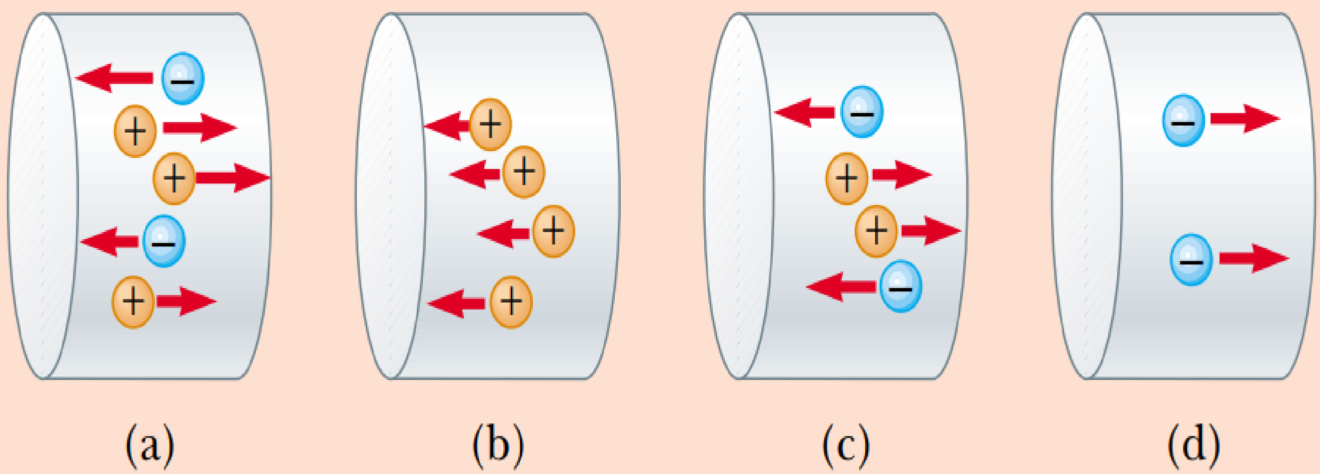


Figure 27.4 (Quick Quiz 27.1) Charges move through four regions.

Quick Quiz 27.2 Electric charge is conserved. As a consequence, when current arrives at a junction of wires, the charges can take either of two paths out of the junction and the numerical sum of the currents in the two paths equals the current that entered the junction. Thus, current is (a) a vector (b) a scalar (c) neither a vector nor a scalar.

Microscopic Model of Current

Example 27.1 Drift Speed in a Copper Wire

The 12-gauge copper wire in a typical residential building has a cross-sectional area of $3.31 \times 10^{-6}\ \text{m}^2$. If it carries a current of 10.0 A, what is the drift speed of the electrons? Assume that each copper atom contributes one free electron to the current. The density of copper is $8.95\ \text{g/cm}^3$.

Solution From the periodic table of the elements in Appendix C, we find that the molar mass of copper is 63.5 g/mol. Recall that 1 mol of any substance contains Avogadro's number of atoms (6.02×10^{23}). Knowing the density of copper, we can calculate the volume occupied by 63.5 g (= 1 mol) of copper:

$$V = \frac{m}{\rho} = \frac{63.5\ \text{g}}{8.95\ \text{g/cm}^3} = 7.09\ \text{cm}^3$$

Because each copper atom contributes one free electron to the current, we have

$$n = \frac{6.02 \times 10^{23}\ \text{electrons}}{7.09\ \text{cm}^3}\left(\frac{1.00 \times 10^6\ \text{cm}^3}{1\ \text{m}^3}\right)$$

$$= 8.49 \times 10^{28}\ \text{electrons/m}^3$$

From Equation 27.4, we find that the drift speed is

$$v_d = \frac{I}{nqA}$$

where q is the absolute value of the charge on each electron. Thus,

$$v_d = \frac{I}{nqA}$$

$$= \frac{10.0\ \text{C/s}}{(8.49 \times 10^{28}\ \text{m}^{-3})(1.60 \times 10^{-19}\ \text{C})(3.31 \times 10^{-6}\ \text{m}^2)}$$

$$= 2.22 \times 10^{-4}\ \text{m/s}$$

This example shows that typical drift speeds are very low. For instance, electrons traveling with a speed of 2.22×10^{-4} m/s would take about 75 min to travel 1 m.

Electrons do not have to travel from the light switch to the light in order for the light to operate. Electrons already in the filament of the lightbulb move in response to the electric field set up by the battery. Notice also that a battery does not provide electrons to the circuit. It establishes the electric field that exerts a force on electrons already in the wires and elements of the circuit.

27.1 Electric Current

Selected Solved Problems (Chapter # 27)

1. In a particular cathode ray tube, the measured beam current is 30.0 μA. How many electrons strike the tube screen every 40.0 s?

11. An aluminum wire having a cross-sectional area of 4.00×10^{-6} m^2 carries a current of 5.00 A. Find the drift speed of the electrons in the wire. The density of aluminum is 2.70 g/cm^3. Assume that one conduction electron is supplied by each atom.

27.2 Resistance

❖ Consider a conductor of cross-sectional area *A* carrying a current *I*. The **current density** *J* in the conductor is defined as the current per unit area.

$$J \equiv \frac{I}{A} = nqv_d$$

where *J* has SI units of A/m^2.

❑ Current density is a vector quantity:

$$\mathbf{J} = nq\mathbf{v}_d$$

J direction is opposite the direction of motion for negative charge carriers.

A current density J and an electric field E are established in a conductor whenever a potential difference is maintained across the conductor.

In some materials, the current density is proportional to the electric field:

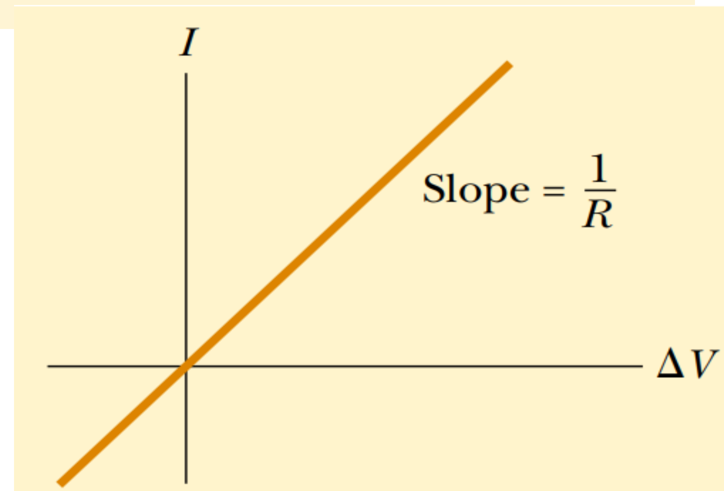
$$\mathbf{J} = \sigma\mathbf{E}$$

The constant of proportionality σ is called the **conductivity** of the conductor.

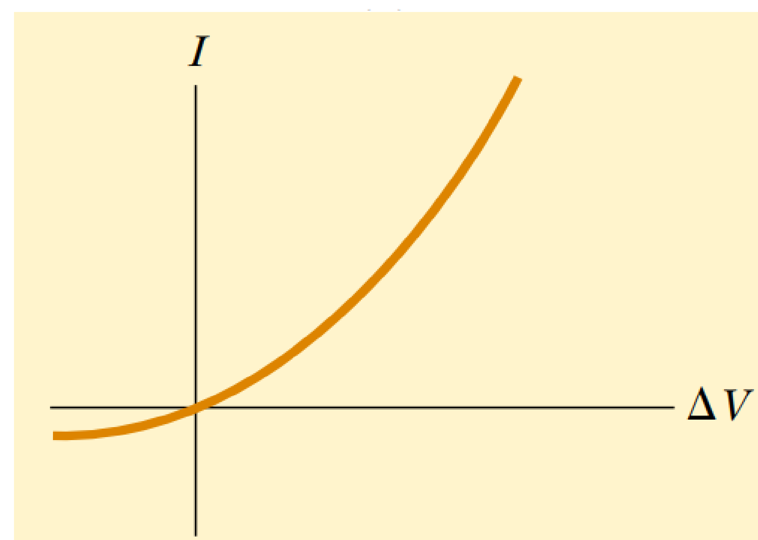
27.2 Resistance

Ohm's law states that

for many materials (including most metals), the ratio of the current density to the electric field is a constant σ that is independent of the electric field producing the current.

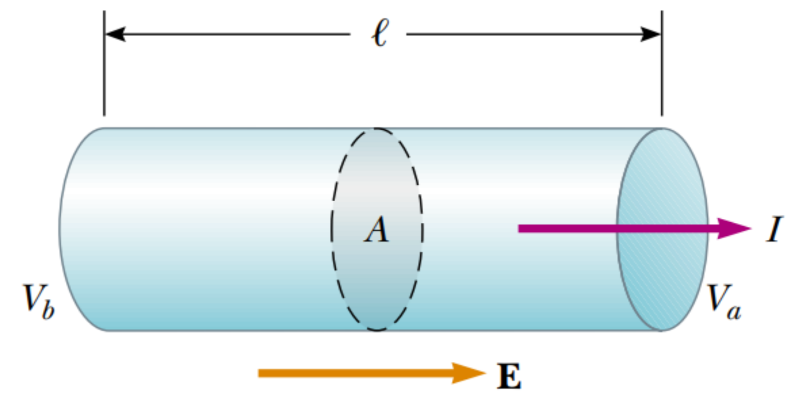


❖ Materials that obey Ohm's law and hence demonstrate this simple relationship between **E** and **J** are said to be ***ohmic***.



❖ Materials and devices that do not obey Ohm's law are said to be ***nonohmic.***

27.2 Resistance



- Consider a segment of straight wire of uniform cross-sectional area A and length l,

- A potential difference $\Delta V = V_b - V_a$ is maintained across the wire, creating in the wire an electric field and a current.

$$\Delta V = E\ell \qquad J = \sigma E = \sigma \frac{\Delta V}{\ell}$$

$$\Delta V = \frac{\ell}{\sigma} J = \left(\frac{\ell}{\sigma A}\right) I = RI \qquad J = I/A$$

The quantity $R = l/\sigma A$ is called the **resistance** of the conductor

$$R \equiv \frac{\Delta V}{I}$$

- Resistance has SI units of volts per ampere. If a potential difference of 1 V across a conductor causes a current of 1 A, the resistance of the conductor is 1 Ω .

$$1\ \Omega \equiv \frac{1\ \text{V}}{1\ \text{A}}$$

27.2 Resistance

$$\rho = \frac{1}{\sigma}$$

❖ The inverse of conductivity is **resistivity ρ**

❖ **ρ** has the units ohm-meters (Ω. m)

Because $R = l/\sigma A$

$$R = \rho \frac{\ell}{A}$$

Resistance of wire is proportional to its length and inversely proportional to its cross-sectional area

❖ Resistivity is property of a *substance*,

❖ Resistance is a property of an *object*.

Resistivities and Temperature Coefficients of Resistivity for Various Materials

Material	Resistivity[a] $(\Omega \cdot m)$	Temperature Coefficient[b] $\alpha[(°C)^{-1}]$
Silver	1.59×10^{-8}	3.8×10^{-3}
Copper	1.7×10^{-8}	3.9×10^{-3}
Gold	2.44×10^{-8}	3.4×10^{-3}
Aluminum	2.82×10^{-8}	3.9×10^{-3}
Tungsten	5.6×10^{-8}	4.5×10^{-3}
Iron	10×10^{-8}	5.0×10^{-3}
Platinum	11×10^{-8}	3.92×10^{-3}
Lead	22×10^{-8}	3.9×10^{-3}
Nichrome[c]	1.50×10^{-6}	0.4×10^{-3}
Carbon	3.5×10^{-5}	-0.5×10^{-3}
Germanium	0.46	-48×10^{-3}
Silicon	640	-75×10^{-3}
Glass	10^{10} to 10^{14}	
Hard rubber	$\sim 10^{13}$	
Sulfur	10^{15}	
Quartz (fused)	75×10^{16}	

27.2 Resistance

Quick Quiz 27.3 Suppose that a current-carrying ohmic metal wire has a cross-sectional area that gradually becomes smaller from one end of the wire to the other. The current must have the same value in each section of the wire so that charge does not accumulate at any one point. How do the drift velocity and the resistance per unit length vary along the wire as the area becomes smaller? (a) The drift velocity and resistance both increase. (b) The drift velocity and resistance both decrease. (c) The drift velocity increases and the resistance decreases. (d) The drift velocity decreases and the resistance increases.

Quick Quiz 27.4 A cylindrical wire has a radius r and length ℓ. If both r and ℓ are doubled, the resistance of the wire (a) increases (b) decreases (c) remains the same.

Quick Quiz 27.5 In Figure 27.7b, as the applied voltage increases, the resistance of the diode (a) increases (b) decreases (c) remains the same.

27.2 Resistance

Example 27.2 The Resistance of a Conductor

Calculate the resistance of an aluminum cylinder that has a length of 10.0 cm and a cross-sectional area of $2.00 \times 10^{-4}\ \text{m}^2$. Repeat the calculation for a cylinder of the same dimensions and made of glass having a resistivity of $3.0 \times 10^{10}\ \Omega \cdot \text{m}$.

Solution From Equation 27.11 and Table 27.1, we can calculate the resistance of the aluminum cylinder as follows:

$$R = \rho \frac{\ell}{A} = (2.82 \times 10^{-8}\ \Omega \cdot \text{m}) \left(\frac{0.100\ \text{m}}{2.00 \times 10^{-4}\ \text{m}^2} \right)$$

$$= \boxed{1.41 \times 10^{-5}\ \Omega}$$

Similarly, for glass we find that

$$R = \rho \frac{\ell}{A} = (3.0 \times 10^{10}\ \Omega \cdot \text{m}) \left(\frac{0.100\ \text{m}}{2.00 \times 10^{-4}\ \text{m}^2} \right)$$

$$= \boxed{1.5 \times 10^{13}\ \Omega}$$

As you might guess from the large difference in resistivities, the resistances of identically shaped cylinders of aluminum and glass differ widely. The resistance of the glass cylinder is 18 orders of magnitude greater than that of the aluminum cylinder.

27.2 Resistance

Example 27.3 The Resistance of Nichrome Wire Interactive

(A) Calculate the resistance per unit length of a 22-gauge Nichrome wire, which has a radius of 0.321 mm.

Solution The cross-sectional area of this wire is

$$A = \pi r^2 = \pi(0.321 \times 10^{-3}\ \text{m})^2 = 3.24 \times 10^{-7}\ \text{m}^2$$

The resistivity of Nichrome is $1.5 \times 10^{-6}\ \Omega \cdot \text{m}$ (see Table 27.1). Thus, we can use Equation 27.11 to find the resistance per unit length:

$$\frac{R}{\ell} = \frac{\rho}{A} = \frac{1.5 \times 10^{-6}\ \Omega \cdot \text{m}}{3.24 \times 10^{-7}\ \text{m}^2} = \boxed{4.6\ \Omega/\text{m}}$$

(B) If a potential difference of 10 V is maintained across a 1.0-m length of the Nichrome wire, what is the current in the wire?

Solution Because a 1.0-m length of this wire has a resistance of 4.6 Ω, Equation 27.8 gives

$$I = \frac{\Delta V}{R} = \frac{10\ \text{V}}{4.6\ \Omega} = \boxed{2.2\ \text{A}}$$

Note from Table 27.1 that the resistivity of Nichrome wire is about 100 times that of copper. A copper wire of the same radius would have a resistance per unit length of only 0.052 Ω/m. A 1.0-m length of copper wire of the same radius would carry the same current (2.2 A) with an applied potential difference of only 0.11 V.

Because of its high resistivity and its resistance to oxidation, Nichrome is often used for heating elements in toasters, irons, and electric heaters.

27.2 Resistance

Selected Solved Problems (Chapter # 27)

12. Calculate the current density in a gold wire at 20°C, if an electric field of 0.740 V/m exists in the wire.

15. A 0.900-V potential difference is maintained across a 1.50-m length of tungsten wire that has a cross-sectional area of 0.600 mm^2. What is the current in the wire?

16. A conductor of uniform radius 1.20 cm carries a current of 3.00 A produced by an electric field of 120 V/m. What is the resistivity of the material?

22. Aluminum and copper wires of equal length are found to have the same resistance. What is the ratio of their radii?

27.4 Resistance and Temperature

Variation of ρ with temperature

❑ Resistivity of a conductor varies approximately linearly with temperature

$$\rho = \rho_0[1 + \alpha(T - T_0)]$$

ρ is the resistivity at some temperature T (in degrees Celsius)
ρ_0 is the resistivity at some reference temperature T_0 (usually taken to be 20°C)
α is the **temperature coefficient of resistivity.** (the rate of change of Resistivity per degree change in the temperature from a substance's original temperature.)

Temperature coefficient of resistivity

$$\alpha = \frac{1}{\rho_0}\frac{\Delta\rho}{\Delta T}$$

$\Delta\rho = \rho - \rho_0$ is the change in resistivity in the temperature interval $\Delta T = T - T_0$.

unit for α is degrees Celsius^{-1} $[(°C)^{-1}]$.

27.4 Resistance and Temperature

- Resistance is proportional to resistivity (Eq. 27.11)

$$R = R_0[1 + \alpha(T - T_0)]$$

Quick Quiz 27.6 When does a lightbulb carry more current: (a) just after it is turned on and the glow of the metal filament is increasing, or (b) after it has been on for a few milliseconds and the glow is steady?

Example 27.6 A Platinum Resistance Thermometer

A resistance thermometer, which measures temperature by measuring the change in resistance of a conductor, is made from platinum and has a resistance of 50.0 Ω at 20.0°C. When immersed in a vessel containing melting indium, its resistance increases to 76.8 Ω. Calculate the melting point of the indium.

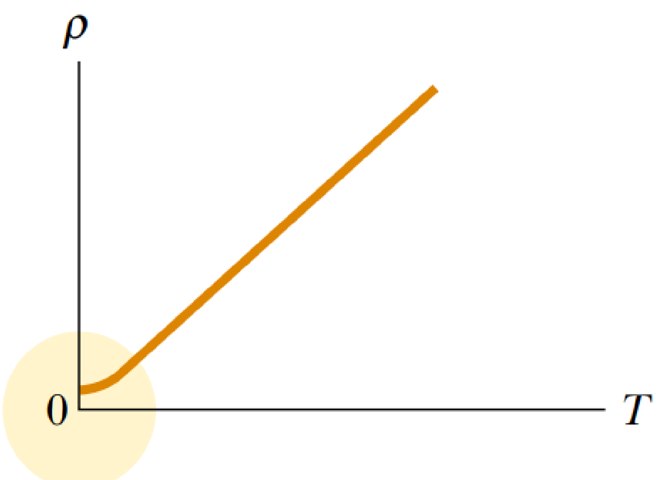
Solution Solving Equation 27.21 for ΔT and using the α value for platinum given in Table 27.1, we obtain

$$\Delta T = \frac{R - R_0}{\alpha R_0} = \frac{76.8\ \Omega - 50.0\ \Omega}{[3.92 \times 10^{-3}(°\mathrm{C})^{-1}](50.0\ \Omega)} = 137°\mathrm{C}$$

Because $T_0 = 20.0°\mathrm{C}$, we find that T, the temperature of the melting indium sample, is 157°C.

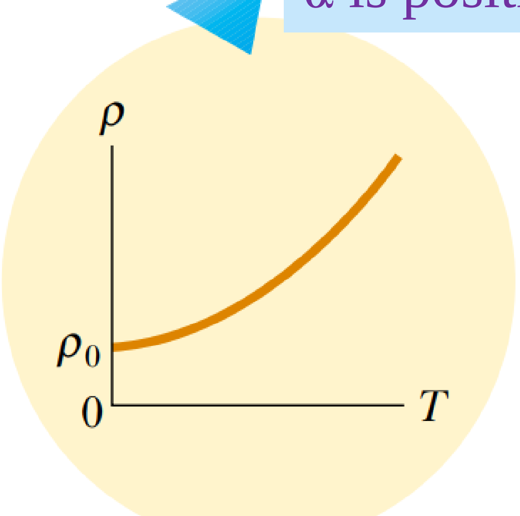
27.4 Resistance and Temperature

❑ Resistivity versus temperature for a metals and semiconductors

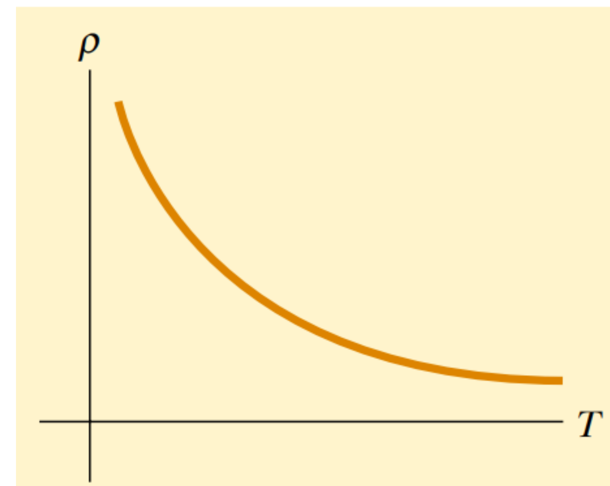


Metals (copper), resistivity is nearly proportional to temperature.

α is positive



Resistivity usually reaches some finite value as the temperature approaches absolute zero.



Pure semiconductor (silicon or germanium), resistivity decreases with increasing temperature.

α is negative

27.4 Resistance and Temperature

Selected Solved Problems (Chapter # 27)

33. What is the fractional change in the resistance of an iron filament when its temperature changes from 25.0°C to 50.0°C?

27.6 Electrical Power

❖To calculate the rate of energy transfer

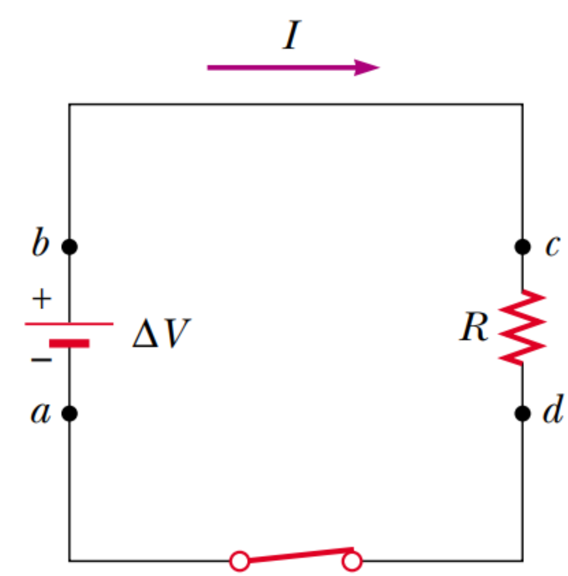
- Battery → Conductor → Electric current → transformation of chemical energy in the battery to kinetic energy of the electrons → Internal energy in the conductor → increase in the temperature of the conductor

- Imagine following a positive quantity of charge Q that is moving clockwise around the circuit.

- As the charge moves from a to b through the battery, the electric potential energy of the system *increases* by an amount $Q\Delta V$ while the chemical potential energy in the battery *decreases* by the same amount

- The charge moves from c to d through the resistor, the system *loses* this electric potential energy during collisions of electrons with atoms in the resistor.

- In this process, the energy is transformed to internal energy corresponding to increased vibrational motion of the atoms in the resistor.



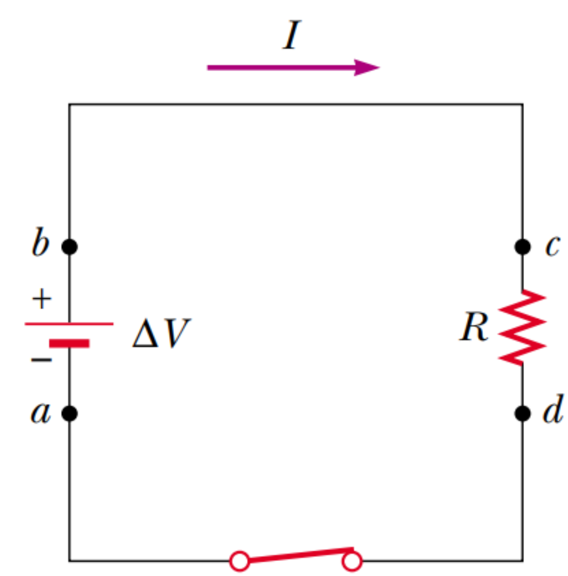
A circuit consisting of a resistor of resistance R and a battery having a potential difference ΔV across its terminals. Positive charge flows in the clockwise direction.

27.6 Electrical Power

❖ The rate at which the system loses electric potential energy as the charge *Q* passes through the resistor:

$$\frac{dU}{dt} = \frac{d}{dt}(Q\,\Delta V) = \frac{dQ}{dt}\Delta V = I\,\Delta V$$

❖ Power represents the rate at which energy is delivered to the resistor



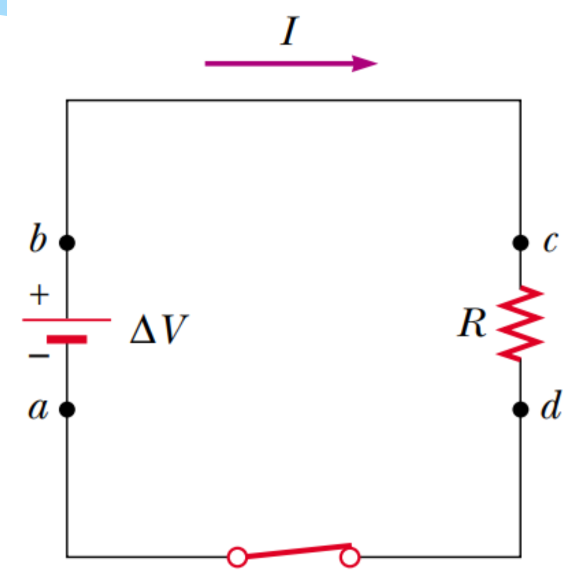
Power delivered to a device

$$\mathcal{P} = I\,\Delta V$$

❖ Used to calculate the power delivered by a voltage source to *any* device carrying a current *I* and having a potential difference Δ*V* between its terminals

27.6 Electrical Power

Power delivered to a resistor



$$\mathcal{P} = I^2R = \frac{(\Delta V)^2}{R}$$

I is expressed in amperes, ΔV in volts, and *R* in ohms

SI unit of power is the watt.

- ❖ Power is lost as internal energy in a conductor of resistance *R* is often called ***joule heating***.

- ❖ Because power = $I\,\Delta V$, the same amount of power can be transported either at high currents and low potential differences or at low currents and high potential differences.
- ❖ Utility companies choose to transport energy at low currents and high potential differences primarily for economic reasons.

27.6 Electrical Power

Quick Quiz 27.7 The same potential difference is applied to the two lightbulbs shown in Figure 27.14. Which one of the following statements is true? (a) The 30-W bulb carries the greater current and has the higher resistance. (b) The 30-W bulb carries the greater current, but the 60-W bulb has the higher resistance. (c) The 30-W bulb has the higher resistance, but the 60-W bulb carries the greater current. (d) The 60-W bulb carries the greater current and has the higher resistance.

George Semple

Figure 27.14 (Quick Quiz 27.7) These lightbulbs operate at their rated power only when they are connected to a 120-V source.

Quick Quiz 27.8 For the two lightbulbs shown in Figure 27.15, rank the current values at points *a* through *f*, from greatest to least.

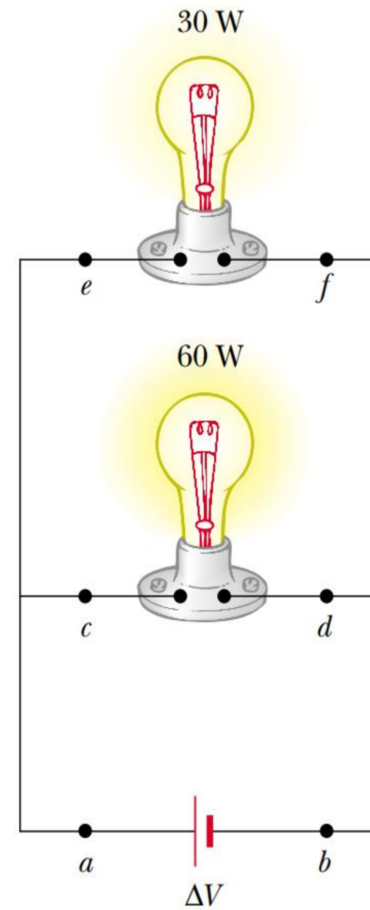


Figure 27.15 (Quick Quiz 27.8) Two lightbulbs connected across the same potential difference.

27.6 Electrical Power

Example 27.7 Power in an Electric Heater

An electric heater is constructed by applying a potential difference of 120 V to a Nichrome wire that has a total resistance of 8.00 Ω. Find the current carried by the wire and the power rating of the heater.

Solution Because $\Delta V = IR$, we have

$$I = \frac{\Delta V}{R} = \frac{120\ \text{V}}{8.00\ \Omega} = \boxed{15.0\ \text{A}}$$

We can find the power rating using the expression $\mathcal{P} = I^2R$:

$$\mathcal{P} = I^2R = (15.0\ \text{A})^2(8.00\ \Omega) = 1.80 \times 10^3\ \text{W}$$

$$\mathcal{P} = \boxed{1.80\ \text{kW}}$$

What If? What if the heater were accidentally connected to a 240-V supply? (This is difficult to do because the shape and orientation of the metal contacts in 240-V plugs are different from those in 120-V plugs.) How would this affect the current carried by the heater and the power rating of the heater?

Answer If we doubled the applied potential difference, Equation 27.8 tells us that the current would double. According to Equation 27.23, $\mathcal{P} = (\Delta V)^2/R$, the power would be four times larger.

27.6 Electrical Power

Selected Solved Problems (Chapter # 27)

36. A toaster is rated at 600 W when connected to a 120-V source. What current does the toaster carry, and what is its resistance?

49. Compute the cost per day of operating a lamp that draws a current of 1.70 A from a 110-V line. Assume the cost of energy from the power company is \$0.060 0/kWh.

56. A high-voltage transmission line with a diameter of 2.00 cm and a length of 200 km carries a steady current of 1 000 A. If the conductor is copper wire with a free charge density of 8.49×10^{28} electrons/m^3, how long does it take one electron to travel the full length of the line?

27.6 Electrical Power

Selected Old exam questions (Chapter # 27)

س1- التيار الناشئ عن مرور 500 بروتون خلال 40 *ns* يساوي:

Q1- The electric current due to the passing of 500 protons during 40 *ns* is:

A) 40 *A* | B) 2 *nA* | C) 0.4 *mA* | D) 20 *μA*

س2- إذا مر تيار كهربي قدره 10 *A* بموصل اسطواني قطره 10 *cm* فإن كثافة التيار تساوي:

Q2- If electric current of 10 *A* passes through a cylindrical conductor having a diameter of 10 *cm*, the current density equals:

A) 10 | B) 3148 | C) 1274 | D) 100

س3- ما مقدار الجهد الكهربي على طرفي مقاومة قدرها 50 Ω عند مرور تيار كهربي بها قدره 5 *A* ؟

Q3- What is the electric potential across a resistor of 50 Ω when a current of 5 *A* passes through it?

A) 250 | B) 500 | C) 10 | D) 0.1

س4- تتناسب كثافة التيار الكهربي في مادة عكسيا مع:

Q4- The current density in a material is inversely proportional to:

A) *Conductivity* | B) *Electric Field* | C) *Resistivity* | D) *Volume*

27.6 Electrical Power

Selected Old exam questions (Chapter # 27)

Q10.In a conductor if the cross-sectional area 4 cm^2, the electron density $3\times10^{22} cm^{-3}$ and the drift speed 2×10^{-5} m/s, then the current density, J, becomes:

10. إن مقدار كثافة التيار الكهربائي المار في موصل حسب المعطيات أعلاه تساوي:

A) 8.4×10^4 A/m B) 2.1×10^4 A/m^2 C) 3.2×10^4 A/m^2 D) 6.4×10^4 A/m^2 (E) 9.6×10^4 A/m^2

Q11. A wire shows resistance of 10Ω at 20°C and 30Ω at 100°C.The temperature coefficient of resistivity of the wire will be:

11. إذا تغيرت مقاومة موصل من 10Ω إلى 30 Ω عند تغير درجة الحرارة من 20°C إلى 100°C، فإن قيمة معامل الحرارة للمقاومة النوعية (المقاومية) تساوي إلى:

(A) $2.5\times10^{-2}\,°C^{-1}$ B) $4.3°C^{-1}$ C) $1.65\times10^{-30}C^{-1}$ D) $3.78°C^{-1}$ E) $5.00\,°C^{-1}$

Q12. An electric heater is operated with a potential difference of 110V to a Tungsten wire that has a resistance of 10Ω. The power of the heateris:

12. يُشغّل سخان كهربائي بفرق جهد كهربائي 110V بسلك تنغستين مقاومته 10Ω، إن قدرة السخان هي:

A) 8.00kW B) 3.25kW C) 5.78kW D) 2.45kW (E) 1.21kW