Chapter 27

Current and Resistance

Electric Current

Most practical applications of electricity deal with electric currents.

- The electric charges move through some region of space.

The *resistor* is a new element added to circuits.

Energy can be transferred to a device in an electric circuit.

The energy transfer mechanism is electrical transmission, $T_{ET.}$

Electric Current

Electric current is the rate of flow of charge through some region of space.

The SI unit of current is the **ampere** (A).

- 1 A = 1 C / s

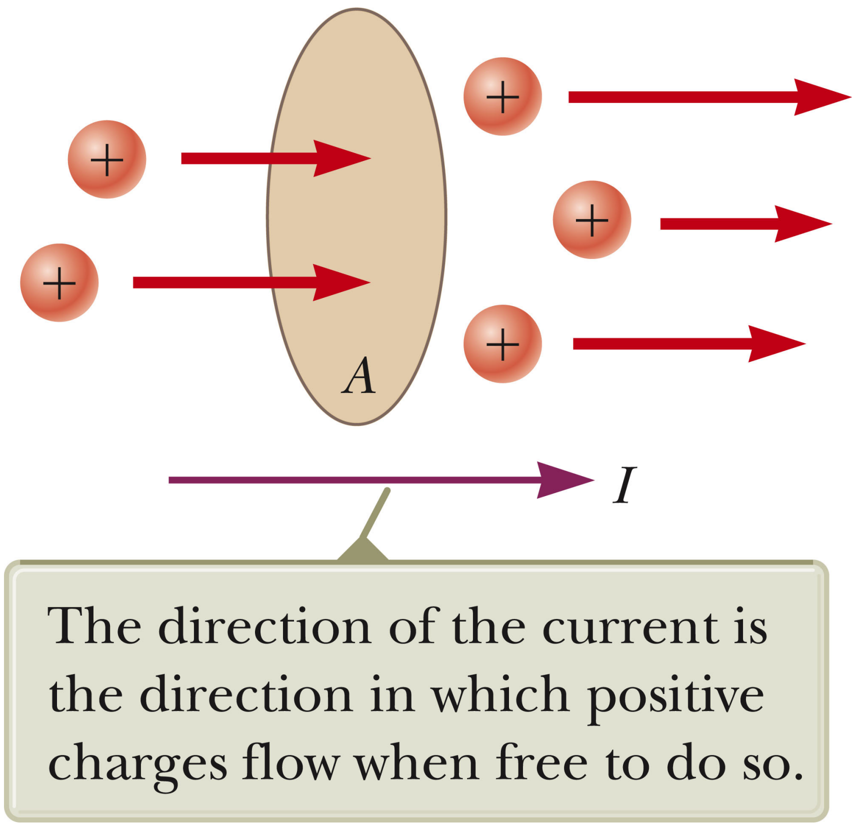
The symbol for electric current is I.

Average Electric Current

Assume charges are moving perpendicular to a surface of area *A*.

If ΔQ is the amount of charge that passes through *A* in time Δt, then the average current is

$$I_{avg} = \frac{\Delta Q}{\Delta t}$$



Instantaneous Electric Current

If the rate at which the charge flows varies with time, the instantaneous current, I, is defined as the differential limit of average current as $\Delta t \to 0$.

$$I \equiv \frac{dQ}{dt}$$

Direction of Current

The charged particles passing through the surface could be positive, negative or both.

It is conventional to assign to the current the same direction as the flow of positive charges.

In an ordinary conductor, the direction of current flow is opposite the direction of the flow of electrons.

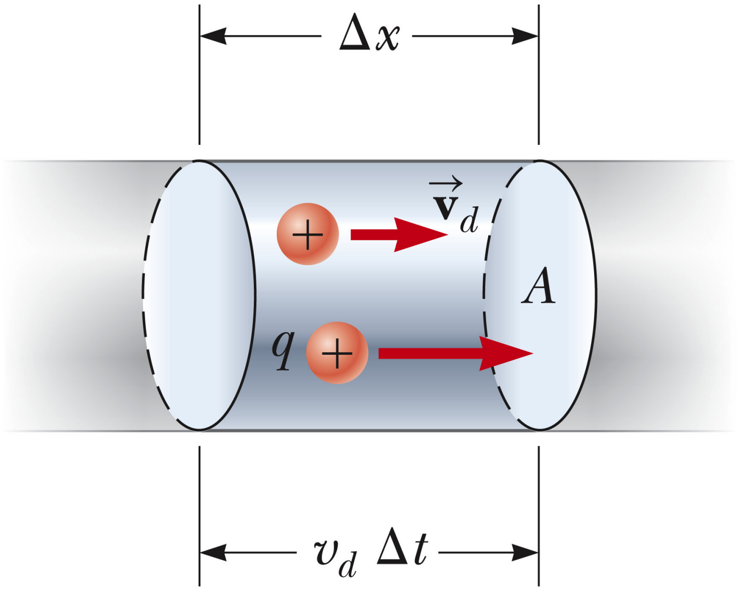
It is common to refer to any moving charge as a *charge carrier.*

Current and Drift Speed

Charged particles move through a cylindrical conductor of cross-sectional area *A.*

n is the number of mobile charge carriers per unit volume.

$nA\Delta x$ is the total number of charge carriers in a segment.



Current and Drift Speed, cont

The total charge is the number of carriers times the charge per carrier, *q.*

- $\Delta Q = (nA\Delta x)q$

Assume the carriers move with a velocity parallel to the axis of the cylinder such that they experience a displacement in the x-direction.

If v_d is the speed at which the carriers move, then

- $v_d = \Delta x / \Delta t$ and $\Delta x = v_d \Delta t$

Rewritten: $\Delta Q = (nAv_d \Delta t)q$

Finally, current, $I_{\text{ave}} = \Delta Q/\Delta t = nqv_dA$

v_d is an average speed called the **drift speed.**

Charge Carrier Motion in a Conductor

When a potential difference is applied across the conductor, an electric field is set up in the conductor which exerts an electric force on the electrons.

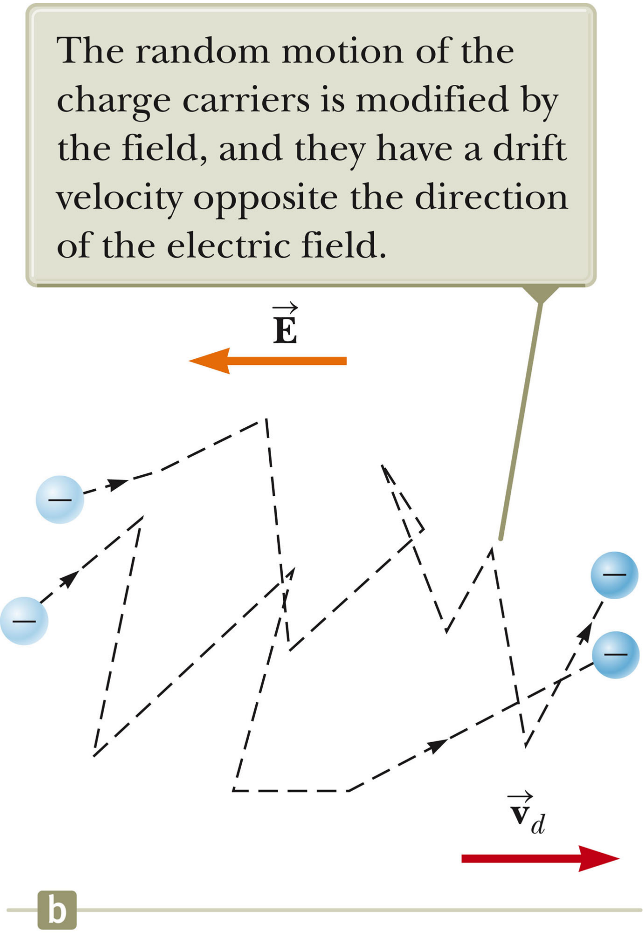
The motion of the electrons is no longer random.

The zigzag black lines represents the motion of a charge carrier in a conductor in the presence of an electric field.

- The net drift speed is small.

The sharp changes in direction are due to collisions.

The net motion of electrons is opposite the direction of the electric field.



Motion of Charge Carriers, cont.

In the presence of an electric field, in spite of all the collisions, the charge carriers slowly move along the conductor with a drift velocity,

$$\vec{\mathbf{v}}_d$$

The electric field exerts forces on the conduction electrons in the wire.

These forces cause the electrons to move in the wire and create a current.

Motion of Charge Carriers, final

The electrons are already in the wire.

They respond to the electric field set up by the battery.

The battery does not supply the electrons, it only establishes the electric field.

Example 27.1 Drift Speed in a Copper Wire

The 12-gauge copper wire in a typical residential building has a cross-sectional area of 3.31 x 10^{-6} m^2. If it carries a current of 10.0 A, what is the drift speed of the electrons? Assume that each copper atom contributes one free electron to the current. The density of copper is 8.95 g/cm^3. The molar mass of copper is 63.5 g/mol. Recall that 1 mol of any substance contains Avogadro's number of atoms (6.02 x 10^{23})

$$V = \frac{m}{\rho} = \frac{63.5\ \mathrm{g}}{8.95\ \mathrm{g/cm^3}} = 7.09\ \mathrm{cm^3}$$

$$\begin{aligned} n &= \frac{6.02 \times 10^{23}\ \text{electrons}}{7.09\ \mathrm{cm^3}} \left(\frac{1.00 \times 10^{6}\ \mathrm{cm^3}}{1\ \mathrm{m^3}} \right) \\ &= 8.49 \times 10^{28}\ \text{electrons/m}^3 \end{aligned}$$

$$\begin{aligned} v_d &= \frac{I}{nqA} \\ &= \frac{10.0\ \mathrm{C/s}}{(8.49 \times 10^{28}\ \mathrm{m^{-3}})(1.60 \times 10^{-19}\ \mathrm{C})(3.31 \times 10^{-6}\ \mathrm{m^2})} \\ &= \boxed{2.22 \times 10^{-4}\ \mathrm{m/s}} \end{aligned}$$

Problem 27.1

In a particular cathode ray tube, the measured beam current is 30.0 μA. How many electrons strike the tube screen every 40.0 s?

$$I = \frac{\Delta Q}{\Delta t} \qquad \Delta Q = I\Delta t = \left(30.0 \times 10^{-6}\ \text{A}\right)\left(40.0\ \text{s}\right) = 1.20 \times 10^{-3}\ \text{C}$$

$$N = \frac{Q}{e} = \frac{1.20 \times 10^{-3}\ \text{C}}{1.60 \times 10^{-19}\ \text{C/electron}} = \boxed{7.50 \times 10^{15}\ \text{electrons}}$$

Problem 27.11

An aluminum wire having a cross-sectional area of 4.00 x 10^{-6} m^2 carries a current of 5.00 A. Find the drift speed of the electrons in the wire. The density of aluminum is 2.70 g/cm^3. Assume that one conduction electron is supplied by each atom.

$$\frac{27.0 \text{ g}}{N_A} = \frac{27.0 \text{ g}}{6.02 \times 10^{23}} = 4.49 \times 10^{-23} \text{ g/atom}.$$

Thus, $$n = \frac{\text{density of aluminum}}{\text{mass per atom}} = \frac{2.70 \text{ g/cm}^3}{4.49 \times 10^{-23} \text{ g/atom}}$$

$$n = 6.02 \times 10^{22} \text{ atoms/cm}^3 = 6.02 \times 10^{28} \text{ atoms/m}^3 .$$

Therefore, $$v_d = \frac{I}{nqA} = \frac{5.00 \text{ A}}{\left(6.02 \times 10^{28} \text{ m}^{-3}\right)\left(1.60 \times 10^{-19} \text{ C}\right)\left(4.00 \times 10^{-6} \text{ m}^2\right)} = 1.30 \times 10^{-4} \text{ m/s}$$

or, $$v_d = \boxed{0.130 \text{ mm/s}}.$$

Current Density

J is the **current density** of a conductor.

It is defined as the current per unit area.

- $J \equiv I / A = nq\mathbf{v}_d$
- This expression is valid only if the current density is uniform and A is perpendicular to the direction of the current.

J has SI units of A/m^2

The current density is in the direction of the positive charge carriers.

Conductivity

A current density and an electric field are established in a conductor whenever a potential difference is maintained across the conductor.

For some materials, the current density is directly proportional to the field.

The constant of proportionality, σ, is called the **conductivity** of the conductor.

Ohm's Law

Ohm's law states that for many materials, the ratio of the current density to the electric field is a constant σ that is independent of the electric field producing the current.

- Most metals obey Ohm's law
- Mathematically, $J = \sigma E$
- Materials that obey Ohm's law are said to be *ohmic*
- Not all materials follow Ohm's law
  - Materials that do not obey Ohm's law are said to be *nonohmic.*

Ohm's law is not a fundamental law of nature.

Ohm's law is an empirical relationship valid only for certain materials.

Georg Simon Ohm

1789 -1854

German physicist

Formulated idea of resistance

Discovered the proportionalities now known as forms of Ohm’s Law

Resistance

In a conductor, the voltage applied across the ends of the conductor is proportional to the current through the conductor.

The constant of proportionality is called the **resistance** of the conductor.

$$R \equiv \frac{\Delta V}{I}$$

SI units of resistance are *ohms* (Ω).

- 1 Ω = 1 V / A

Resistance in a circuit arises due to collisions between the electrons carrying the current with the fixed atoms inside the conductor.

Resistors

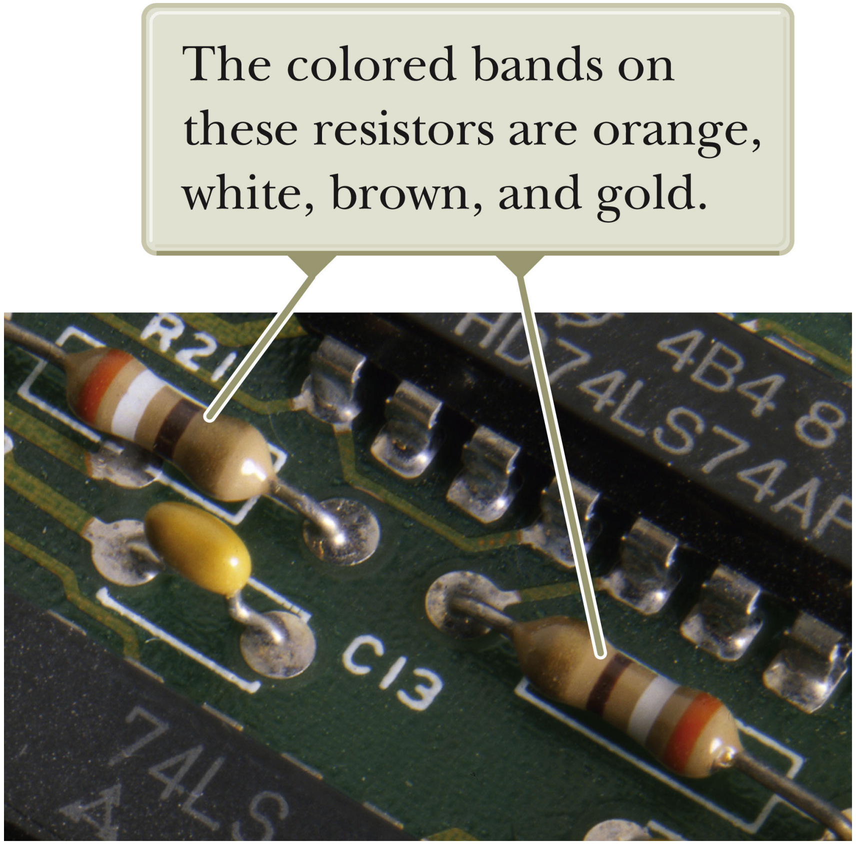
Most electric circuits use circuit elements called **resistors** to control the current in the various parts of the circuit.

Stand-alone resistors are widely used.

- Resistors can be built into integrated circuit chips.

Values of resistors are normally indicated by colored bands.

- The first two bands give the first two digits in the resistance value.
- The third band represents the power of ten for the multiplier band.
- The last band is the tolerance.



Resistor Color Codes

TABLE 27.1 *Color Coding for Resistors*

Color	Number	Multiplier	Tolerance
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		10^{-1}	5%
Silver		10^{-2}	10%
Colorless			20%

Resistor Color Code Example

Red (=2) and blue (=6) give the first two digits: 26

Green (=5) gives the power of ten in the multiplier: 10^5

The value of the resistor then is $26 \times 10^5\ \Omega$ (or $2.6\ M\Omega$)

The tolerance is 10% (silver = 10%) or $2.6 \times 10^5\ \Omega$

Resistivity

The inverse of the conductivity is the **resistivity**:

- $\rho = 1 / \sigma$

Resistivity has SI units of ohm-meters ($\Omega \cdot \mathrm{m}$)

Resistance is also related to resistivity:

$$R = \rho \frac{\ell}{A}$$

Resistivity Values

TABLE 27.2 *Resistivities and Temperature Coefficients of Resistivity for Various Materials*

Material	Resistivity[a] ($\Omega \cdot m$)	Temperature Coefficient[b] $\alpha[(^\circ C)^{-1}]$
Silver	1.59×10^{-8}	3.8×10^{-3}
Copper	1.7×10^{-8}	3.9×10^{-3}
Gold	2.44×10^{-8}	3.4×10^{-3}
Aluminum	2.82×10^{-8}	3.9×10^{-3}
Tungsten	5.6×10^{-8}	4.5×10^{-3}
Iron	10×10^{-8}	5.0×10^{-3}
Platinum	11×10^{-8}	3.92×10^{-3}
Lead	22×10^{-8}	3.9×10^{-3}
Nichrome[c]	1.00×10^{-6}	0.4×10^{-3}
Carbon	3.5×10^{-5}	-0.5×10^{-3}
Germanium	0.46	-48×10^{-3}
Silicon[d]	2.3×10^{3}	-75×10^{-3}
Glass	10^{10} to 10^{14}	
Hard rubber	$\sim 10^{13}$	
Sulfur	10^{15}	
Quartz (fused)	75×10^{16}	

[a] All values at 20°C. All elements in this table are assumed to be free of impurities.
[b] See Section 27.4.
[c] A nickel–chromium alloy commonly used in heating elements. The resistivity of Nichrome varies with composition and ranges between 1.00×10^{-6} and 1.50×10^{-6} $\Omega \cdot m$.
[d] The resistivity of silicon is very sensitive to purity. The value can be changed by several orders of magnitude when it is doped with other atoms.

Resistance and Resistivity, Summary

Every ohmic material has a characteristic resistivity that depends on the properties of the material and on temperature.

- Resistivity is a property of substances.

The resistance of a material depends on its geometry and its resistivity.

- Resistance is a property of an object.

An ideal conductor would have zero resistivity.

An ideal insulator would have infinite resistivity.

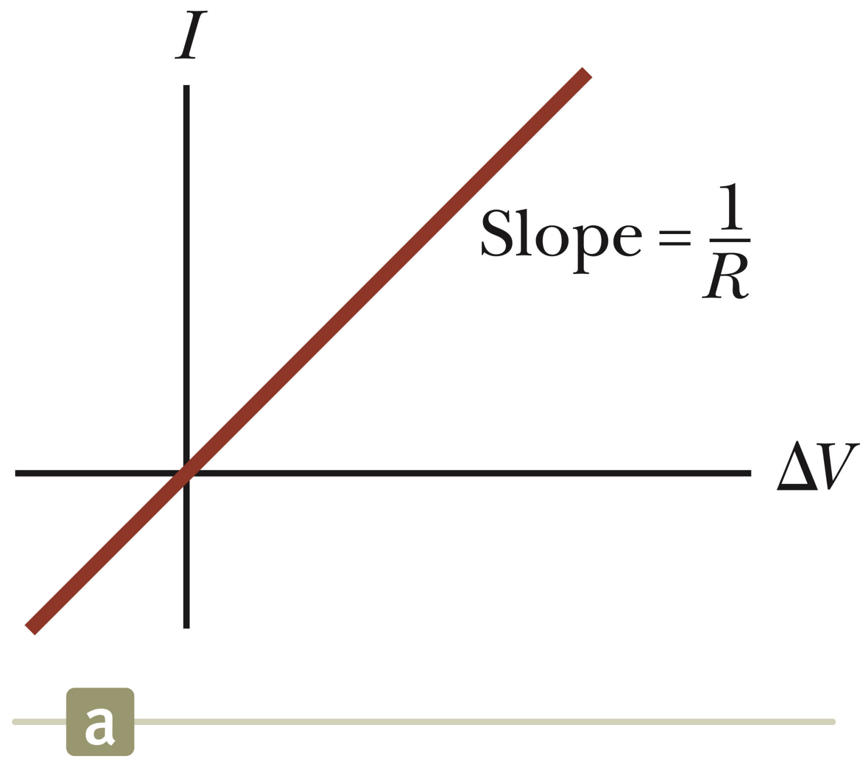
Ohmic Material, Graph

An ohmic device

The resistance is constant over a wide range of voltages.

The relationship between current and voltage is linear.

The slope is related to the resistance.

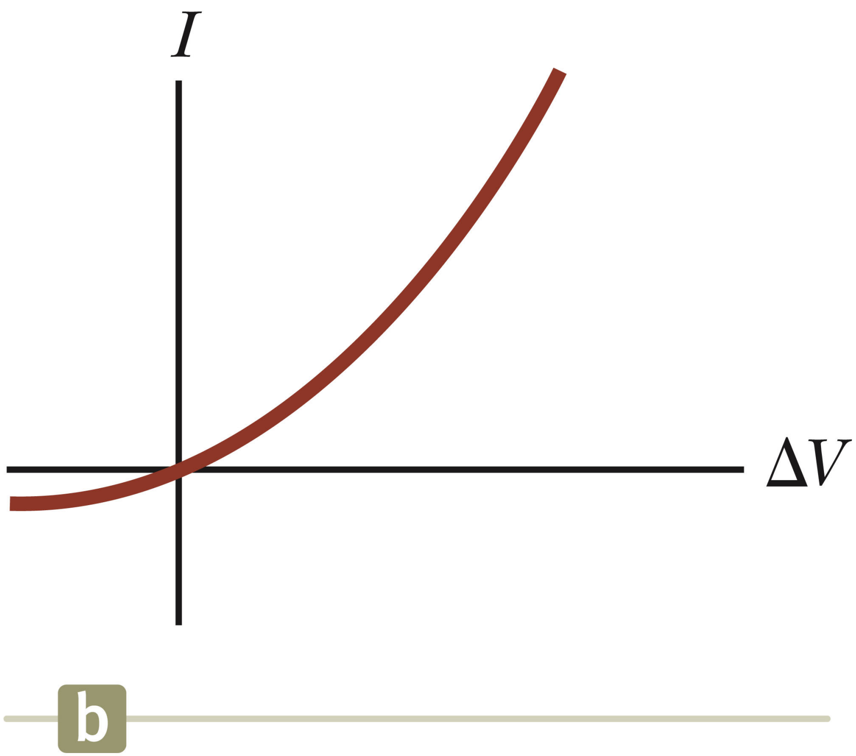


Nonohmic Material, Graph

Nonohmic materials are those whose resistance changes with voltage or current.

The current-voltage relationship is nonlinear.

A junction diode is a common example of a nonohmic device.

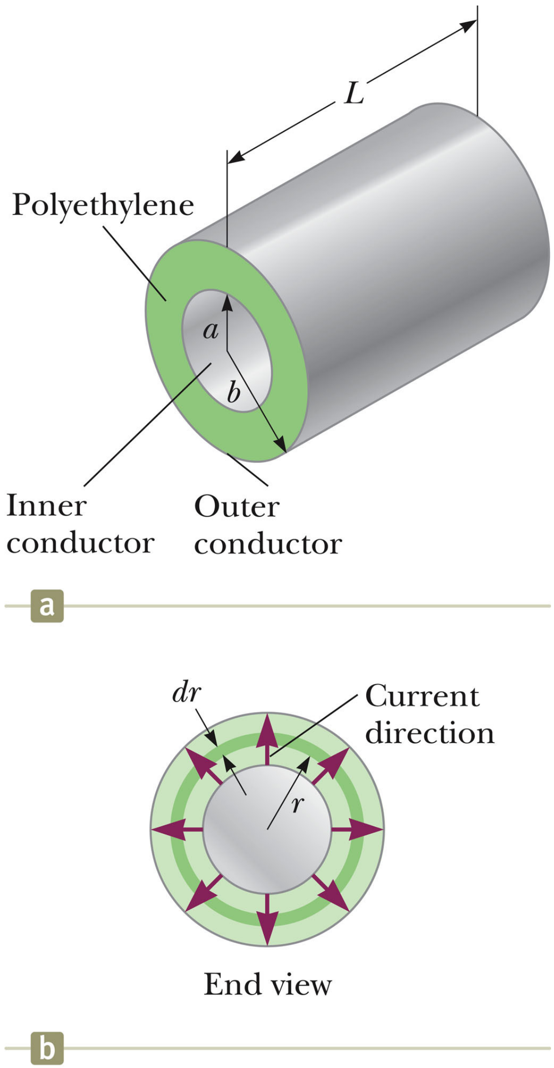


Resistance of a Cable, Example

Assume the silicon between the conductors to be concentric elements of thickness *dr.*

The resistance of the hollow cylinder of silicon is

$$dR = \frac{\rho}{2\pi r L} dr$$



Resistance of a Cable, Example, cont.

The total resistance across the entire thickness is

$$R = \int_a^b dR = \frac{\rho}{2\pi L} \ln\left(\frac{b}{a}\right)$$

This is the radial resistance of the cable.

The calculated value is fairly high, which is desirable since you want the current to flow along the cable and not radially out of it.

Example 27.2 The Resistance of a Conductor

Calculate the resistance of an aluminum cylinder that has a length of 10.0 cm and a cross-sectional area of 2.00 x 10^{-4} m^2. Repeat the calculation for a cylinder of the same dimensions and made of glass having a resistivity of 3.0 x 10^{10} Ω.m.

For aluminum

$$R = \rho \frac{\ell}{A} = (2.82 \times 10^{-8}\ \Omega\cdot\mathrm{m})\left(\frac{0.100\ \mathrm{m}}{2.00 \times 10^{-4}\ \mathrm{m}^2}\right)$$
$$= \boxed{1.41 \times 10^{-5}\ \Omega}$$

For glass

$$R = \rho \frac{\ell}{A} = (3.0 \times 10^{10}\ \Omega\cdot\mathrm{m})\left(\frac{0.100\ \mathrm{m}}{2.00 \times 10^{-4}\ \mathrm{m}^2}\right)$$
$$= \boxed{1.5 \times 10^{13}\ \Omega}$$

Example 27.3 The Resistance of Nichrome Wire

Calculate the resistance per unit length of a 22-gauge Nichrome wire, which has a radius of 0.321 mm.

$$A = \pi r^2 = \pi(0.321 \times 10^{-3}\text{ m})^2 = 3.24 \times 10^{-7}\text{ m}^2$$

$$\frac{R}{\ell} = \frac{\rho}{A} = \frac{1.5 \times 10^{-6}\ \Omega\cdot\text{m}}{3.24 \times 10^{-7}\text{ m}^2} = \boxed{4.6\ \Omega/\text{m}}$$

If a potential difference of 10 V is maintained across a 1.0-m length of the Nichrome wire, what is the current in the wire?

$$I = \frac{\Delta V}{R} = \frac{10\text{ V}}{4.6\ \Omega} = \boxed{2.2\text{ A}}$$

Problem 27.12

Calculate the current density in a gold wire at 20° C, if an electric field of 0.740 V/m exists in the wire.

$$J = \sigma E = \frac{E}{\rho} = \frac{0.740\ \mathrm{V/m}}{2.44 \times 10^{-8}\ \Omega \cdot \mathrm{m}} \left(\frac{1\ \Omega \cdot \mathrm{A}}{1\ \mathrm{V}} \right) = \boxed{3.03 \times 10^{7}\ \mathrm{A/m^2}}$$

Problem 27.15

A 0.900-V potential difference is maintained across a 1.50-m length of tungsten wire that has a cross-sectional area of 0.600 mm². What is the current in the wire?

$$\Delta V = IR$$

and $\quad R = \frac{\rho \ell}{A}: \qquad A = (0.600\ \text{mm})^2 \left(\frac{1.00\ \text{m}}{1\,000\ \text{mm}} \right)^2 = 6.00 \times 10^{-7}\ \text{m}^2$

$$\Delta V = \frac{I \rho \ell}{A}: \qquad I = \frac{\Delta V A}{\rho \ell} = \frac{(0.900\ \text{V})\left(6.00 \times 10^{-7}\ \text{m}^2\right)}{\left(5.60 \times 10^{-8}\ \Omega \cdot \text{m}\right)(1.50\ \text{m})}$$

$$I = \boxed{6.43\ \text{A}}$$

Problem 27.16

A conductor of uniform radius 1.20 cm carries a current of 3.00 A produced by an electric field of 120 V/m. What is the resistivity of the material?

$$J = \frac{I}{\pi r^2} = \sigma E = \frac{3.00\ \text{A}}{\pi(0.012\,0\ \text{m})^2} = \sigma(120\ \ \text{N/C})$$

$$\sigma = 55.3(\Omega\cdot\text{m})^{-1} \qquad \rho = \frac{1}{\sigma} = \boxed{0.018\,1\ \Omega\cdot\text{m}}$$

Problem 27.22

Aluminum and copper wires of equal length are found to have the same resistance. What is the ratio of their radii?

$$\frac{\rho_{\mathrm{Al}}\ell}{\pi(r_{\mathrm{Al}})^2} = \frac{\rho_{\mathrm{Cu}}\ell}{\pi(r_{\mathrm{Cu}})^2}$$

$$\frac{r_{\mathrm{Al}}}{r_{\mathrm{Cu}}} = \sqrt{\frac{\rho_{\mathrm{Al}}}{\rho_{\mathrm{Cu}}}} = \sqrt{\frac{2.82\times10^{-8}}{1.70\times10^{-8}}} = \boxed{1.29}$$

Resistance and Temperature

Over a limited temperature range, the resistivity of a conductor varies approximately linearly with the temperature.

$$\rho = \rho_o[1 + \alpha(T - T_o)]$$

- ρ_o is the resistivity at some reference temperature T_o
  - T_o is usually taken to be 20° C
  - α is the **temperature coefficient of resistivity**
    - SI units of α are $^oC^{-1}$

The temperature coefficient of resistivity can be expressed as

$$\alpha = \frac{1}{\rho_o}\frac{\Delta\rho}{\Delta T}$$

Temperature Variation of Resistance

Since the resistance of a conductor with uniform cross sectional area is proportional to the resistivity, you can find the effect of temperature on resistance.

$$R = R_o[1 + \alpha(T - T_o)]$$

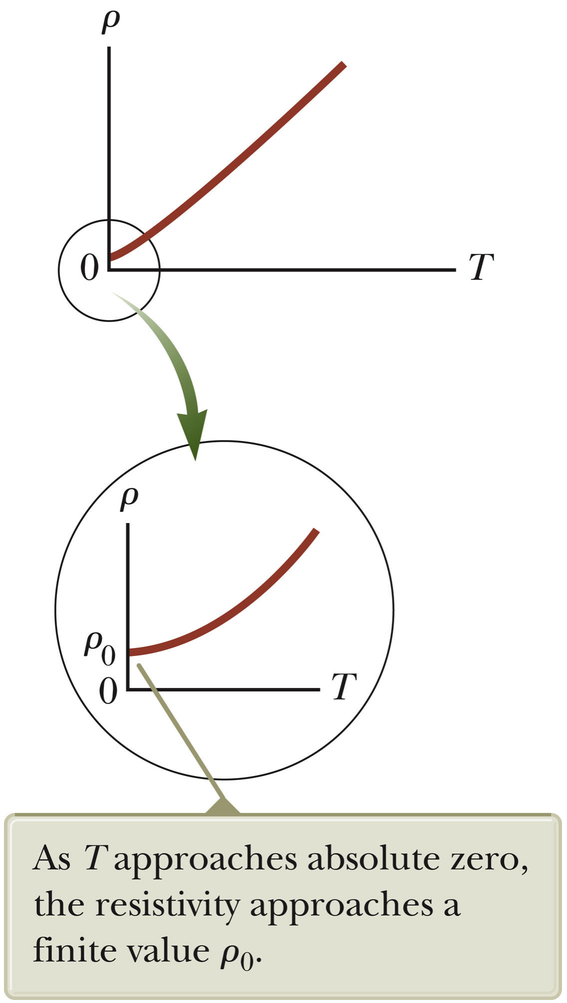
Use of this property enables precise temperature measurements through careful monitoring of the resistance of a probe made from a particular material.

Resistivity and Temperature, Graphical View

For some metals, the resistivity is nearly proportional to the temperature.

A nonlinear region always exists at very low temperatures.

The resistivity usually reaches some finite value as the temperature approaches absolute zero.



Residual Resistivity

The residual resistivity near absolute zero is caused primarily by the collisions of electrons with impurities and imperfections in the metal.

High temperature resistivity is predominantly characterized by collisions between the electrons and the metal atoms.

- This is the linear range on the graph.

Semiconductors

Semiconductors are materials that exhibit a decrease in resistivity with an increase in temperature.

α is negative

There is an increase in the density of charge carriers at higher temperatures.

Example 27.6 A Platinum Resistance Thermometer

A resistance thermometer, which measures temperature by measuring the change in resistance of a conductor, is made from platinum and has a resistance of 50.0 Ω at 20.0° C. When immersed in a vessel containing melting indium, its resistance increases to 76.8 Ω. Calculate the melting point of the indium.

$$\Delta T = \frac{R - R_0}{\alpha R_0} = \frac{76.8\ \Omega - 50.0\ \Omega}{[3.92 \times 10^{-3}(^\circ\mathrm{C})^{-1}](50.0\ \Omega)}$$

$$= 137^\circ\mathrm{C}$$

Because $T_0 = 20.0^\circ\mathrm{C}$, we find that T, the temperature of the melting indium sample, is $157^\circ\mathrm{C}$.

Problem 27.32

An aluminum rod has a resistance of 1.234 Ω at 20.0° C. Calculate the resistance of the rod at 120° C by accounting for the changes in both the resistivity and the dimensions of the rod.

For aluminum,

$$\alpha_E = 3.90 \times 10^{-3}\ {}^\circ\mathrm{C}^{-1} \qquad \text{(Table 27.1)}$$

$$\alpha = 24.0 \times 10^{-6}\ {}^\circ\mathrm{C}^{-1} \qquad \text{(Table 19.1)}$$

$$R = \frac{\rho\ell}{A} = \frac{\rho_0(1+\alpha_E\Delta T)\ell(1+\alpha\Delta T)}{A(1+\alpha\Delta T)^2} = R_0\frac{(1+\alpha_E\Delta T)}{(1+\alpha\Delta T)} = (1.234\ \Omega)\left(\frac{1.39}{1.002\,4}\right) = \boxed{1.71\ \Omega}$$

Problem 27.33

What is the fractional change in the resistance of an iron filament when its temperature changes from 25.0° C to 50.0° C?

$$R = R_0[1 + \alpha T]$$

$$R - R_0 = R_0 \alpha \Delta T$$

$$\frac{R - R_0}{R_0} = \alpha \Delta T = \left(5.00 \times 10^{-3}\right)25.0 = \boxed{0.125}$$

Electrical Power

Assume a circuit as shown

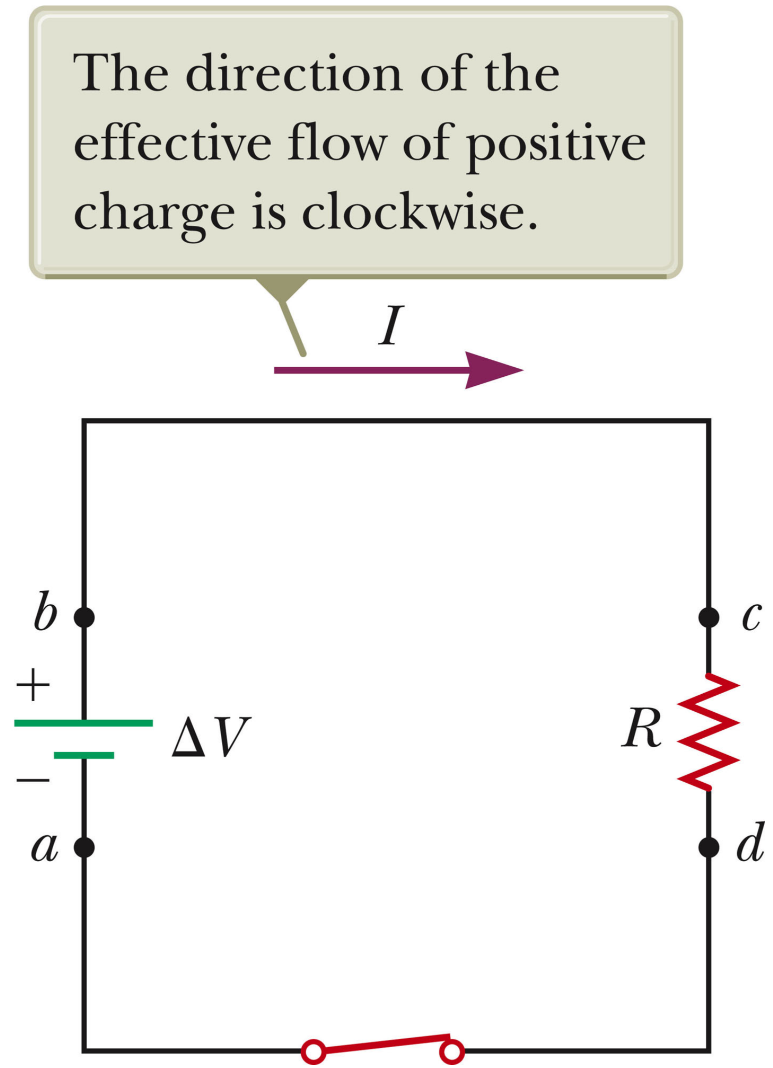
The entire circuit is the system.

As a charge moves from *a* to *b*, the electric potential energy of the system increases by $Q\Delta V$.

- The chemical energy in the battery must decrease by this same amount.

This electric potential energy is transformed into internal energy in the resistor.

- Corresponds to increased vibrational motion of the atoms in the resistor



Electric Power, 2

The resistor is normally in contact with the air, so its increased temperature will result in a transfer of energy by heat into the air.

The resistor also emits thermal radiation.

After some time interval, the resistor reaches a constant temperature.

- The input of energy from the battery is balanced by the output of energy by heat and radiation.

The rate at which the system's potential energy decreases as the charge passes through the resistor is equal to the rate at which the system gains internal energy in the resistor.

The **power** is the rate at which the energy is delivered to the resistor.

Electric Power, final

The power is given by the equation P = I ΔV.

Applying Ohm's Law, alternative expressions can be found:

$$P = I\,\Delta V = I^2 R = \frac{(\Delta V)^2}{R}$$

Units: I is in A, R is in Ω, ΔV is in V, and P is in W

Some Final Notes About Current

A single electron is moving at the drift velocity in the circuit.

- It may take hours for an electron to move completely around a circuit.

The current is the same everywhere in the circuit.

- Current is not "used up" anywhere in the circuit

The charges flow in the same rotational sense at all points in the circuit.

Electric Power Transmission

Real power lines have resistance.

Power companies transmit electricity at high voltages and low currents to minimize power losses.

Example 27.7 Power in an Electric Heater

An electric heater is constructed by applying a potential difference of 120 V to a Nichrome wire that has a total resistance of 8.00 Ω. Find the current carried by the wire and the power rating of the heater.

$$I = \frac{\Delta V}{R} = \frac{120\ \text{V}}{8.00\ \Omega} = \boxed{15.0\ \text{A}}$$

$$\mathcal{P} = I^2 R = (15.0\ \text{A})^2 (8.00\ \Omega) = 1.80 \times 10^3\ \text{W}$$

$$\mathcal{P} = \boxed{1.80\ \text{kW}}$$

Example 27.8 Linking Electricity and Thermodynamics

(A) What is the required resistance of an immersion heater that will increase the temperature of 1.50 kg of water from 10.0° C to 50.0° C in 10.0 min while operating at 110 V?

(B) Estimate the cost of heating the water.

$$Q = mc\,\Delta T.$$

$$\mathcal{P} = \frac{(\Delta V)^2}{R} = \frac{Q}{\Delta t}$$

$$\frac{(\Delta V)^2}{R} = \frac{mc\,\Delta T}{\Delta t} \longrightarrow R = \frac{(\Delta V)^2\,\Delta t}{mc\,\Delta T}$$

$$R = \frac{(110\text{ V})^2(600\text{ s})}{(1.50\text{ kg})(4186\text{ J/kg}\cdot{}^\circ\text{C})(50.0^\circ\text{C} - 10.0^\circ\text{C})}$$

$$= \boxed{28.9\ \Omega}$$

$$\mathcal{P}\,\Delta t = \frac{(\Delta V)^2}{R}\Delta t = \frac{(110\text{ V})^2}{28.9\ \Omega}(10.0\text{ min})\left(\frac{1\text{ h}}{60.0\text{ min}}\right)$$

$$= 69.8\text{ Wh} = 0.069\,8\text{ kWh}$$

If the energy is purchased at an estimated price of 10.0¢ per kilowatt-hour, the cost is

$$\text{Cost} = (0.069\,8\text{ kWh})(\$0.100/\text{kWh}) = \$0.006\,98$$

$$\approx \boxed{0.7\ ¢}$$

Problem 27.36

A toaster is rated at 600 W when connected to a 120-V source. What current does the toaster carry, and what is its resistance?

$$I = \frac{\mathcal{P}}{\Delta V} = \frac{600\ \text{W}}{120\ \text{V}} = \boxed{5.00\ \text{A}}$$

$$\text{and}\ \ R = \frac{\Delta V}{I} = \frac{120\ \text{V}}{5.00\ \text{A}} = \boxed{24.0\ \Omega}\ .$$

Problem 27.49

Compute the cost per day of operating a lamp that draws a current of 1.70 A from a 110-V line. Assume the cost of energy from the power company is \$0.060 0/kWh.

$$\mathcal{P} = I(\Delta V) = (1.70\ \text{A})(110\ \text{V}) = 187\ \text{W}$$

$$\text{Energy used in a 24-hour day} = (0.187\ \text{kW})(24.0\ \text{h}) = 4.49\ \text{kWh}$$

$$\therefore \text{cost} = 4.49\ \text{kWh}\left(\frac{\$0.060\,0}{\text{kWh}}\right) = \$0.269 = \boxed{26.9¢}$$