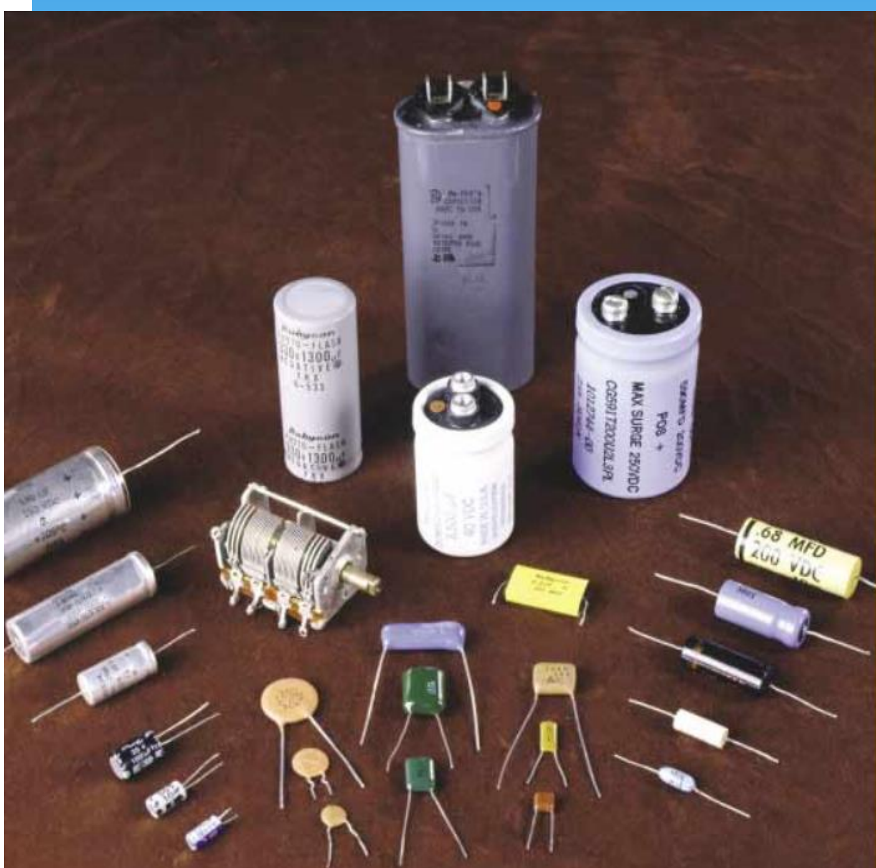
Chapter 26

Capacitance and Dielectrics



CHAPTER OUTLINE

26.1 Definition of Capacitance

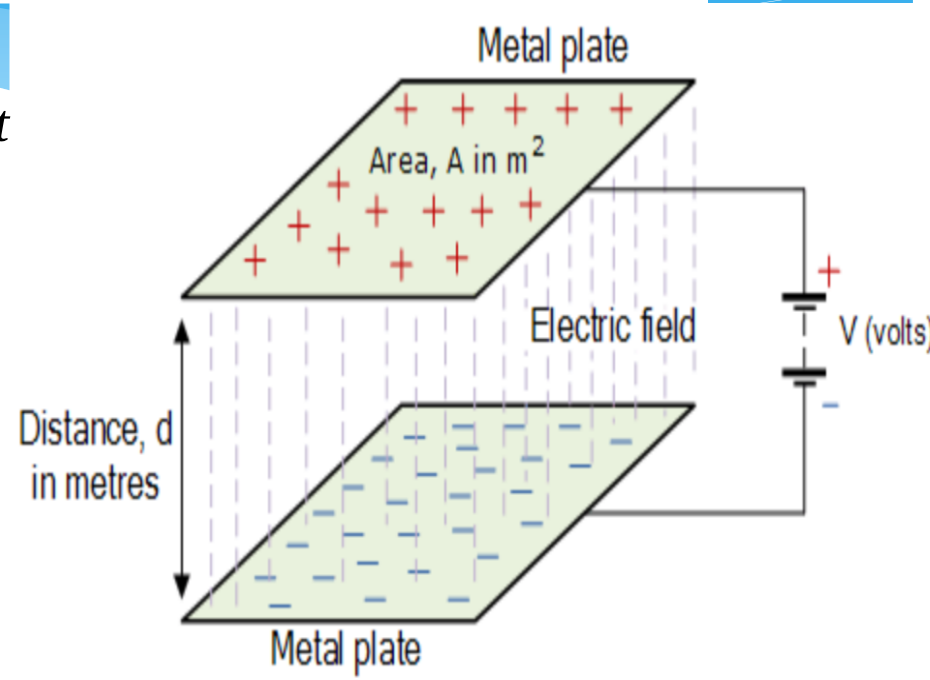
❖ *A capacitor is one type of circuit element that we can combine with others to make electric circuits.*

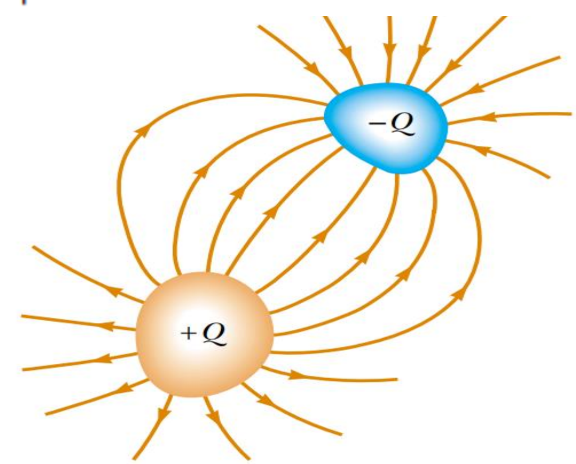
❖ A capacitor consists of two conductors separated by an insulator

❖ The capacitance of a given capacitor depends on its geometry and on the material—called a *dielectric*— hat separates the conductors

❖ *Capacitance* of a capacitor is the amount of charge the capacitor can store per unit of potential difference.

❖ A potential difference ΔV exists between the conductors due to the presence of the charges.





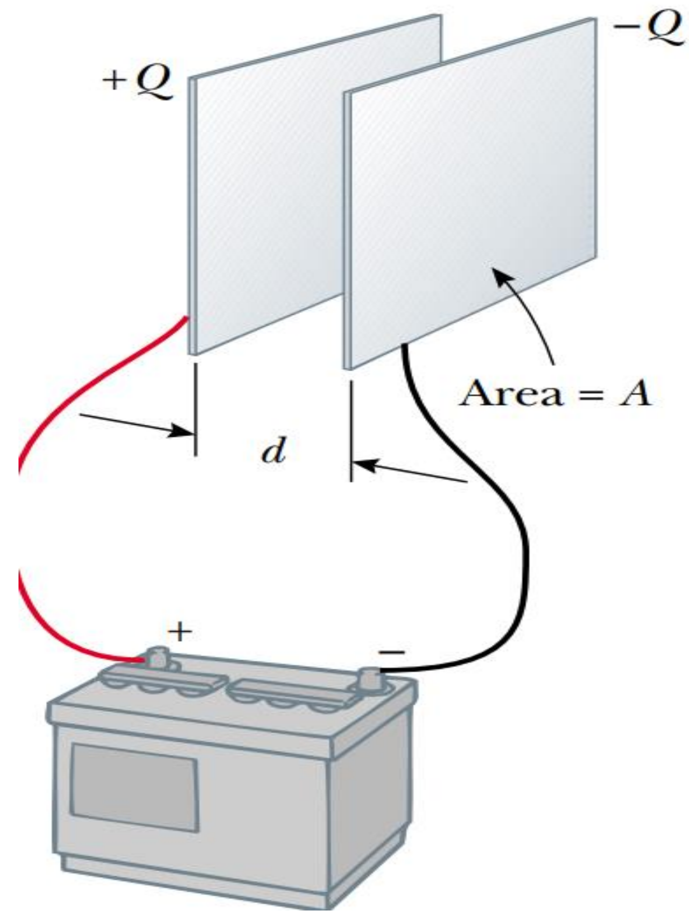
When the capacitor is charged, the conductors carry charges of equal magnitude and opposite sign.

26.1 Definition of Capacitance

The **capacitance** C of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between the conductors:

$$C \equiv \frac{Q}{\Delta V} \tag{26.1}$$

- ❑ Capacitance will always be a positive quantity (because we take the magnitude of the charge on either conductor)
- ❑ The capacitance of a given capacitor is constant



The capacitance is a measure of the capacitor's ability to store charge

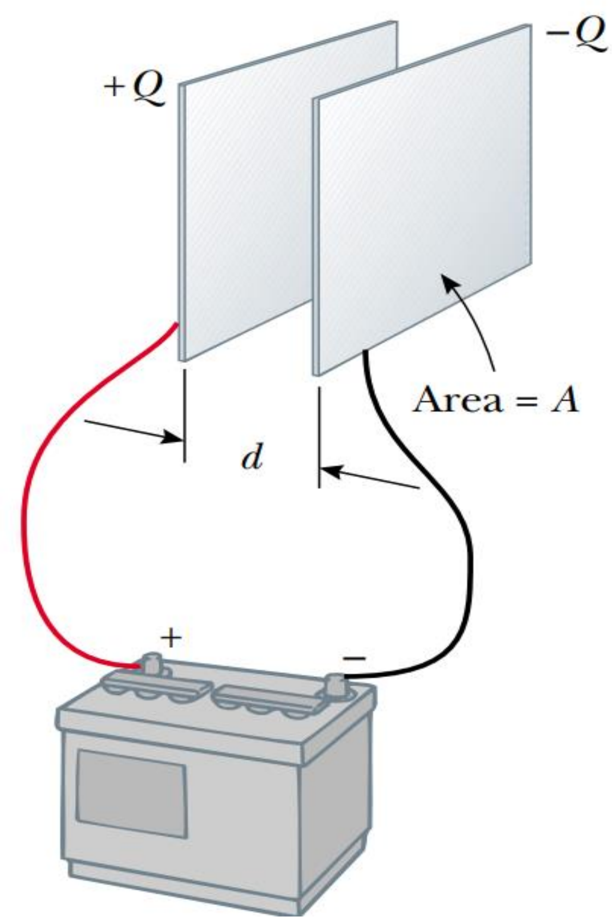
SI Unit of capacitance(***C***)**:** farad (F), 1F = 1 C/V
The farad is an extremely large unit,
in practice:

- ➢ microfarads (μF=10^{-6}F),
- ➢ nanofarads (nF=10^{-9}F),
- ➢ picofarads (pF=10^{-12}F)

26.1 Definition of Capacitance

- Consider a capacitor formed from a pair of parallel plates (two parallel metal plates)
- Each plate is connected to one terminal of a battery, which acts as a source of potential difference.

- When the plates are connected to the positive and negative terminals of a battery,
- electrons are pulled off one of the plates, leaving it with a charge of **+*Q***, and are transferred through the battery to the other plate, leaving it with a charge of **–*Q***. The transfer of charge **stops** when ΔV (between the plates) = ΔV (between the terminals)



- A capacitor rated at 4pF. This rating means that the capacitor can store 4pC of charge for each volt of potential difference between the two conductors

- If a 9-V battery is connected across this capacitor, one of the conductor ends up with a net charge of ***-36pC*** and the other ends up with a net charge of ***+36pC***

26.1 Definition of Capacitance

Quick Quiz 26.1 A capacitor stores charge Q at a potential difference ΔV. If the voltage applied by a battery to the capacitor is doubled to $2\Delta V$, (a) the capacitance falls to half its initial value and the charge remains the same (b) the capacitance and the charge both fall to half their initial values (c) the capacitance and the charge both double (d) the capacitance remains the same and the charge doubles.

1. (a) How much charge is on each plate of a 4.00-μF capacitor when it is connected to a 12.0-V battery? (b) If this same capacitor is connected to a 1.50-V battery, what charge is stored?

2. Two conductors having net charges of $+10.0\ \mu$C and $-10.0\ \mu$C have a potential difference of 10.0 V between them. (a) Determine the capacitance of the system. (b) What is the potential difference between the two conductors if the charges on each are increased to $+100\ \mu$C and $-100\ \mu$C?

26.2 Calculating Capacitance

Capacitance of an isolated charged sphere

- Single conductor also has a capacitance.

- From last chapter:

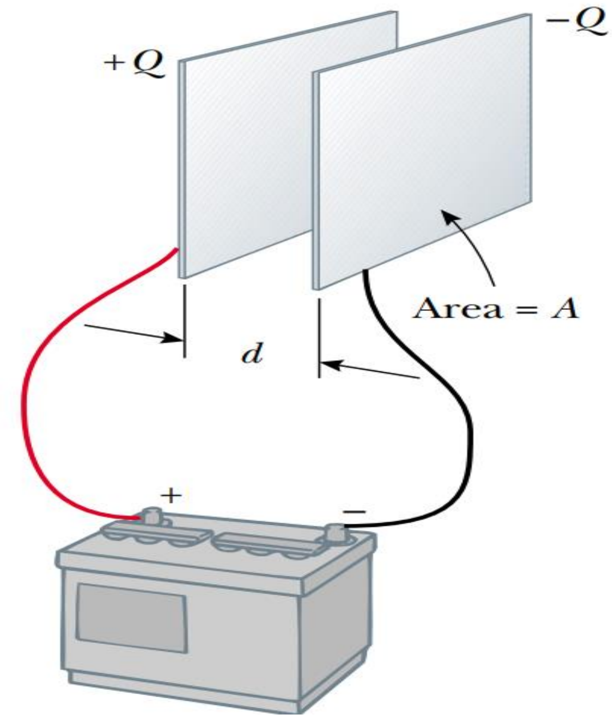
- The electric potential of the sphere of radius *R* is simply (*keQ* /*R)* and setting *V* = 0 for the infinitely large shell

$$C = \frac{Q}{\Delta V} = \frac{Q}{k_e Q / R} = \frac{R}{k_e} = 4\pi\epsilon_0 R$$

- The capacitance of an isolated charged sphere is *proportional* to its radius and is ***independent*** of both the charge on the sphere and the potential difference.

26.2 Calculating Capacitance

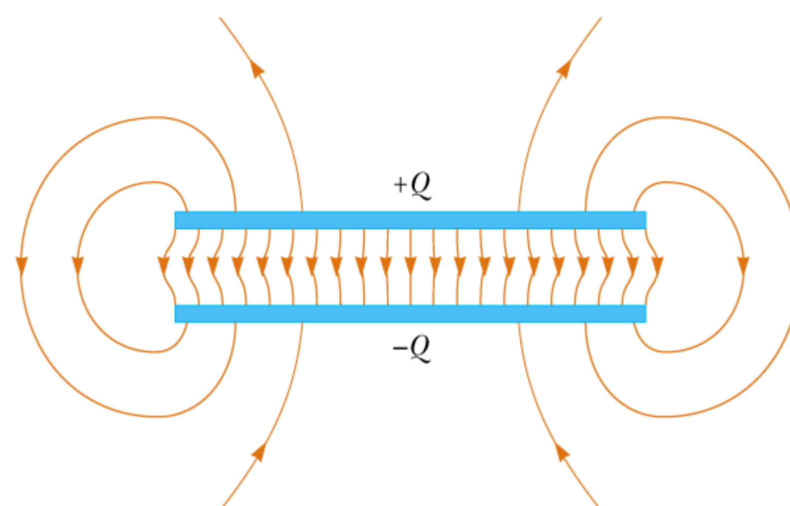
Parallel-Plate Capacitors



- Two parallel metallic plates of equal area *A* are separated by a distance *d*, with One plate carries a charge *Q* , and the other carries a charge -*Q* .

- As a capacitor is being charged by a battery, electrons flow into the negative plate and out of the positive plate.

- Amount of charge that can be stored on a plate for a given potential difference increases as the plate area is increased. Thus, we expect the capacitance to be proportional to the plate area *A*.



(a)

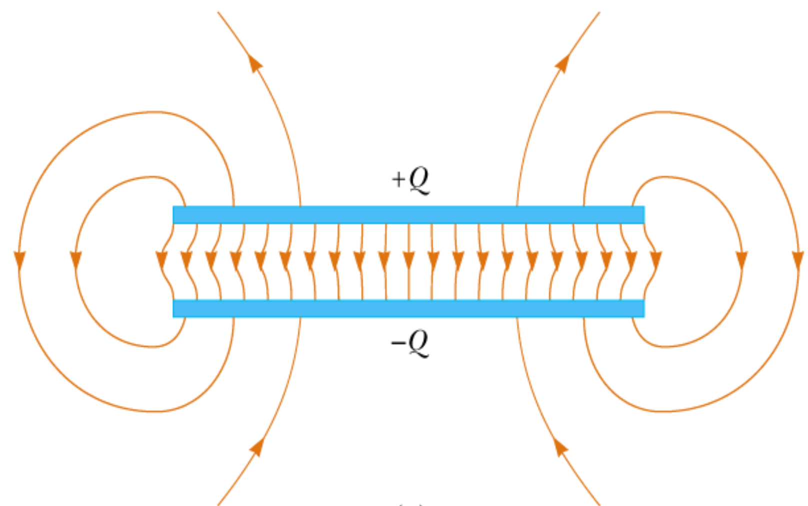
- The region that separates the plates moving the plates closer together causes the charge on the capacitor to increase. If *d* is increased, the charge decreases. As a result, we expect the capacitance of the pair of plates to be inversely proportional to *d*.

26.2 Calculating Capacitance

$$C = \frac{Q}{\Delta V} \qquad \Delta V = Ed \qquad E = \frac{\sigma}{\varepsilon_o}$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{Ed} = \frac{Q}{(\frac{\sigma}{\varepsilon_o})d} = \frac{Q\varepsilon_o}{(\frac{Q}{A})d}$$

$$C = \frac{\varepsilon_o A}{d}$$



(a)

- The capacitance of a parallel-plate capacitor is proportional to the area of its plates and inversely proportional to the plate separation.

- This means plates with larger area can store more charge

- Electric field is uniform in the central region between the plates, but is nonuniform at the edges of the plate

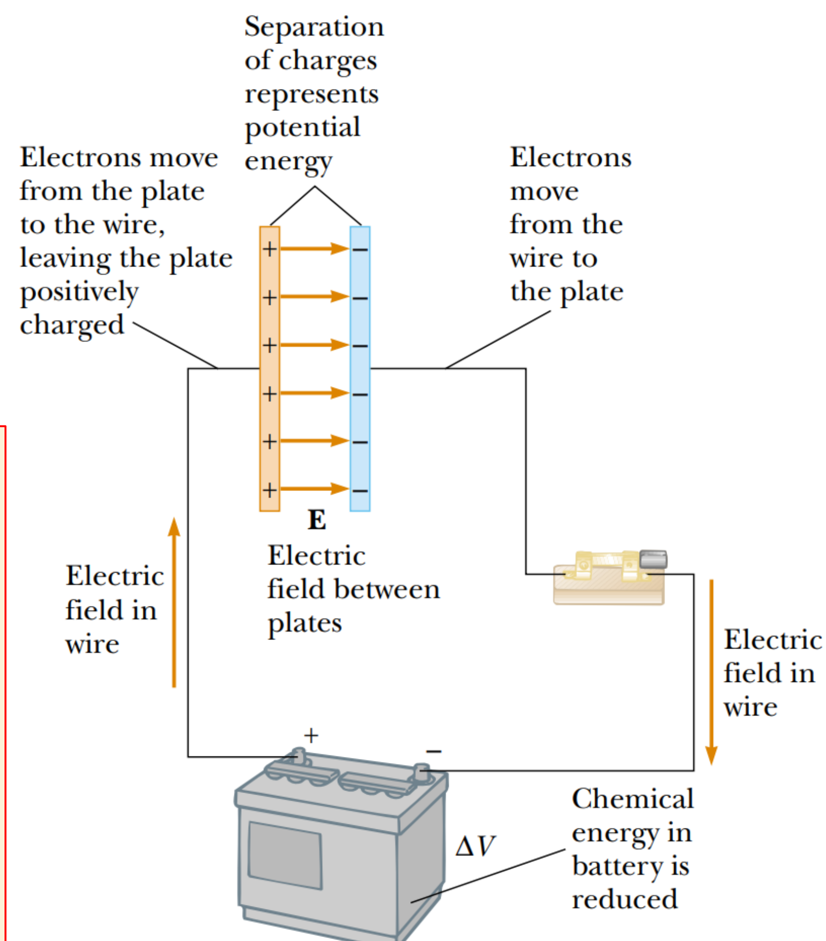
26.2 Calculating Capacitance

❖ Capacitor as a device that stores energy & charge

❖ When switch is open, energy is stored as chemical energy in the battery.

❖ When the switch is closed, the battery establishes an electric field in the wires and charges flow between the wires and the capacitor.

❖ Energy is transformed during the chemical reaction that occurs within the battery when it is operating in an electric circuit some of the chemical energy in the battery is converted to electric potential energy related to the separation of positive and negative charges on the plates.



26.2 Calculating Capacitance

Quick Quiz 26.2 Many computer keyboard buttons are constructed of capacitors, as shown in Figure 26.5. When a key is pushed down, the soft insulator between the movable plate and the fixed plate is compressed. When the key is pressed, the capacitance (a) increases, (b) decreases, or (c) changes in a way that we cannot determine because the complicated electric circuit connected to the keyboard button may cause a change in ΔV.

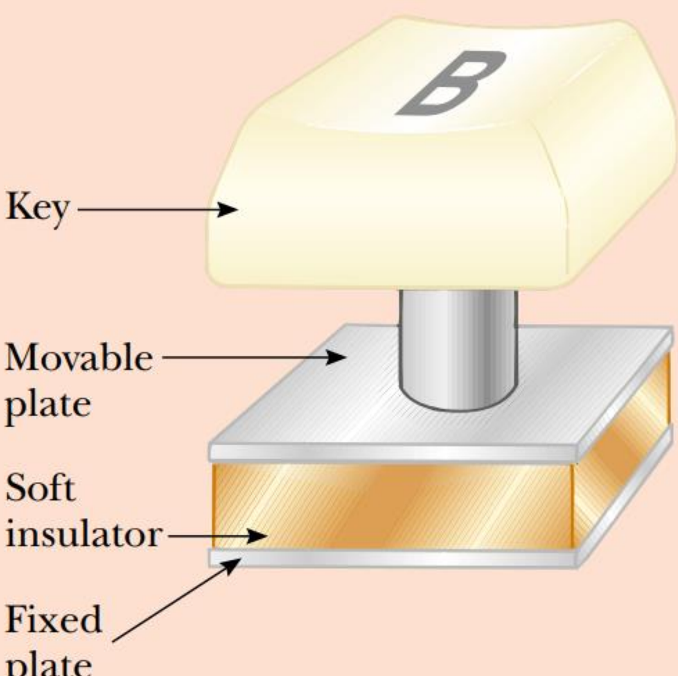


Figure 26.5 (Quick Quiz 26.2) One type of computer keyboard button.

26.2 Calculating Capacitance

Example 26.1 Parallel-Plate Capacitor

A parallel-plate capacitor with air between the plates has an area $A = 2.00 \times 10^{-4}\ \text{m}^2$ and a plate separation $d = 1.00$ mm. Find its capacitance.

Solution From Equation 26.3, we find that

$$C = \frac{\epsilon_0 A}{d} = \frac{(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.00 \times 10^{-4}\ \text{m}^2)}{1.00 \times 10^{-3}\ \text{m}}$$

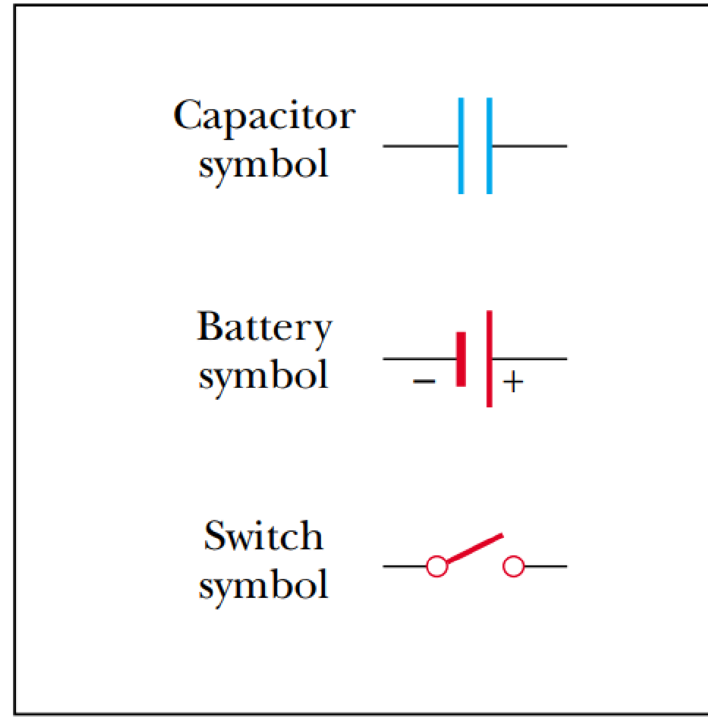
$$= 1.77 \times 10^{-12}\ \text{F} = \boxed{1.77\ \text{pF}}$$

7. An air-filled capacitor consists of two parallel plates, each with an area of 7.60 cm^2, separated by a distance of 1.80 mm. A 20.0-V potential difference is applied to these plates. Calculate (a) the electric field between the plates, (b) the surface charge density, (c) the capacitance, and (d) the charge on each plate.

9. When a potential difference of 150 V is applied to the plates of a parallel-plate capacitor, the plates carry a surface charge density of 30.0 nC/cm^2. What is the spacing between the plates?

26.3 Combinations of Capacitors

- In electric circuits, more capacitors are often combined. How to calculate the equivalent capacitance !!!!



Circuit symbols for capacitors, batteries, and switches

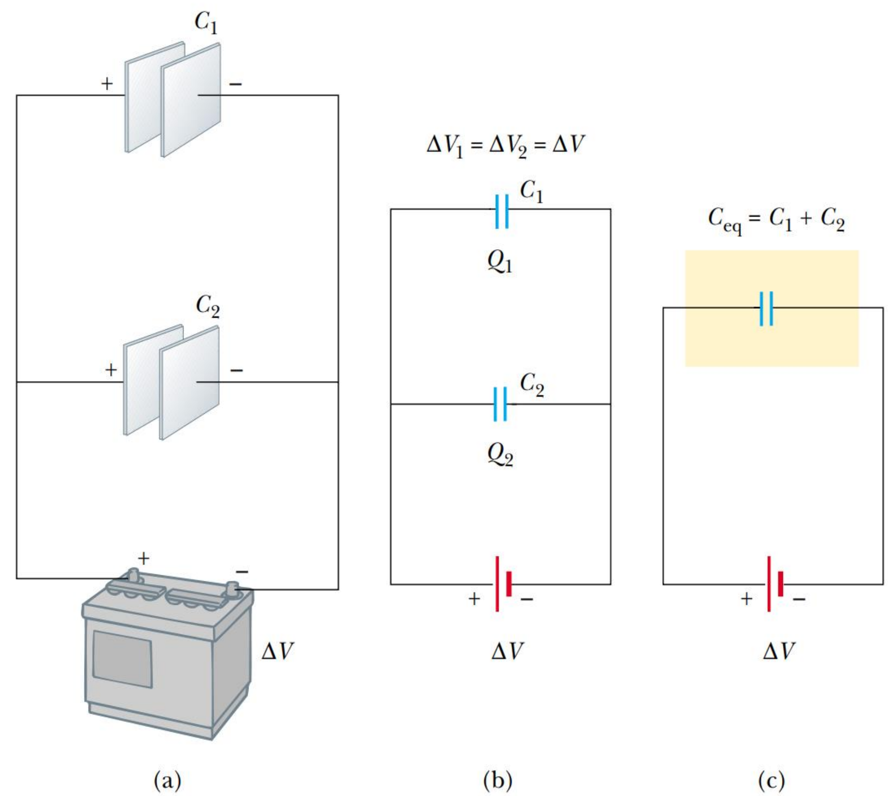
- The symbol for the capacitor reflects the geometry of the most common model for a capacitor — a pair of parallel plates.

- Assume that the capacitors to be combined are initially uncharged.

26.3 Combinations of Capacitors

Parallel Combination

- The individual potential differences across capacitors connected in parallel are the same And are equal to the potential difference applied across the combination.



- Capacitors are first connected in the circuit, electrons are transferred between the wires and the plates; this transfer leaves the left plates positively charged and the right plates negatively charged

Left plates of the capacitors are connected by to the positive terminal of the battery and the same for right plates connected to negative terminal. Therefore both at the same potential and the voltage applied across the combination is the terminal voltage of the battery .

- *Total charge Q* stored by the two capacitors is **the sum of the charges on the individual capacitors.**

$$Q_{total} = Q_1 + Q_2$$

Because the voltages across the capacitors are the same, the charges that they carry are

$$Q_1 = C_1 \Delta V \qquad Q_2 = C_2 \Delta V$$

26.3 Combinations of Capacitors

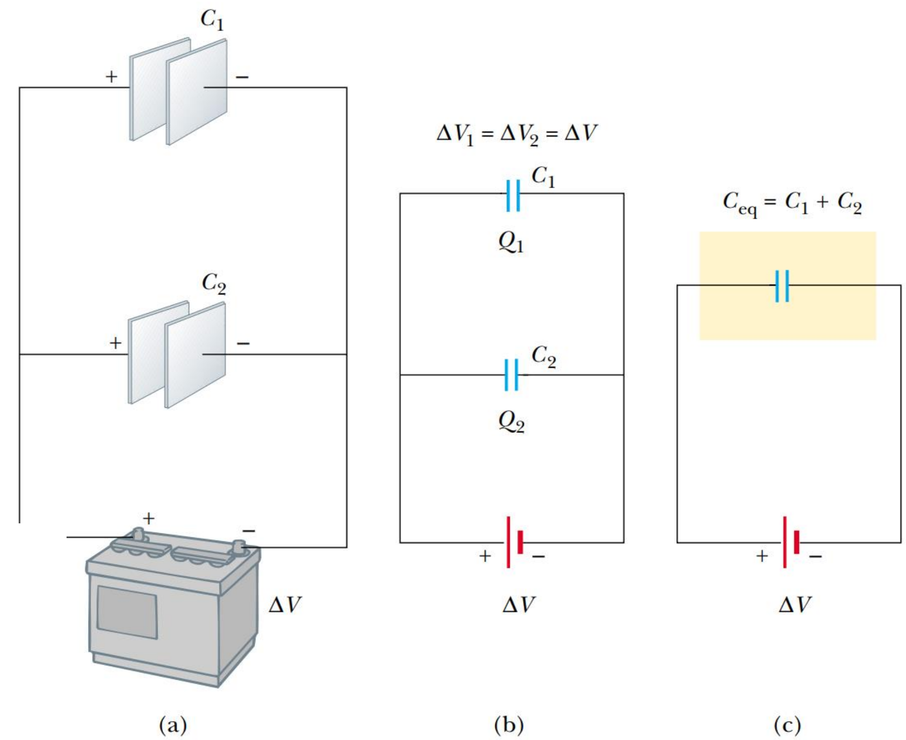
Parallel Combination

- Let's replace these two capacitors by one *equivalent capacitor* having a capacitance C_{eq},

$$Q = C_{eq}\,\Delta V$$

$$C_{eq}\,\Delta V = C_1\,\Delta V + C_2\,\Delta V$$

$$C_{eq} = C_1 + C_2 \qquad \text{(parallel combination)}$$



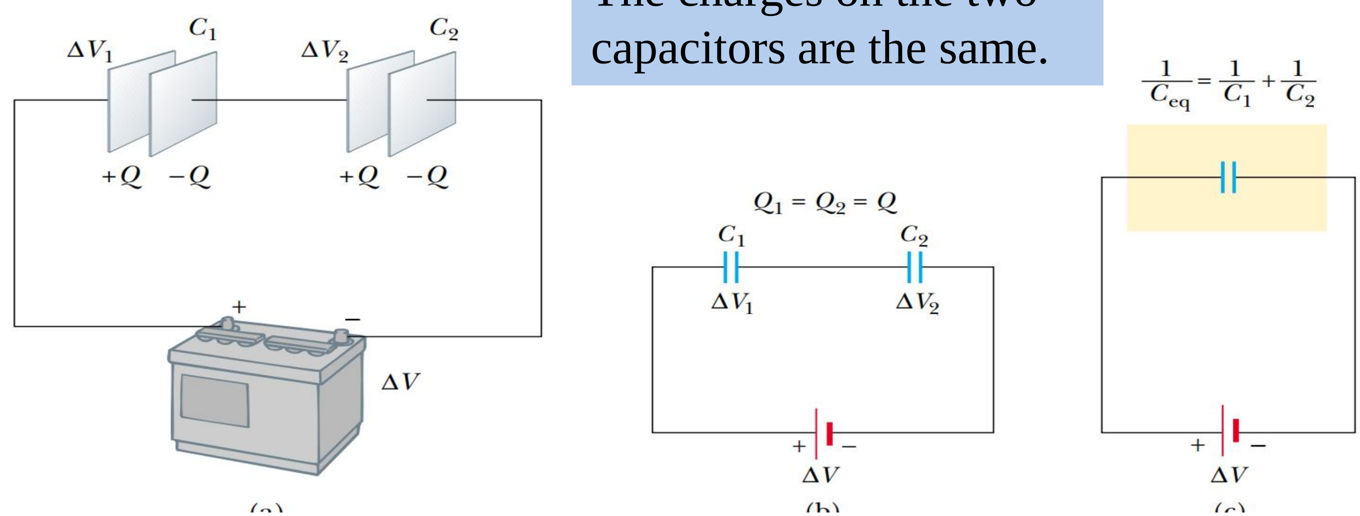
$$C_{eq} = C_1 + C_2 + C_3 + \cdots \qquad \text{(parallel combination)}$$

The equivalent capacitance of a parallel combination of capacitors is the algebraic sum of the individual capacitances and is greater than any of the individual capacitances.

26.3 Combinations of Capacitors

Series Combination

The charges on the two capacitors are the same.



Left plate of capacitor 1 and the right plate of capacitor 2 are connected to the terminals of a battery. other two plates are connected to each other and to nothing else.

The charges on capacitors connected in series are the same

$$\Delta V = \Delta V_1 + \Delta V_2$$

$$\Delta V_1 = \frac{Q}{C_1} \qquad \Delta V_2 = \frac{Q}{C_2}$$

$$\Delta V = \frac{Q}{C_{eq}}$$

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \qquad \text{(series combination)}$$

26.3 Combinations of Capacitors

Quick Quiz 26.3 Two capacitors are identical. They can be connected in series or in parallel. If you want the *smallest* equivalent capacitance for the combination, do you connect them in (a) series, in (b) parallel, or (c) do the combinations have the same capacitance?

Quick Quiz 26.4 Consider the two capacitors in Quick Quiz 26.3 again. Each capacitor is charged to a voltage of 10 V. If you want the largest combined potential difference across the combination, do you connect them in (a) series, in (b) parallel, or (c) do the combinations have the same potential difference?

PROBLEM-SOLVING HINTS

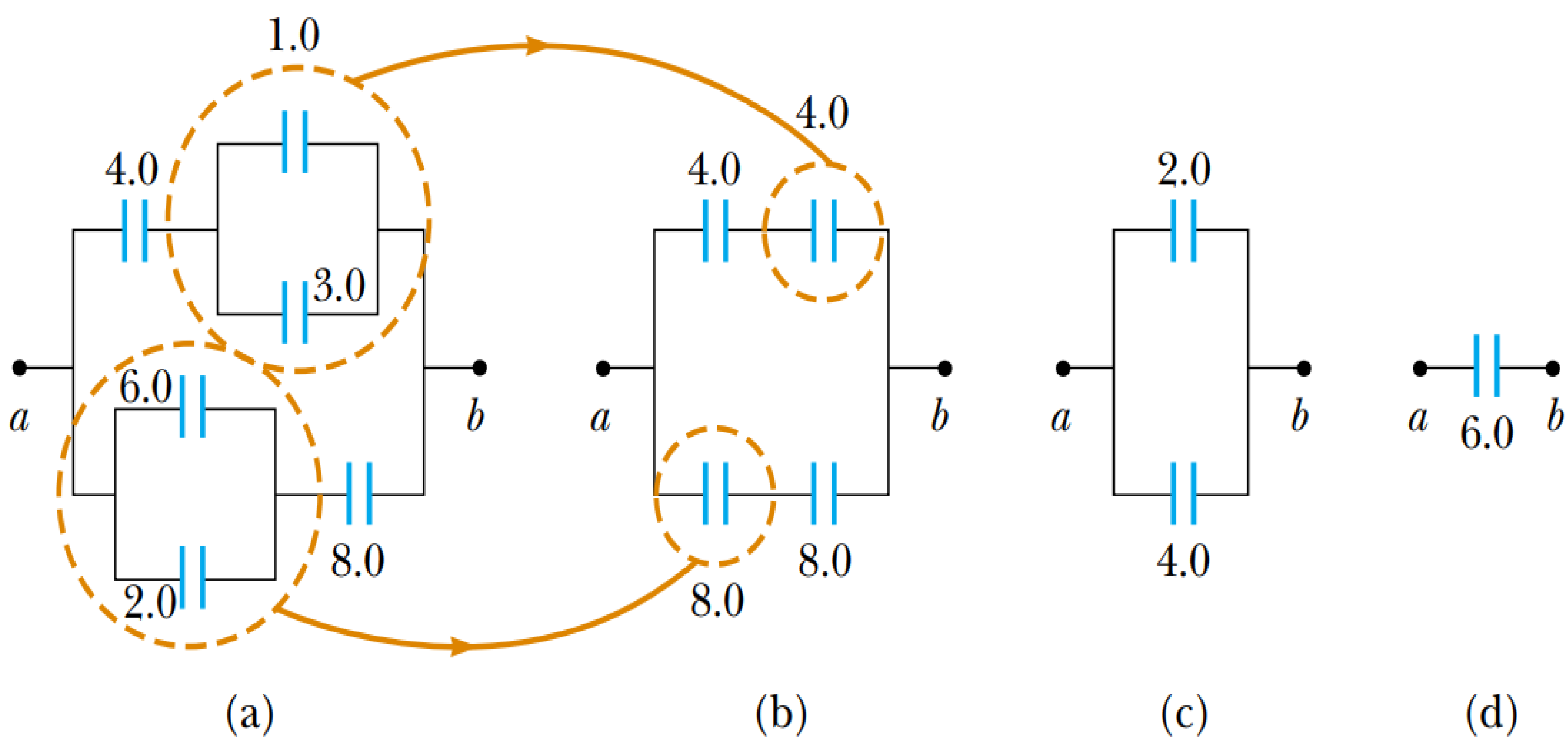
Capacitors

- Be careful with units. When you calculate capacitance in farads, make sure that distances are expressed in meters. When checking consistency of units, remember that the unit for electric fields can be either N/C or V/m.
- When two or more capacitors are connected in parallel, the potential difference across each is the same. The charge on each capacitor is proportional to its capacitance; hence, the capacitances can be added directly to give the equivalent capacitance of the parallel combination. The equivalent capacitance is always larger than the individual capacitances.
- When two or more capacitors are connected in series, they carry the same charge, and the sum of the potential differences equals the total potential difference applied to the combination. The sum of the reciprocals of the capacitances equals the reciprocal of the equivalent capacitance, which is always less than the capacitance of the smallest individual capacitor.

26.3 Combinations of Capacitors

Example 26.4 Equivalent Capacitance

Find the equivalent capacitance between a and b for the combination of capacitors shown in Figure 26.11a. All capacitances are in microfarads.



29. Find the equivalent capacitance between points *a* and *b* in the combination of capacitors shown in Figure P26.29.

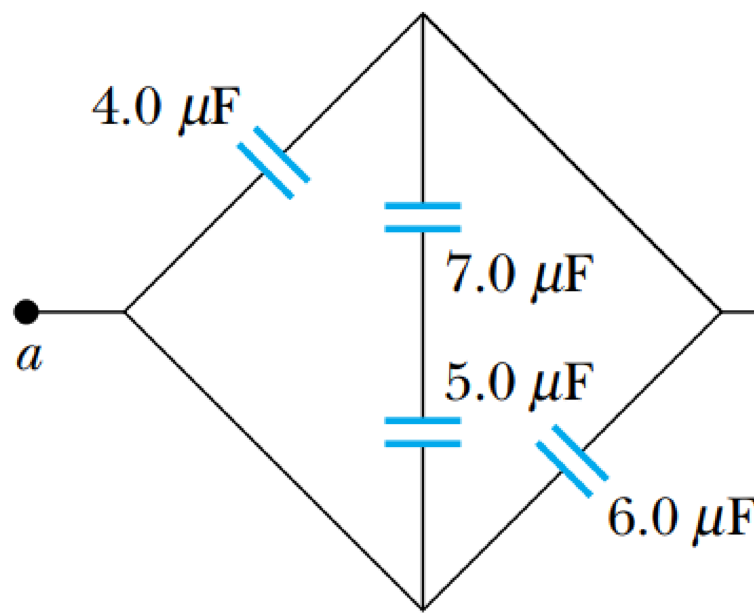


Figure P26.29

26.4 Energy Stored in a Charged Capacitor

❑ To calculate the energy stored in the capacitor

- We imagine that the charge is transferred mechanically through the space between the plates (grab a small amount of positive charge on the plate connected to the negative terminal and apply a force that causes this positive charge to move over to the plate connected to the positive terminal).

- No work is required to transfer a small amount of charge *dq* from one plate to the other. once this charge has been transferred, a small ΔV exists between the plates. Therefore, work must be done to move additional charge through this potential difference. As more and more charge is transferred from one plate to the other, the ΔV increases, and more work is required

- Suppose that *q* is the charge during the charging, and $\Delta V = q/C$, *so*, work necessary to transfer an increment of charge *dq* from the plate carrying charge -*q* to the plate carrying charge *q*

$$dW = \Delta V\, dq = \frac{q}{C}\, dq$$

26.4 Energy Stored in a Charged Capacitor

➢ The total work required to charge the capacitor from $q = 0$ to some final charge $q = Q$ is

$$W = \int_0^Q \frac{q}{C}\, dq = \frac{1}{C}\int_0^Q q\, dq = \frac{Q^2}{2C}$$

Energy stored in a charged capacitor

$$U = \frac{Q^2}{2C} = \tfrac{1}{2}Q\,\Delta V = \tfrac{1}{2}C(\Delta V)^2$$

➢ Stored energy increases as the ***Q*** increases and as the ***ΔV*** increases. This formula applies to any geometry

❖ For a parallel-plate capacitor geometry, sorted energy can be written as

$$U = \tfrac{1}{2}\frac{\epsilon_0 A}{d}(E^2 d^2) = \tfrac{1}{2}(\epsilon_0 A d)E^2$$

❖ *Energy density* (*energy per unit volume* $u_E = U/Ad$)

$$u_E = \tfrac{1}{2}\epsilon_0 E^2$$

u_E in any electric field is proportional to the square of the magnitude of the electric field at a given point.

26.4 Energy Stored in a Charged Capacitor

Quick Quiz 26.5 You have three capacitors and a battery. In which of the following combinations of the three capacitors will the maximum possible energy be stored when the combination is attached to the battery? (a) series (b) parallel (c) Both combinations will store the same amount of energy.

Quick Quiz 26.6 You charge a parallel-plate capacitor, remove it from the battery, and prevent the wires connected to the plates from touching each other. When you pull the plates apart to a larger separation, do the following quantities increase, decrease, or stay the same? (a) C; (b) Q; (c) E between the plates; (d) ΔV; (e) energy stored in the capacitor.

Quick Quiz 26.7 Repeat Quick Quiz 26.6, but this time answer the questions for the situation in which the battery remains connected to the capacitor while you pull the plates apart.

26.5 Capacitors with Dielectrics

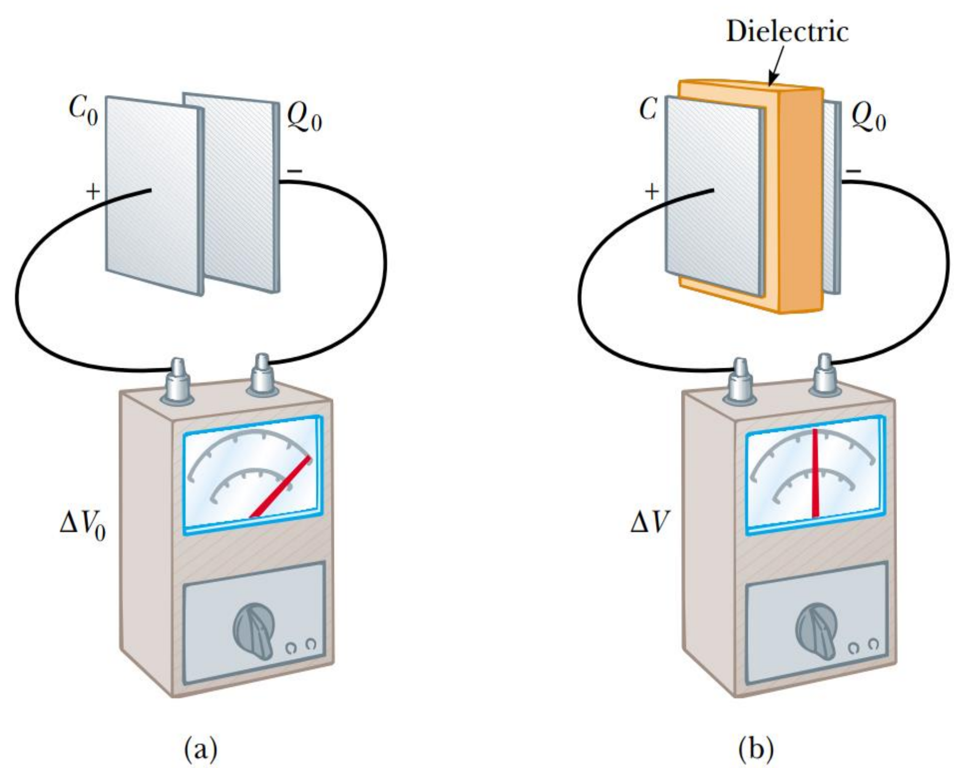
A **dielectric** is a nonconducting material, such as rubber, glass, or waxed paper.

When a dielectric is inserted between the plates of a capacitor, the capacitance increases. If the dielectric completely fills the space between the plates, the capacitance increases by a dimensionless factor κ, (called the **dielectric constant)** of the material which varies from one material to another.

- parallel-plate capacitor that without a dielectric has a charge Q_0 and a capacitance C_0

- potential difference across the capacitor is $\Delta V_0 = Q_0/C_0$. ΔV is measured by a *voltmeter*

- When a dielectric is now inserted between the plates, the voltmeter indicates that the voltage between the plates decreases to a value ΔV.



Charged capacitor (a) before and (b) after insertion of a dielectric between the plates.

26.5 Capacitors with Dielectrics

The voltages with and without the dielectric are related by the factor κ as follows:

$$\Delta V = \frac{\Delta V_0}{\kappa}$$

Because $\Delta V < \Delta V_0$, we see that $\kappa > 1$

Capacitance of a capacitor filled with a material of dielectric constant κ

$$C = \frac{Q_0}{\Delta V} = \frac{Q_0}{\Delta V_0/\kappa} = \kappa \frac{Q_0}{\Delta V_0}$$

$$C = \kappa C_0$$

Capacitance *increases* by the factor κ when the dielectric completely fills the region between the plates

parallel-plate capacitor,

$$C = \kappa \frac{\epsilon_0 A}{d}$$

26.5 Capacitors with Dielectrics

- **Dielectric strength** (maximum electric field)

If the magnitude of the electric field in the dielectric exceeds the dielectric strength, then the insulating properties break down and the dielectric begins to conduct.

Dielectric breakdown in air. Sparks are produced when the high voltage between the wires causes the electric field to exceed the dielectric strength of air

- *working voltage, breakdown voltage,* and *rated voltage of capacitor* represents the largest voltage that can be applied to the capacitor without exceeding the dielectric strength of the dielectric material in the capacitor.

Dielectric provides the following advantages:

- Increase in capacitance
- Increase in maximum operating voltage
- Possible mechanical support between the plates, which allows the plates to be close together without touching, thereby decreasing *d* and increasing *C*

26.5 Capacitors with Dielectrics Types of Capacitors

Approximate Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature

Material	Dielectric Constant κ	Dielectric Strength[a] (10^6 V/m)
Air (dry)	1.000 59	3
Bakelite	4.9	24
Fused quartz	3.78	8
Mylar	3.2	7
Neoprene rubber	6.7	12
Nylon	3.4	14
Paper	3.7	16
Paraffin-impregnated paper	3.5	11
Polystyrene	2.56	24
Polyvinyl chloride	3.4	40
Porcelain	6	12
Pyrex glass	5.6	14
Silicone oil	2.5	15
Strontium titanate	233	8
Teflon	2.1	60
Vacuum	1.000 00	—
Water	80	—

The dielectric strength equals the maximum electric field that can exist in a dielectric without electrical breakdown.

26.5 Capacitors with Dielectrics

Quick Quiz 26.9 A fully charged parallel-plate capacitor remains connected to a battery while you slide a dielectric between the plates. Do the following quantities increase, decrease, or stay the same? (a) C; (b) Q; (c) E between the plates; (d) ΔV.

Example 26.6 A Paper-Filled Capacitor

A parallel-plate capacitor has plates of dimensions 2.0 cm by 3.0 cm separated by a 1.0-mm thickness of paper.

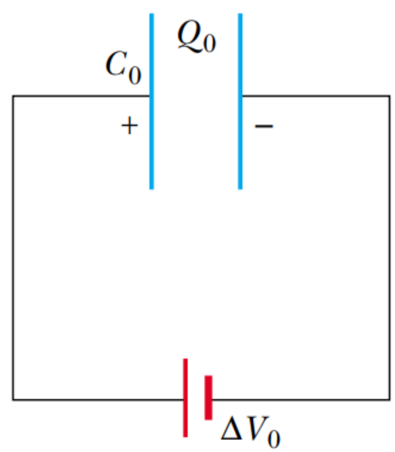
(A) Find its capacitance.

(B) What is the maximum charge that can be placed on the capacitor?

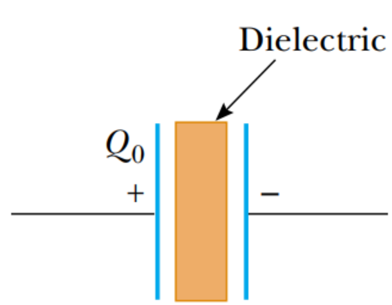
26.5 Capacitors with Dielectrics

Example 26.7 Energy Stored Before and After

A parallel-plate capacitor is charged with a battery to a charge Q_0, as shown in Figure 26.20a. The battery is then removed, and a slab of material that has a dielectric constant κ is inserted between the plates, as shown in Figure 26.20b. Find the energy stored in the capacitor before and after the dielectric is inserted.



(a)



(b)

Solution From Equation 26.11, we see that the energy stored in the absence of the dielectric is

$$U_0 = \frac{Q_0^2}{2C_0}$$

After the battery is removed and the dielectric inserted, the *charge on the capacitor remains the same.* Hence, the energy stored in the presence of the dielectric is

$$U = \frac{Q_0^2}{2C}$$

But the capacitance in the presence of the dielectric is $C = \kappa C_0$, so U becomes

$$U = \frac{Q_0^2}{2\kappa C_0} = \frac{U_0}{\kappa}$$

18. Evaluate the equivalent capacitance of the configuration shown in Figure P26.18. All the capacitors are identical, and each has capacitance C.

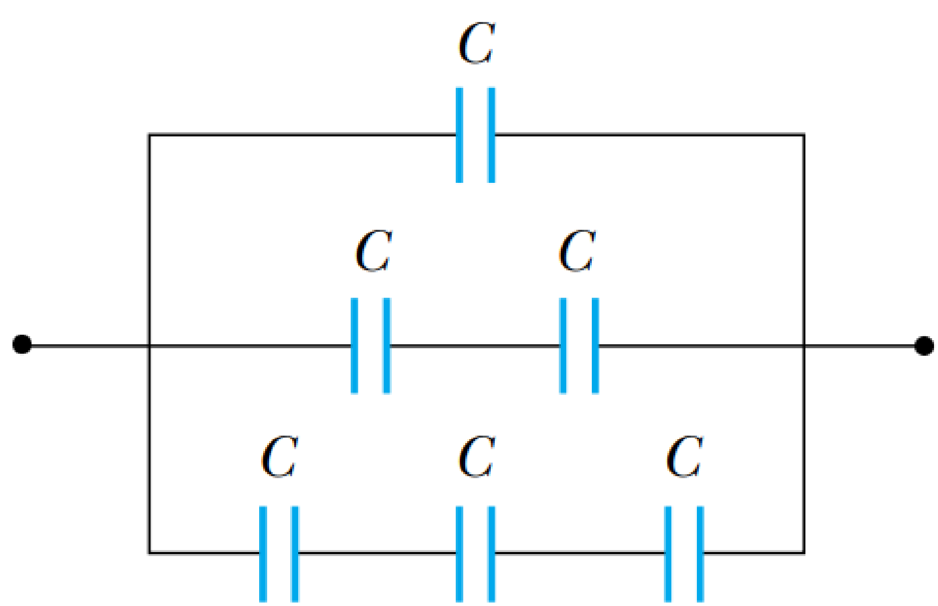


Figure P26.18

21. Four capacitors are connected as shown in Figure P26.21. (a) Find the equivalent capacitance between points a and b. (b) Calculate the charge on each capacitor if $\Delta V_{ab} = 15.0$ V.

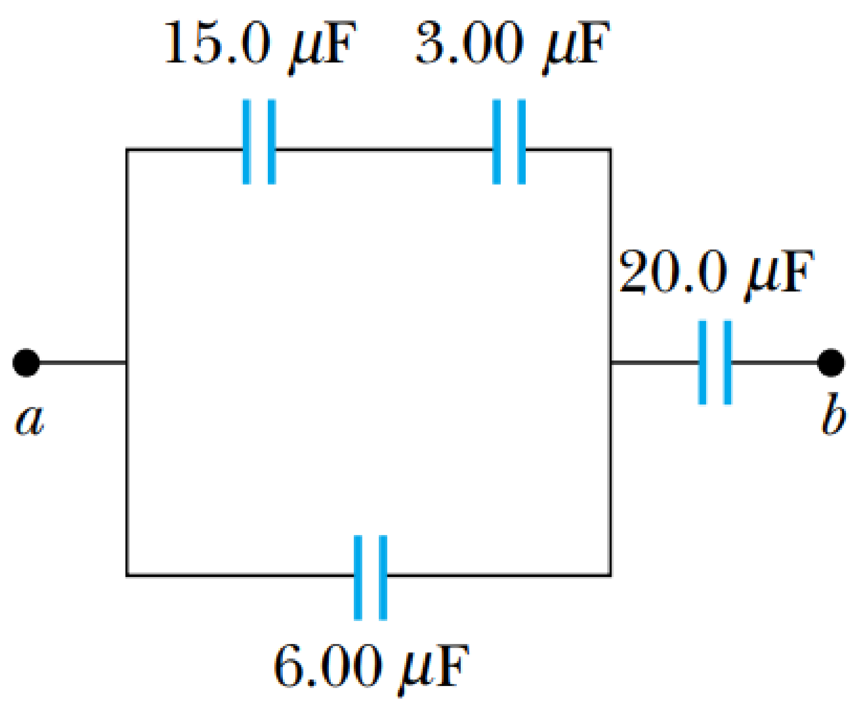


Figure P26.21

Section 26.4 Energy Stored in a Charged Capacitor

31. (a) A 3.00-μF capacitor is connected to a 12.0-V battery. How much energy is stored in the capacitor? (b) If the capacitor had been connected to a 6.00-V battery, how much energy would have been stored?

36. A uniform electric field $E = 3\,000$ V/m exists within a certain region. What volume of space contains an energy equal to 1.00×10^{-7} J? Express your answer in cubic meters and in liters.

54. For the system of capacitors shown in Figure P26.54, find (a) the equivalent capacitance of the system, (b) the potential across each capacitor, (c) the charge on each capacitor, and (d) the total energy stored by the group.

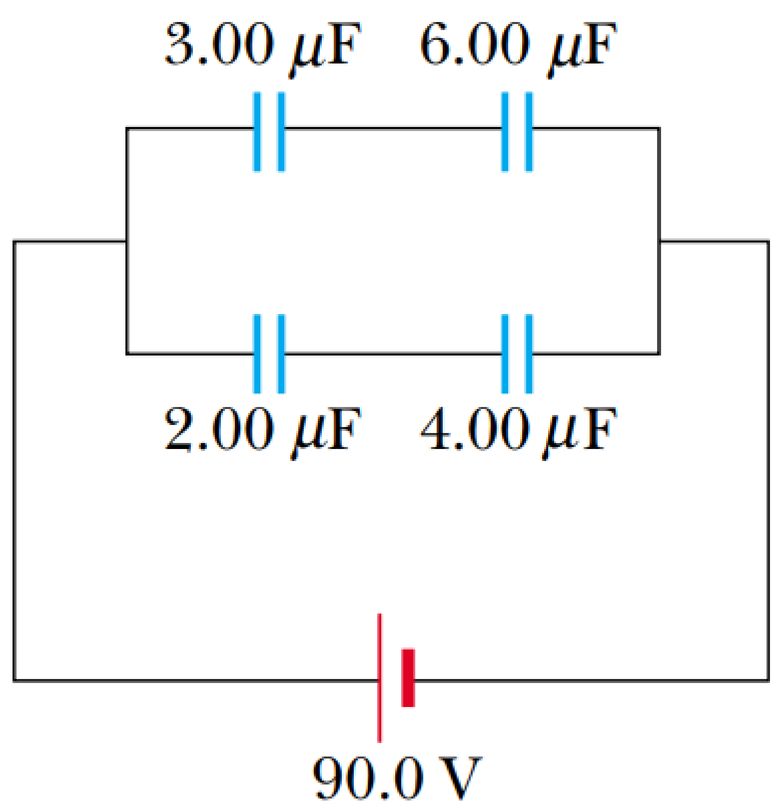


Figure P26.54