Chapter 25

Electric Potential

Electric Potential

Electromagnetism has been connected to the study of forces in previous chapters.

In this chapter, electromagnetism will be linked to energy.

By using an energy approach, problems could be solved that were insoluble using forces.

The concept of potential energy is of great value in the study of electricity.

Because the electrostatic force is conservative, electrostatic phenomena can be conveniently described in terms of an electric potential energy.

This will enable the definition of *electric potential.*

Electrical Potential Energy

When a test charge is placed in an electric field, it experiences a force.

- $\vec{\mathbf{F}}_e = q_o\vec{\mathbf{E}}$
- The force is conservative.

If the test charge is moved in the field by some external agent, the work done by the field is the negative of the work done by the external agent.

$d\vec{\mathbf{s}}$ is an infinitesimal displacement vector that is oriented tangent to a path through space.

- The path may be straight or curved and the integral performed along this path is called either a *path integral* or a *line integral.*

Electric Potential Energy, cont

The work done within the charge-field system by the electric field on the charge is

$$\vec{\mathbf{F}} \cdot d\vec{\mathbf{s}} = q_o \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

As this work is done by the field, the potential energy of the charge-field system is changed by $\Delta U = -q_o \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$

For a finite displacement of the charge from A to B, the change in potential energy of the system is

$$\Delta U = U_B - U_A = -q_o \int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

Because the force is conservative, the line integral does not depend on the path taken by the charge.

Electric Potential

The potential energy per unit charge, U/q_o, is the **electric potential.**

- The potential is characteristic of the field only.
  - The potential energy is characteristic of the charge-field system.
- The potential is independent of the value of q_o.
- The potential has a value at every point in an electric field.

The electric potential is

$$V = \frac{U}{q_o}$$

Electric Potential, cont.

The potential is a scalar quantity.

- Since energy is a scalar

As a charged particle moves in an electric field, it will experience a change in potential.

$$\Delta V = \frac{\Delta U}{q_o} = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

The infinitesimal displacement is interpreted as the displacement between two points in space rather than the displacement of a point charge.

Electric Potential, final

The difference in potential is the meaningful quantity.

We often take the value of the potential to be zero at some convenient point in the field.

Electric potential is a scalar characteristic of an electric field, independent of any charges that may be placed in the field.

The potential difference between two points exists solely because of a source charge and depends on the source charge distribution.

- For a potential energy to exist, there must be a system of two or more charges.
- The potential energy belongs to the system and changes only if a charge is moved relative to the rest of the system.

Work and Electric Potential

Assume a charge moves in an electric field without any change in its kinetic energy.

The work performed on the charge is

$$W = \Delta U = q\,\Delta V$$

Units: $1\ \text{V} \equiv 1\ \text{J/C}$

- V is a volt.
- It takes one joule of work to move a 1-coulomb charge through a potential difference of 1 volt.

In addition, $1\ \text{N/C} = 1\ \text{V/m}$

- This indicates we can interpret the electric field as a measure of the rate of change of the electric potential with respect to position.

Voltage

Electric potential is described by many terms.

The most common term is *voltage.*

A voltage applied to a device or across a device is the same as the potential difference across the device.

- The voltage is not something that moves through a device.

Electron-Volts

Another unit of energy that is commonly used in atomic and nuclear physics is the electron-volt.

One ***electron-volt*** is defined as the energy a charge-field system gains or loses when a charge of magnitude *e* (an electron or a proton) is moved through a potential difference of 1 volt.

- 1 eV = 1.60×10^{-19} J

Problem 25.2

An ion accelerated through a potential difference of 115 V experiences an increase in kinetic energy of 7.37×10^{-17} J. Calculate the charge on the ion.

$$\Delta K = q|\Delta V| \qquad 7.37 \times 10^{-17} = q(115)$$

$$\boxed{q = 6.41 \times 10^{-19}\ \text{C}}$$

Problem 25.3

(a) Calculate the speed of a proton that is accelerated from rest through a potential difference of 120 V. (b) Calculate the speed of an electron that is accelerated through the same potential difference.

(a) Energy of the proton-field system is conserved as the proton moves from high to low potential, which can be defined for this problem as moving from 120 V down to 0 V.

$$K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f \qquad 0 + qV + 0 = \frac{1}{2}mv_p^2 + 0$$

$$\left(1.60 \times 10^{-19}\ \text{C}\right)(120\ \text{V})\left(\frac{1\ \text{J}}{1\ \text{V}\cdot\text{C}}\right) = \frac{1}{2}\left(1.67 \times 10^{-27}\ \text{kg}\right)v_p^2$$

$$v_p = \boxed{1.52 \times 10^5\ \text{m/s}}$$

(b) The electron will gain speed in moving the other way,

from $V_i = 0$ to $V_f = 120$ V: $\qquad K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f$

$$0 + 0 + 0 = \frac{1}{2}mv_e^2 + qV$$

$$0 = \frac{1}{2}\left(9.11 \times 10^{-31}\ \text{kg}\right)v_e^2 + \left(-1.60 \times 10^{-19}\ \text{C}\right)(120\ \text{J/C})$$

$$v_e = \boxed{6.49 \times 10^6\ \text{m/s}}$$

Potential Difference in a Uniform Field

The equations for electric potential between two points A and B can be simplified if the electric field is uniform:

$$V_B - V_A = \Delta V = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -E\int_A^B d\mathbf{s} = -Ed$$

The displacement points from A to B and is parallel to the field lines.

The negative sign indicates that the electric potential at point *B* is lower than at point *A.*

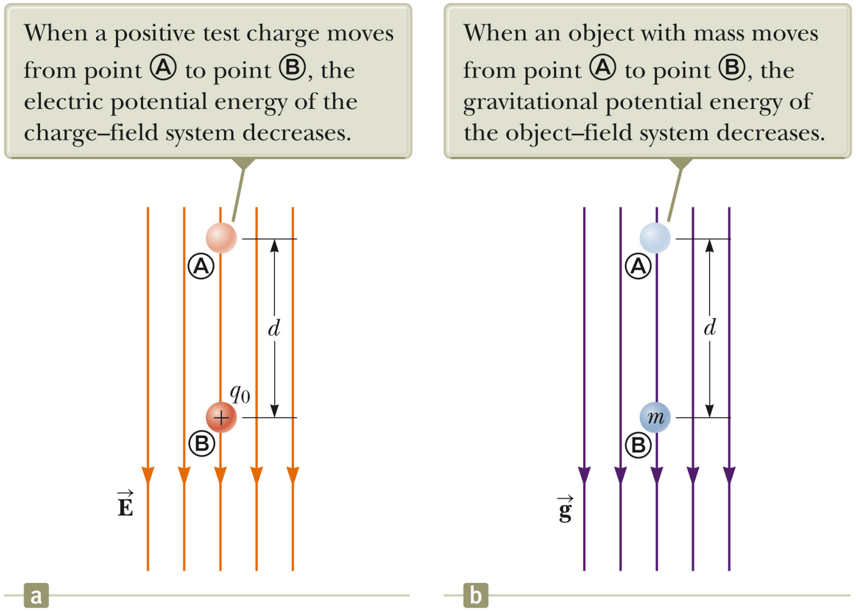
- Electric field lines always point in the direction of decreasing electric potential.

Energy and the Direction of Electric Field

When the electric field is directed downward, point *B* is at a lower potential than point *A.*

When a positive test charge moves from *A* to *B*, the charge-field system loses potential energy.

Electric field lines always point in the direction of decreasing electric potential.



More About Directions

A system consisting of a positive charge and an electric field **loses** electric potential energy when the charge moves in the direction of the field.

- An electric field does work on a positive charge when the charge moves in the direction of the electric field.

The charged particle gains kinetic energy and the potential energy of the charge-field system decreases by an equal amount.

- Another example of Conservation of Energy

Directions, cont.

If q_o is negative, then ΔU is positive.

A system consisting of a negative charge and an electric field *gains* potential energy when the charge moves in the direction of the field.

- In order for a negative charge to move in the direction of the field, an external agent must do positive work on the charge.

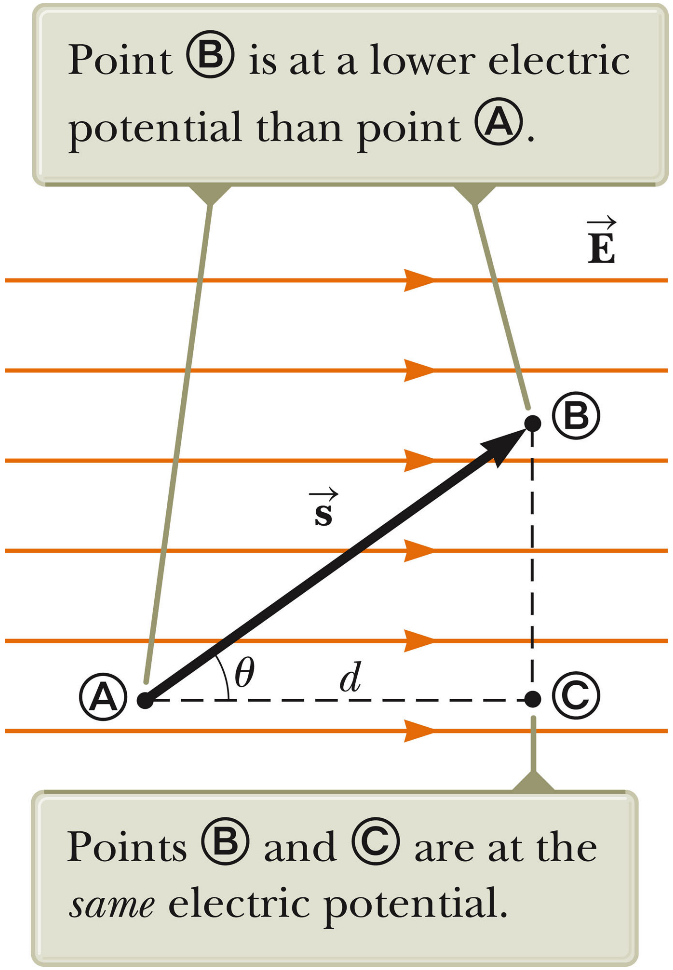
Equipotentials

Point *B* is at a lower potential than point *A.*

Points *B* and *C* are at the same potential.

- All points in a plane perpendicular to a uniform electric field are at the same electric potential.

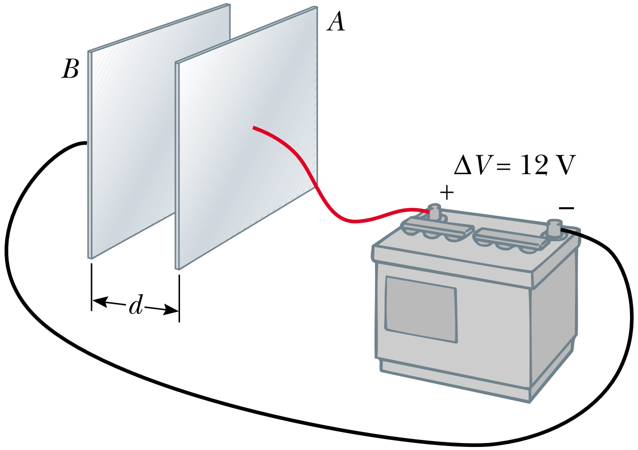
The name **equipotential surface** is given to any surface consisting of a continuous distribution of points having the same electric potential.



Example 25.1 The Electric Field Between Two Parallel Plates of Opposite Charge

The separation between the plates is

d = 0.3 cm. Find the magnitude of the electric field between the plates.



Sol:

We use

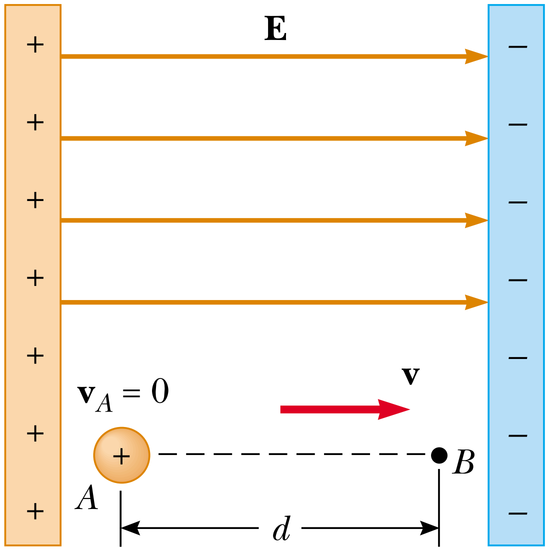
$$V_B - V_A = \Delta V = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -E\int_A^B d\mathbf{s} = -Ed$$

to get

$$E = \frac{|V_B - V_A|}{d} = \frac{12 \text{ V}}{0.30 \times 10^{-2} \text{ m}} = 4.0 \times 10^3 \text{ V/m}$$

Charged Particle in a Uniform Field, Example

A proton is released from rest in a uniform electric field that has a magnitude of 8.0 x 10^4 V/m. The proton undergoes a displacement of 0.50 m in the direction of **E**.



(A) Find the change in electric potential between points A and B.

$$\Delta V = -Ed = -(8.0 \times 10^4 \text{ V/m})(0.50 \text{ m}) = -4.0 \times 10^4 \text{ V}$$

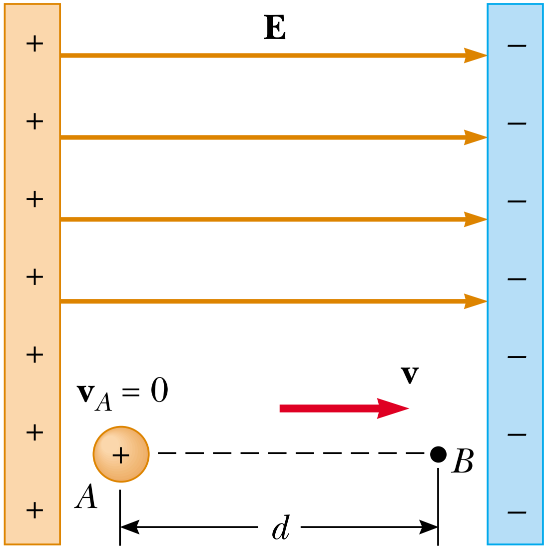
(B) Find the change in potential energy of the proton–field system for this displacement.

$$\Delta U = q_0 \, \Delta V = e \, \Delta V$$
$$= (1.6 \times 10^{-19} \text{ C})(-4.0 \times 10^4 \text{ V})$$
$$= -6.4 \times 10^{-15} \text{ J}$$

Charged Particle in a Uniform Field, Example

A proton is released from rest in a uniform electric field that has a magnitude of 8.0 x 10^4 V/m. The proton undergoes a displacement of 0.50 m in the direction of **E**.

(C) Find the speed of the proton after completing the 0.50 m displacement in the electric field.



$$\Delta K + \Delta U = 0$$

$$(\tfrac{1}{2}mv^2 - 0) + e\,\Delta V = 0$$

$$v = \sqrt{\frac{-(2e\,\Delta V)}{m}}$$

$$= \sqrt{\frac{-2(1.6 \times 10^{-19}\ \mathrm{C})(-4.0 \times 10^4\ \mathrm{V})}{1.67 \times 10^{-27}\ \mathrm{kg}}}$$

$$= 2.8 \times 10^6\ \mathrm{m/s}$$

Problem 25.6

The difference in potential between the accelerating plates in the electron gun of a TV picture tube is about 25 000 V. If the distance between these plates is 1.50 cm, what is the magnitude of the uniform electric field in this region?

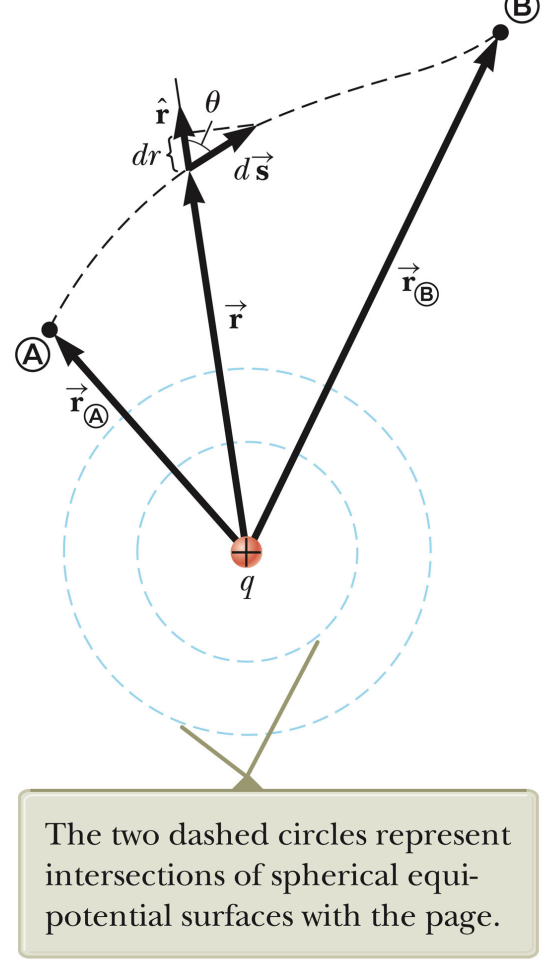
$$E = \frac{|\Delta V|}{d} = \frac{25.0 \times 10^{3}\ \text{J/C}}{1.50 \times 10^{-2}\ \text{m}} = 1.67 \times 10^{6}\ \text{N/C} = \boxed{1.67\ \text{MN/C}}$$

Potential and Point Charges

An isolated positive point charge produces a field directed radially outward.

The potential difference between points *A* and *B* will be

$$V_B - V_A = k_e q\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$



Potential and Point Charges, cont.

The electric potential is independent of the path between points *A* and *B.*

It is customary to choose a reference potential of $V = 0$ at $r_A = \infty$.

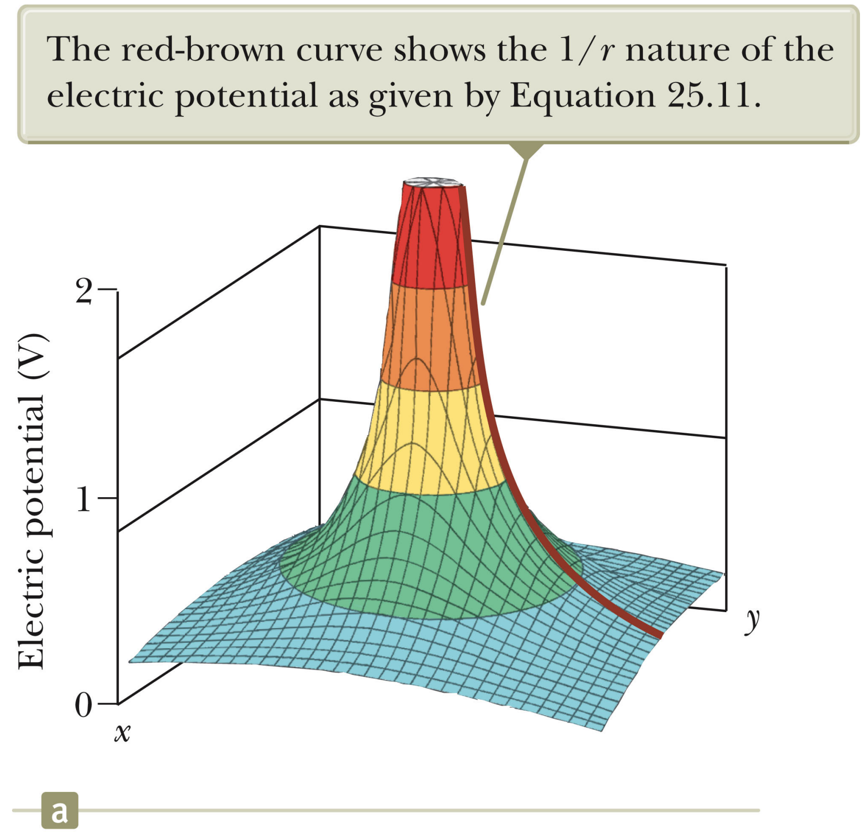
Then the potential due to a point charge at some point *r* is

$$V = k_e \frac{q}{r}$$

Electric Potential of a Point Charge

The electric potential in the plane around a single point charge is shown.

The red line shows the $1/r$ nature of the potential.



Electric Potential with Multiple Charges

The electric potential due to several point charges is the sum of the potentials due to each individual charge.

- This is another example of the superposition principle.
- The sum is the algebraic sum

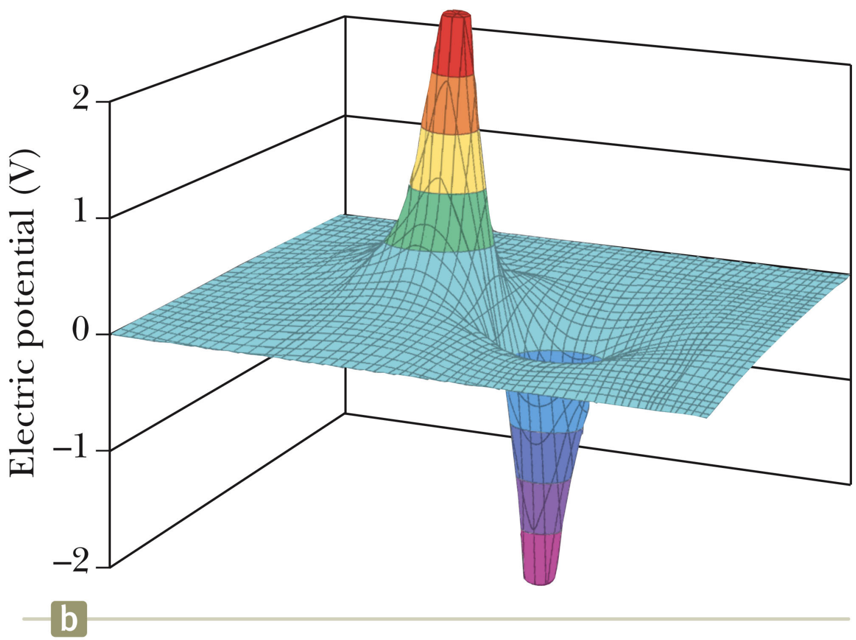
$$V = k_e \sum_i \frac{q_i}{r_i}$$

  - $V = 0$ at $r = \infty$

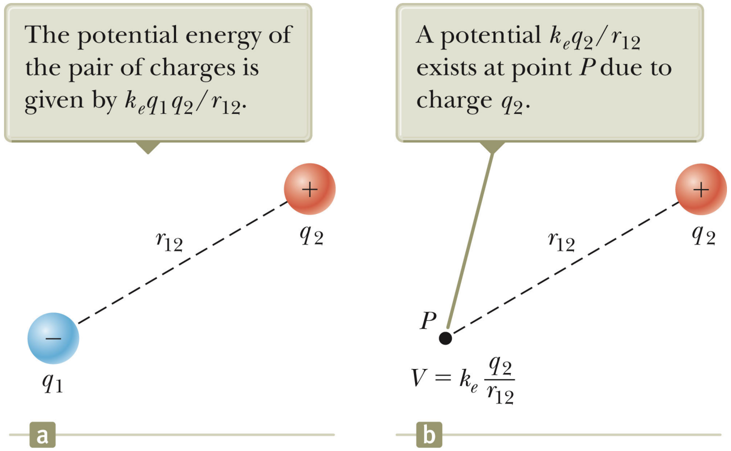
Electric Potential of a Dipole

The graph shows the potential (y-axis) of an electric dipole.

The steep slope between the charges represents the strong electric field in this region.



Potential Energy of Multiple Charges



The potential energy of the system is $U = k_e \dfrac{q_1 q_2}{r_{12}}$.

If the two charges are the same sign, U is positive and work must be done to bring the charges together.

If the two charges have opposite signs, U is negative and work is done to keep the charges apart.

U with Multiple Charges, final

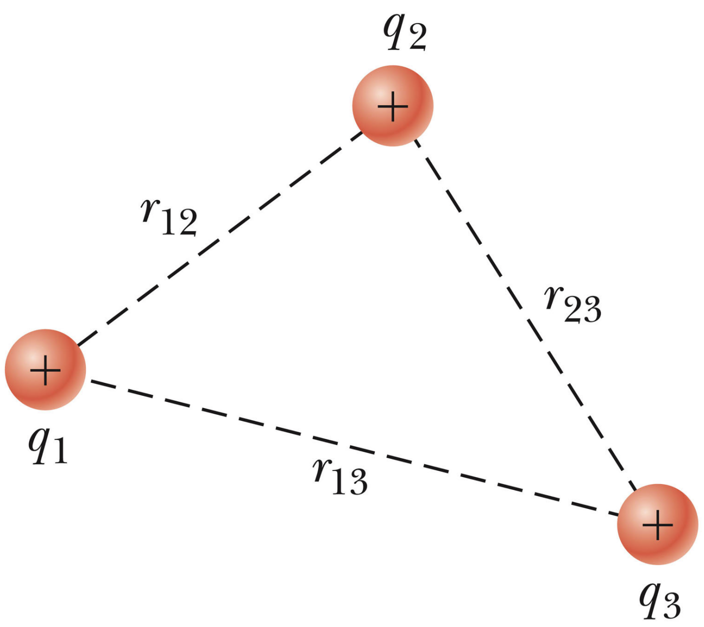
If there are more than two charges, then find U for each pair of charges and add them.

For three charges:

$$U = k_e\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)$$

- The result is independent of the order of the charges.

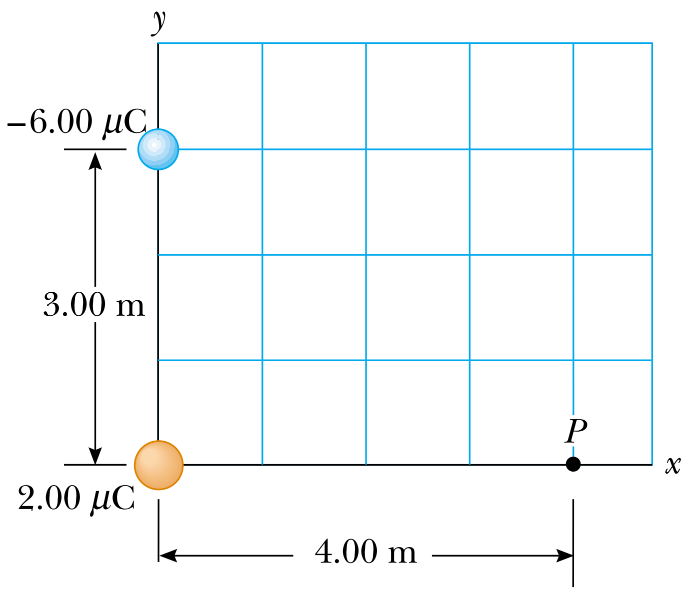
The potential energy of this system of charges is given by Equation 25.14.



Example 25.3 The Electric Potential Due to Two Point Charges

A charge q_1= 2.00 μC is located at the origin, and a charge q_2 = - 6.00 μC is located at (0, 3.00) m, as shown in the Figure.

(A) Find the total electric potential due to these charges at the point P, whose coordinates are (4.00, 0) m.



$$V_P = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

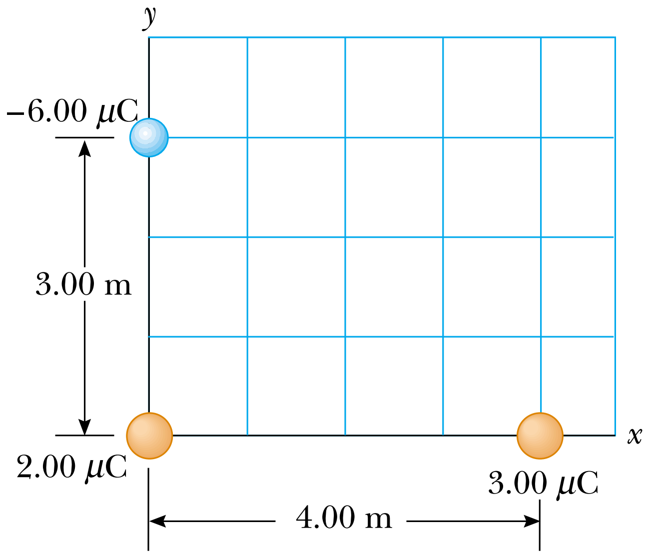
$$V_P = (8.99 \times 10^9\ \mathrm{N\cdot m^2/C^2}) \times \left(\frac{2.00 \times 10^{-6}\ \mathrm{C}}{4.00\ \mathrm{m}} - \frac{6.00 \times 10^{-6}\ \mathrm{C}}{5.00\ \mathrm{m}}\right)$$

$$= -6.29 \times 10^3\ \mathrm{V}$$

Example 25.3 The Electric Potential Due to Two Point Charges

A charge q_1= 2.00 μC is located at the origin, and a charge q_2 = - 6.00 μC is located at (0, 3.00) m, as shown in the Figure.

(A) Find the change in potential energy of the system of two charges plus a charge q_3 = 3.00 μC as the latter charge moves from infinity to point P.



$$\Delta U = q_3 V_P - 0 = (3.00 \times 10^{-6}\ \mathrm{C})(-6.29 \times 10^{3}\ \mathrm{V})$$

$$= -1.89 \times 10^{-2}\ \mathrm{J}$$

Problem 25.16

Given two 2.00-μC charges, as shown in Figure P25.16, and a positive test charge $q = 1.28 \times 10^{-18}$ C at the origin, (a) what is the net force exerted by the two 2.00-μC charges on the test charge q? (b) What is the electric field at the origin due to the two 2.00-μC charges? (c) What is the electric potential at the origin due to the two 2.00-μC charges?

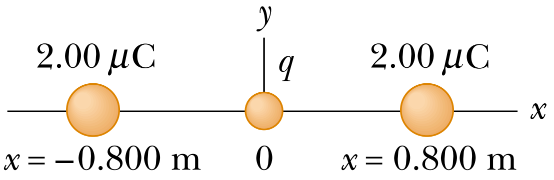


Figure P25.16

(a) Since the charges are equal and placed symmetrically, $\boxed{F = 0}$.

(b) Since $F = qE = 0$, $\boxed{E = 0}$.

(c) $$V = 2k_e \frac{q}{r} = 2\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(\frac{2.00 \times 10^{-6} \text{ C}}{0.800 \text{ m}}\right)$$

$$V = 4.50 \times 10^4 \text{ V} = \boxed{45.0 \text{ kV}}$$

Problem 25.17

At a certain distance from a point charge, the magnitude of the electric field is 500 V/m and the electric potential is −3.00 kV. (a) What is the distance to the charge? (b) What is the magnitude of the charge?

(a)

$$E = \frac{|Q|}{4\pi \epsilon_0 r^2}$$

$$V = \frac{Q}{4\pi \epsilon_0 r}$$

$$r = \frac{|V|}{|E|} = \frac{3\,000 \text{ V}}{500 \text{ V/m}} = \boxed{6.00 \text{ m}}$$

(b)

$$V = -3\,000 \text{ V} = \frac{Q}{4\pi \epsilon_0 (6.00 \text{ m})}$$

$$Q = \frac{-3\,000 \text{ V}}{\left(8.99 \times 10^9 \text{ V} \cdot \text{m/C}\right)}(6.00 \text{ m}) = \boxed{-2.00\ \mu\text{C}}$$

Problem 25.20

Two point charges, $Q_1 = +5.00$ nC and $Q_2 = -3.00$ nC, are separated by 35.0 cm. (a) What is the potential energy of the pair? What is the significance of the algebraic sign of your answer? (b) What is the electric potential at a point midway between the charges?

(a) $$U = \frac{qQ}{4\pi \in_0 r} = \frac{(5.00 \times 10^{-9}\ \text{C})(-3.00 \times 10^{-9}\ \text{C})(8.99 \times 10^{9}\ \text{V} \cdot \text{m/C})}{(0.350\ \text{m})} = \boxed{-3.86 \times 10^{-7}\ \text{J}}$$

The minus sign means it takes 3.86×10^{-7} J to pull the two charges apart from 35 cm to a much larger separation.

(b) $$V = \frac{Q_1}{4\pi \in_0 r_1} + \frac{Q_2}{4\pi \in_0 r_2}$$

$$= \frac{(5.00 \times 10^{-9}\ \text{C})(8.99 \times 10^{9}\ \text{V} \cdot \text{m/C})}{0.175\ \text{m}} + \frac{(-3.00 \times 10^{-9}\ \text{C})(8.99 \times 10^{9}\ \text{V} \cdot \text{m/C})}{0.175\ \text{m}}$$

$$V = \boxed{103\ \text{V}}$$