Introduction to Energy

A variety of problems can be solved with Newton's Laws and associated principles.

Some problems that could theoretically be solved with Newton's Laws are very difficult in practice.

- These problems can be made easier with other techniques.

The concept of energy is one of the most important topics in science and engineering.

Every physical process that occurs in the Universe involves energy and energy transfers or transformations.

Energy is not easily defined.

Work

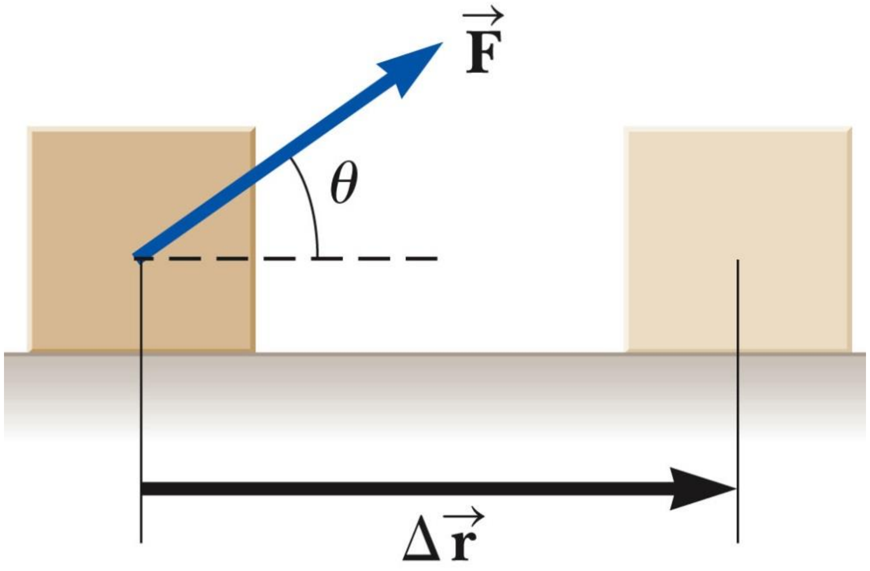
The **work**, W, done on a system by an agent exerting a constant force on the system is the product of the magnitude F of the force, the magnitude Δr of the displacement of the point of application of the force, and cos θ, where θ is the angle between the force and the displacement vectors.

- The meaning of the term *work* is distinctly different in physics than in everyday meaning.
- Work is done *by* some part of the environment that is interacting directly with the system.
- Work is done *on* the system.

Work, cont.

$W = F \Delta r \cos \theta$

- The displacement is that of the point of application of the force.
- A force does no work on the object if the force does not move through a displacement.
- The work done by a force on a moving object is zero when the force applied is perpendicular to the displacement of its point of application.



Displacement in the Work Equation

The displacement is that of the point of application of the force.

If the force is applied to a rigid object that can be modeled as a particle, the displacement is the same as that of the particle.

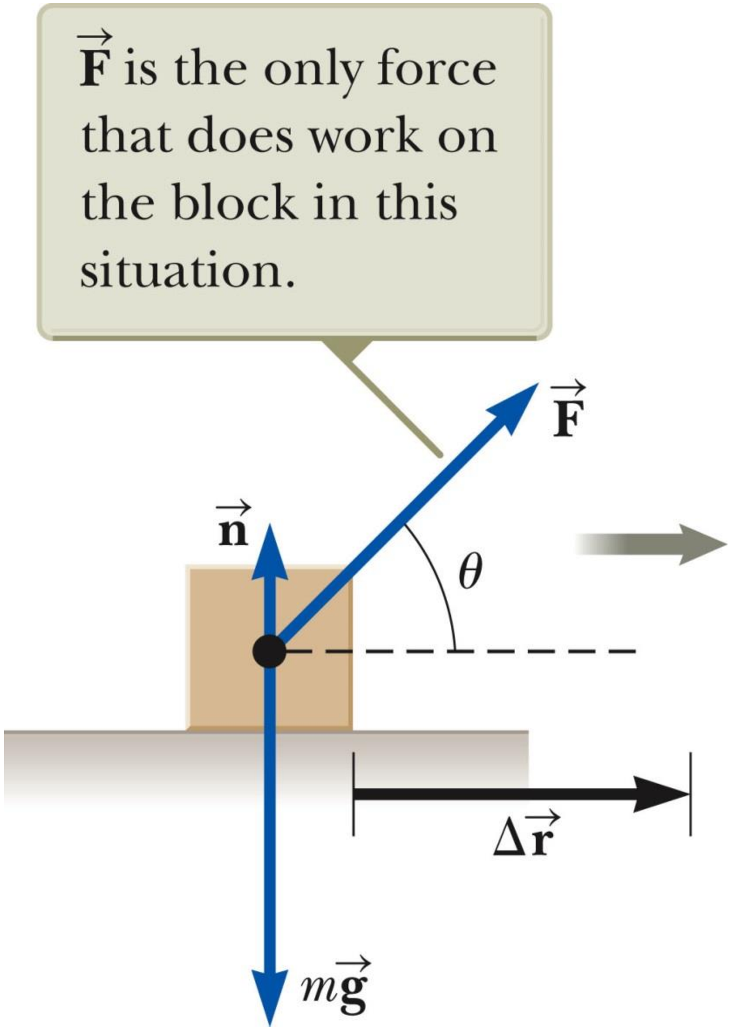
For a deformable system, the displacement of the object generally is not the same as the displacement associated with the forces applied.

Work Example

The normal force and the gravitational force do no work on the object.

- $\cos\theta = \cos 90^\circ = 0$

The force $\vec{\mathbf{F}}$ is the only force that does work on the object.



More About Work

The sign of the work depends on the direction of the force relative to the displacement.

- Work is positive when projection of $\vec{\mathbf{F}}$ onto $\Delta\vec{\mathbf{r}}$ is in the same direction as the displacement.
- Work is negative when the projection is in the opposite direction.

The work done by a force can be calculated, but that force is not necessarily the cause of the displacement.

Work is a scalar quantity.

The unit of work is a joule (J)

- 1 joule = 1 newton · 1 meter = $kg \cdot m^2 / s^2$
- $J = N \cdot m$

Work Is An Energy Transfer

This is important for a system approach to solving a problem.

If the work is done on a system and it is positive, energy is transferred to the system.

If the work done on the system is negative, energy is transferred from the system.

If a system interacts with its environment, this interaction can be described as a transfer of energy across the system boundary.

- This will result in a change in the amount of energy stored in the system.

Scalar Product of Two Vectors

The scalar product of two vectors is written as $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$.

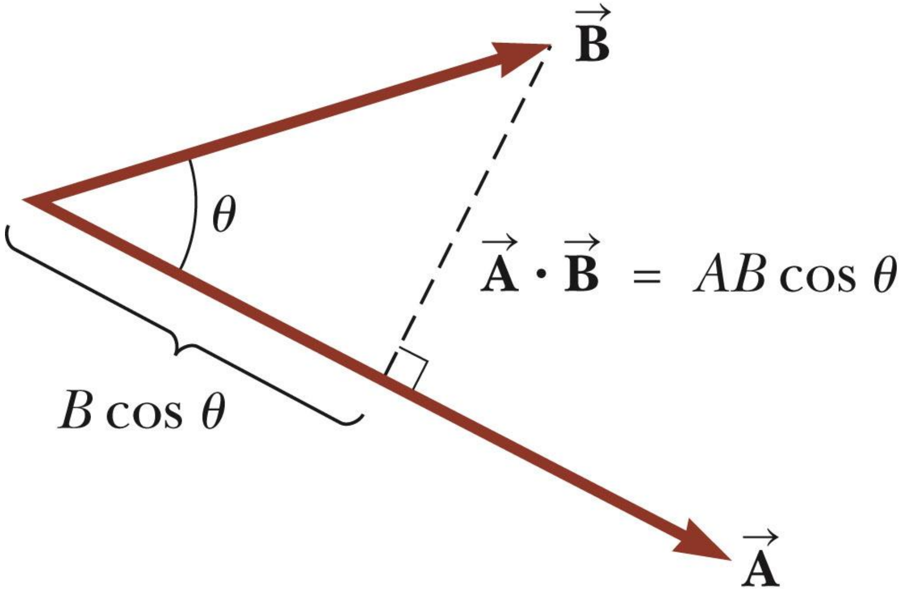
- It is also called the dot product.

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} \equiv A\,B\cos\theta$$

- θ is the angle *between* *A* and *B*

Applied to work, this means

$$W = F\Delta r\cos\theta = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}}$$



Scalar Product, cont

The scalar product is commutative.

- $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}}$

The scalar product obeys the distributive law of multiplication.

- $\vec{\mathbf{A}} \cdot \left(\vec{\mathbf{B}} + \vec{\mathbf{C}} \right) = \vec{\mathbf{A}} \cdot \vec{\mathbf{B}} + \vec{\mathbf{A}} \cdot \vec{\mathbf{C}}$

Dot Products of Unit Vectors

$$\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{k}} = 1$$

$$\hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{i}} \cdot \hat{\mathbf{k}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{k}} = 0$$

Using component form with vectors:

$$\vec{\mathbf{A}} = A_x \hat{\mathbf{i}} + A_y \hat{\mathbf{j}} + A_z \hat{\mathbf{k}}$$

$$\vec{\mathbf{B}} = B_x \hat{\mathbf{i}} + B_y \hat{\mathbf{j}} + B_z \hat{\mathbf{k}}$$

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = A_x B_x + A_y B_y + A_z B_z$$

In the special case where

$$\vec{\mathbf{A}} = \vec{\mathbf{B}};$$

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{A}} = A_x^2 + A_y^2 + A_z^2 = A^2$$

Work Done by a Varying Force

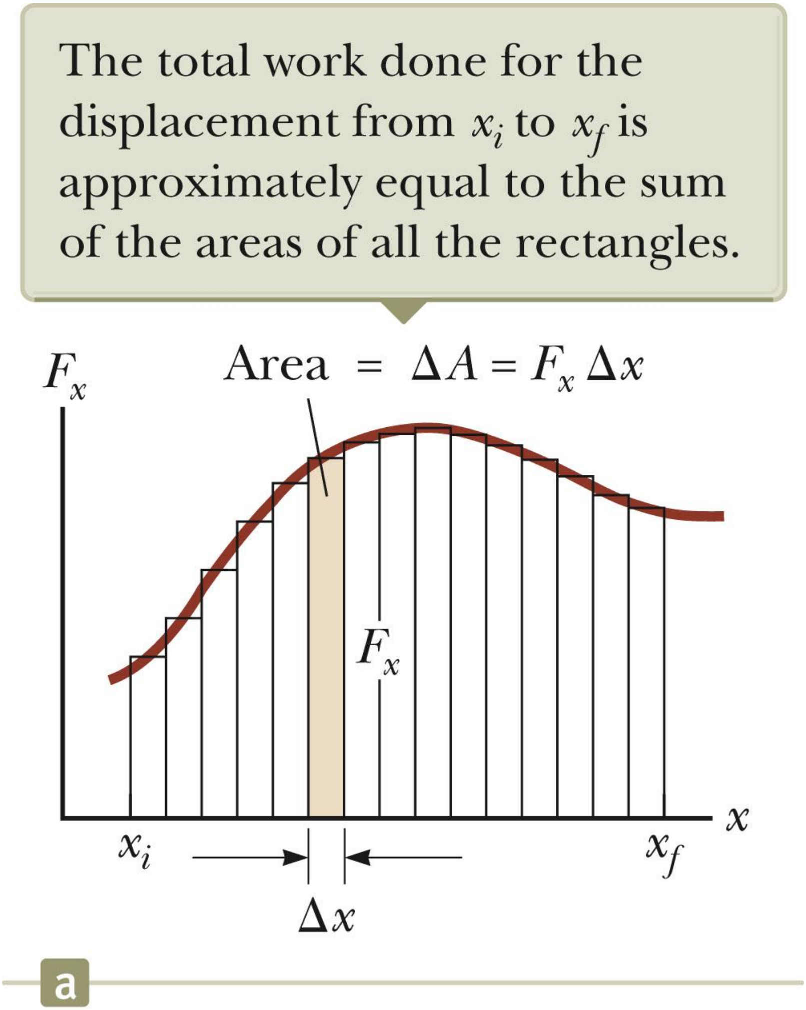
To use W = F Δ r cos θ, the force must be constant, so the equation cannot be used to calculate the work done by a varying force.

Assume that during a very small displacement, Δx, F is constant.

For that displacement, $W \sim F \Delta x$

For all of the intervals,

$$W \approx \sum_{x_i}^{x_f} F_x \Delta x$$



Work Done by a Varying Force, cont.

Let the size of the small displacements approach zero .

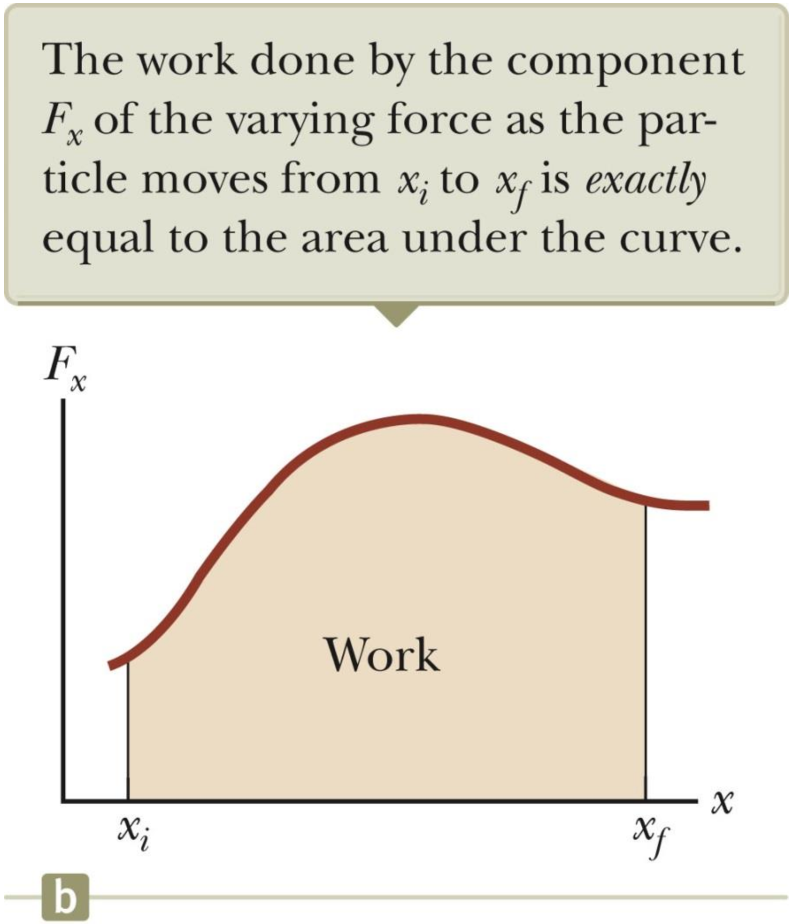
Since

$$\lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x dx$$

Therefore,

$$W = \int_{x_i}^{x_f} F_x dx$$

The work done is equal to the area under the curve between x_i and x_f.



Work Done By Multiple Forces

If more than one force acts on a system *and the system can be modeled as a particle*, the total work done on the system is the work done by the net force.

$$\sum W = W_{ext} = \int_{x_i}^{x_f} \left(\sum F_x \right) dx$$

In the general case of a net force whose magnitude and direction may vary.

$$\sum W = W_{ext} = \int_{x_i}^{x_f} \left(\sum \vec{\mathbf{F}} \right) d\vec{\mathbf{r}}$$

The subscript “ext” indicates the work is done by an *external* agent on the system.

Work Done by Multiple Forces, cont.

If the system cannot be modeled as a particle, then the total work is equal to the algebraic sum of the work done by the individual forces.

$$\sum W = W_{\text{ext}} = \sum_{forces} \left(\vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} \right)$$

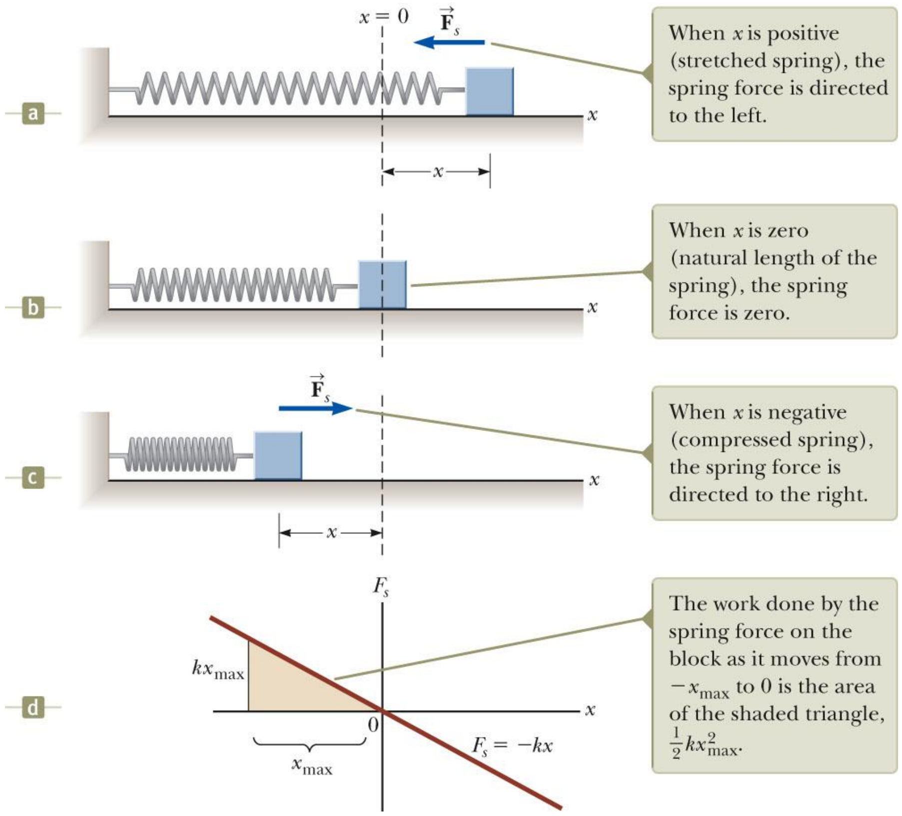
- Remember work is a scalar, so this is the algebraic sum.

Work Done By A Spring

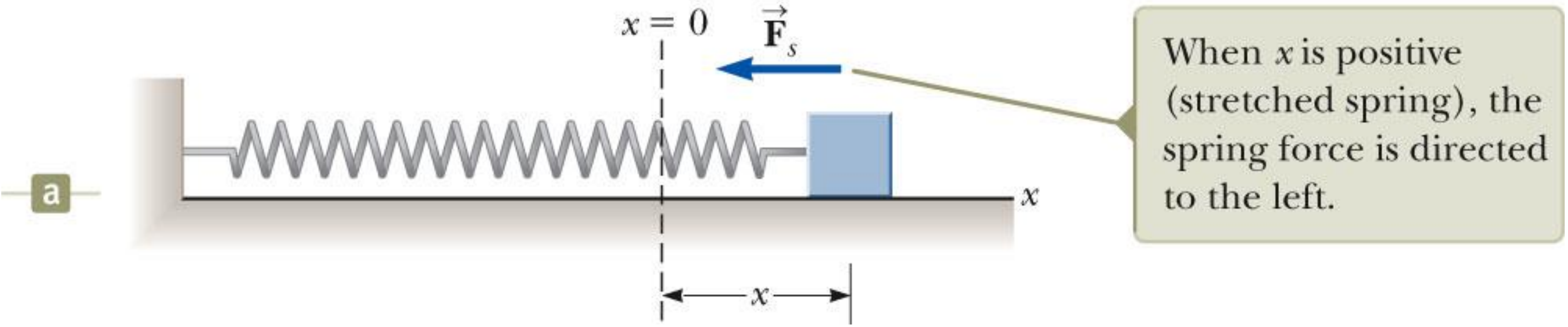
A model of a common physical system for which the force varies with position.

The block is on a horizontal, frictionless surface.

Observe the motion of the block with various values of the spring constant.



Spring Force (Hooke's Law)



The force exerted by the spring is

$$F_s = - kx$$

- x is the position of the block with respect to the equilibrium position ($x = 0$).
- k is called the spring constant or force constant and measures the stiffness of the spring.
  - k measures the *stiffness* of the spring.

This is called Hooke's Law.

Hooke's Law, cont.

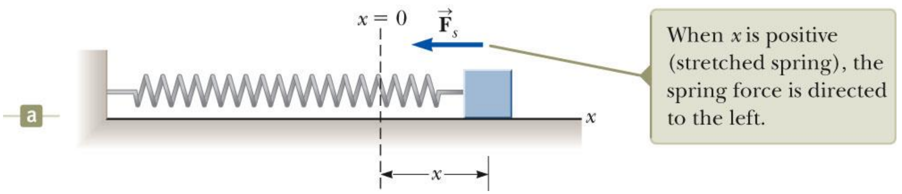
The vector form of Hooke's Law is

$$\vec{\mathbf{F}}_s = F_x\hat{\mathbf{i}} = -kx\hat{\mathbf{i}}$$

When x is positive (spring is stretched), F is negative

When x is 0 (at the equilibrium position), F is 0

When x is negative (spring is compressed), F is positive



Hooke's Law, final

The force exerted by the spring is always directed opposite to the displacement from equilibrium.

The spring force is sometimes called the *restoring force.*

If the block is released it will oscillate back and forth between $-x$ and x.

Work Done by a Spring

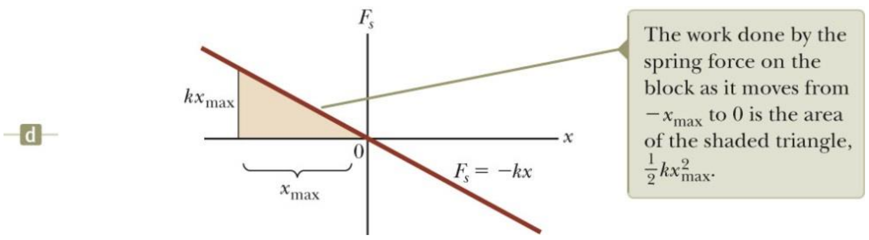
Identify the block as the system.

Calculate the work as the block moves from $x_i = -x_{max}$ to $x_f = 0$.

$$W_s = \int \vec{\mathbf{F}}_s \cdot d\vec{\mathbf{r}} = \int_{x_i}^{x_f} \left(-kx\hat{\mathbf{i}}\right) \cdot \left(dx\hat{\mathbf{i}}\right)$$

$$= \int_{-x_{max}}^{0} \left(-kx\right) dx = \frac{1}{2} k x_{max}^2$$

The net work done as the block moves from $-x_{max}$ to x_{max} is zero



Work Done by a Spring, cont.

Assume the block undergoes an arbitrary displacement from $x = x_i$ to $x = x_f$.

The work done by the spring on the block is

$$W_s = \int_{x_i}^{x_f} (-kx)\,dx = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

- If the motion ends where it begins, W = 0

Spring with an Applied Force

Suppose an external agent, F_{app}, stretches the spring.

The applied force is equal and opposite to the spring force.

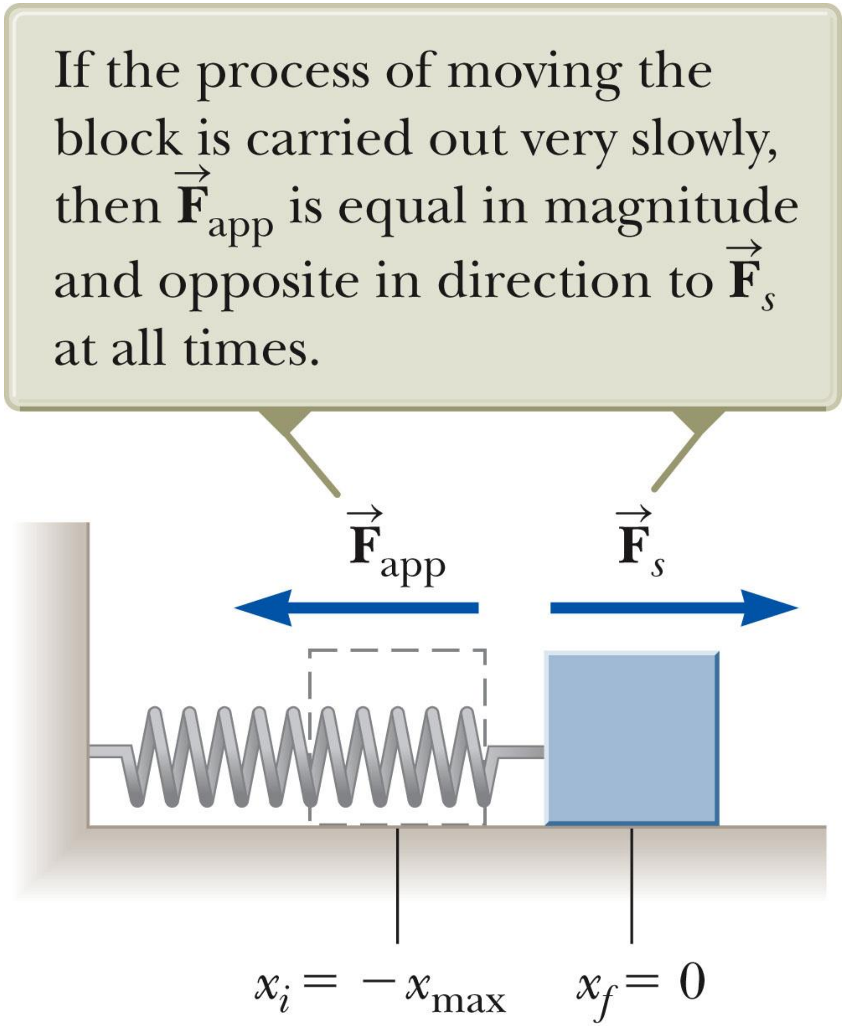
$$\vec{\mathbf{F}}_{app} = F_{app}\hat{\mathbf{i}} = -\vec{\mathbf{F}}_s = -\left(-kx\hat{\mathbf{i}}\right) = kx\hat{\mathbf{i}}$$

Work done by F_{app} as the block moves from $-x_{max}$ to $x = 0$ is equal to

$-\frac{1}{2} kx^2_{max}$

For any displacement, the work done by the applied force is

$$W_{app} = \int_{x_i}^{x_f} (kx)\,dx = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$



Kinetic Energy

One possible result of work acting as an influence on a system is that the system changes its speed.

The system could possess *kinetic energy.*

Kinetic Energy is the energy of a particle due to its motion.

- $K = \frac{1}{2} mv^2$
  - K is the kinetic energy
  - m is the mass of the particle
  - v is the speed of the particle

A change in kinetic energy is one possible result of doing work to transfer energy into a system.

Kinetic Energy, cont

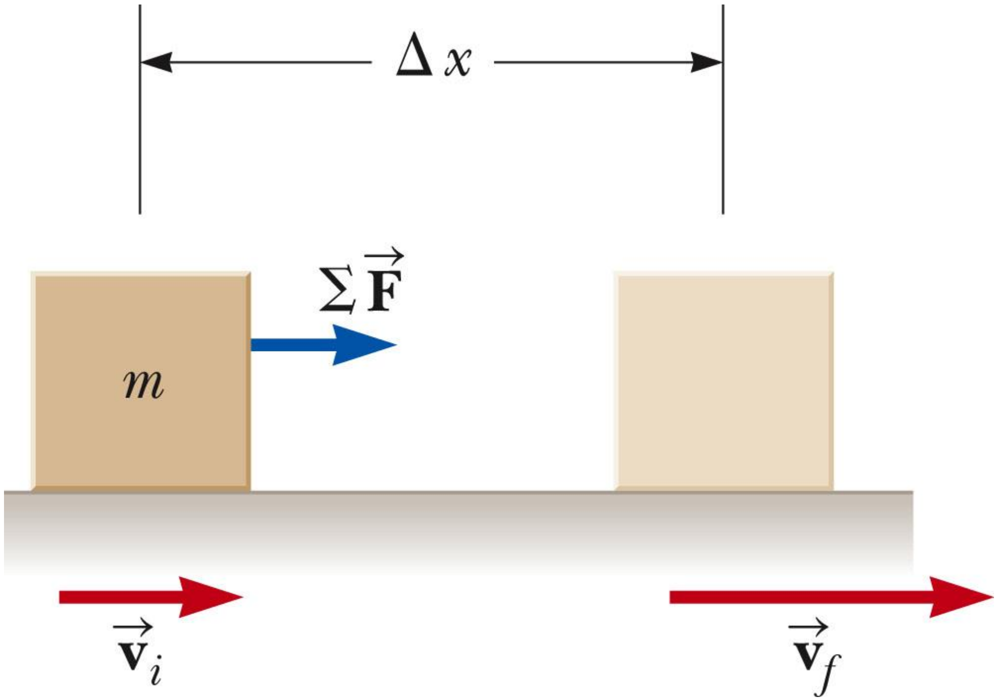
Calculating the work:

$$W_{ext} = \int_{x_i}^{x_f} \sum F\, dx = \int_{x_i}^{x_f} ma\, dx$$

$$W_{ext} = \int_{v_i}^{v_f} mv\, dv$$

$$W_{ext} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$W_{ext} = K_f - K_i = \Delta K$$



Work-Kinetic Energy Theorem

The Work-Kinetic Energy Theorem states $W_{ext} = K_f - K_i = \Delta K$

When work is done on a system and the only change in the system is in its speed, the net work done on the system equals the change in kinetic energy of the system.

- The speed of the system increases if the work done on it is positive.
- The speed of the system decreases if the net work is negative.
- Also valid for changes in rotational speed

The work-kinetic energy theorem is not valid if other changes (besides its speed) occur in the system or if there are other interactions with the environment besides work.

The work-kinetic energy theorem applies to the speed of the system, not its velocity.

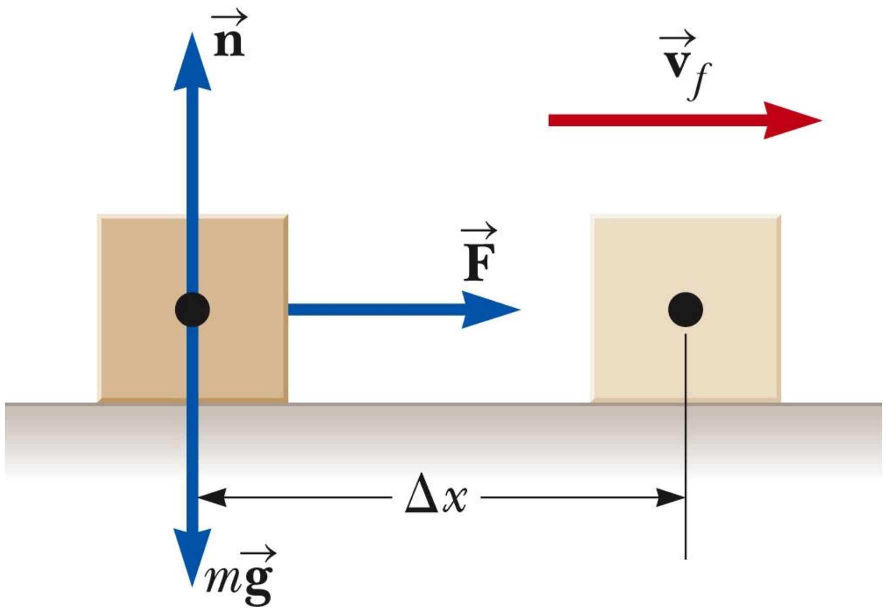
Work-Kinetic Energy Theorem – Example

The block is the system and three external forces act on it.

The normal and gravitational forces do no work since they are perpendicular to the direction of the displacement.

$W_{ext} = \Delta K = ½\ mv_f^2 - 0$

The answer could be checked by modeling the block as a particle and using the kinematic equations.



Potential Energy

Potential energy is energy determined by the configuration of a system in which the components of the system interact by forces.

- The forces are internal to the system.
- Can be associated with only specific types of forces acting between members of a system

Gravitational Potential Energy

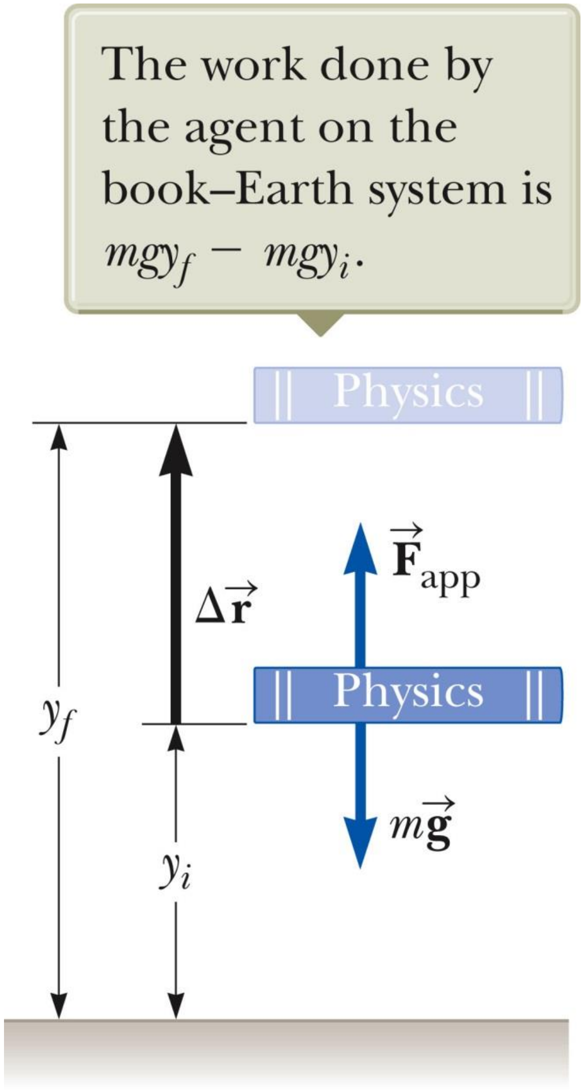
The system is the Earth and the book.

Do work on the book by lifting it slowly through a vertical displacement.

$$\Delta\vec{\mathbf{r}} = (y_f - y_i)\,\hat{\mathbf{j}}$$

The work done on the system must appear as an increase in the energy of the system.

The energy storage mechanism is called *potential energy.*



Gravitational Potential Energy, cont

Assume the book in fig. 7.15 is allowed to fall.

There is no change in kinetic energy since the book starts and ends at rest.

Gravitational potential energy is the energy associated with an object at a given location above the surface of the Earth.

$$W_{ext} = \left(\vec{\mathbf{F}}_{\text{app}}\right) \cdot \Delta\vec{\mathbf{r}}$$

$$W_{ext} = (mg\hat{\mathbf{j}}) \cdot \left[\left(y_f - y_i\right)\hat{\mathbf{j}}\right]$$

$$W_{ext} = mgy_f - mgy_i$$

Gravitational Potential Energy, final

The quantity *mgy* is identified as the gravitational potential energy, $U_{g.}$

- $U_g = mgy$

Units are joules (J)

Is a scalar

Work may change the gravitational potential energy of the system.

- $W_{\text{ext}} = \Delta u_{\text{g}}$

Potential energy is always associated with a system of two or more interacting objects.

Gravitational Potential Energy, Problem Solving

The gravitational potential energy depends only on the vertical height of the object above Earth's surface.

In solving problems, you must choose a reference configuration for which the gravitational potential energy is set equal to some reference value, normally zero.

- The choice is arbitrary because you normally need the difference in potential energy, which is independent of the choice of reference configuration.

Often having the object on the surface of the Earth is a convenient zero gravitational potential energy configuration.

The problem may suggest a convenient configuration to use.

Elastic Potential Energy

Elastic Potential Energy is associated with a spring.

The force the spring exerts (on a block, for example) is $F_s = -kx$

The work done by an external applied force on a spring-block system is

- $W = \frac{1}{2} kx_f^2 - \frac{1}{2} kx_i^2$
- The work is equal to the difference between the initial and final values of an expression related to the configuration of the system.

Elastic Potential Energy, cont.

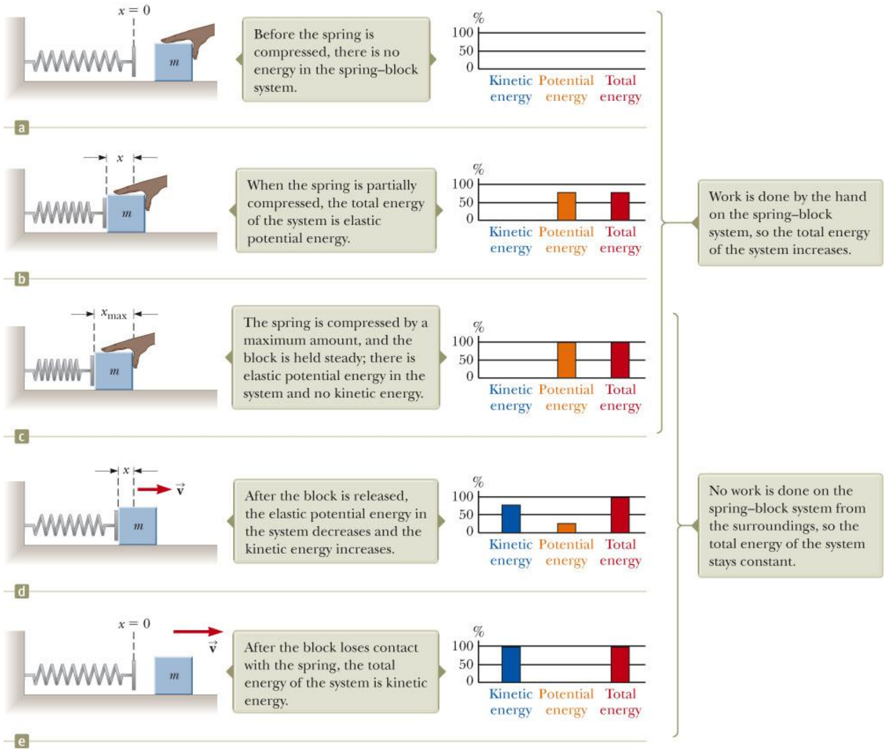
This expression is the elastic potential energy:

$$U_s = \frac{1}{2} kx^2$$

The elastic potential energy can be thought of as the energy stored in the deformed spring.

The stored potential energy can be converted into kinetic energy.

Observe the effects of different amounts of compression of the spring.



Elastic Potential Energy, final

The elastic potential energy stored in a spring is zero whenever the spring is not deformed ($U = 0$ when $x = 0$).

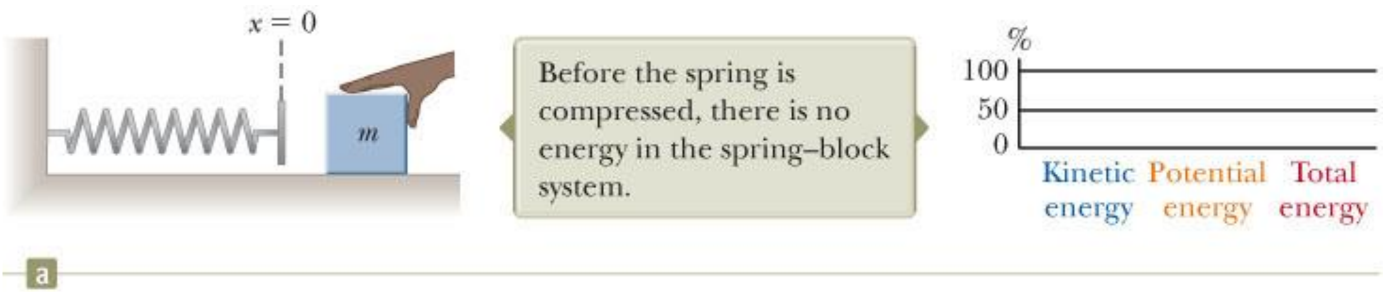
- The energy is stored in the spring only when the spring is stretched or compressed.

The elastic potential energy is a maximum when the spring has reached its maximum extension or compression.

The elastic potential energy is always positive.

- x^2 will always be positive.

Energy Bar Chart Example



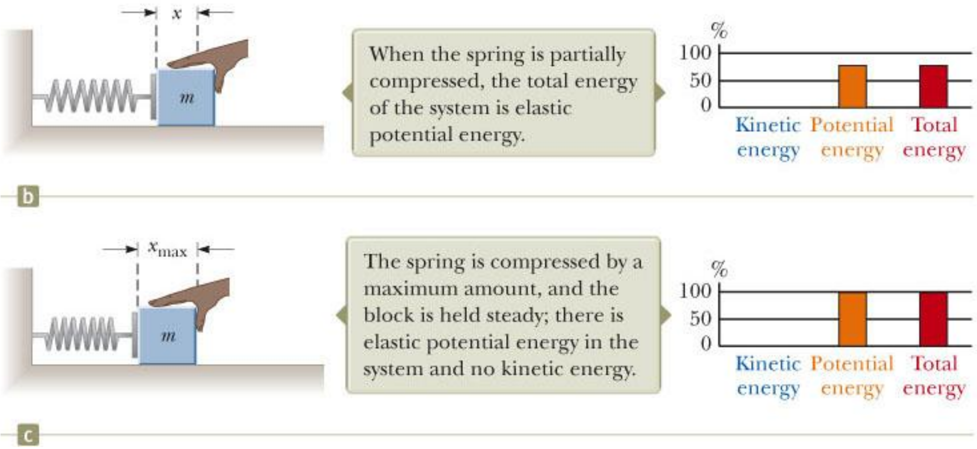
An energy bar chart is an important graphical representation of information related to the energy of a system.

- The vertical axis represents the amount of energy of a given type in the system.
- The horizontal axis shows the types of energy in the system.

In a, there is no energy.

- The spring is relaxed, the block is not moving

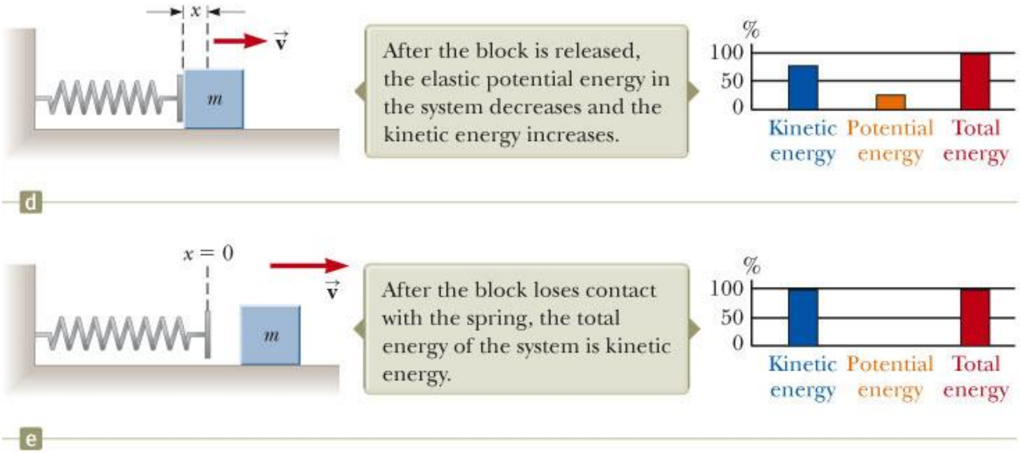
Energy Bar Chart Example, cont.



Between b and c, the hand has done work on the system.

- The spring is compressed.
- There is elastic potential energy in the system.
- There is no kinetic energy since the block is held steady.

Energy Bar Chart Example, final



In d, the block has been released and is moving to the right while still in contact with the spring.

- The elastic potential energy of the system decreases while the kinetic energy increases.

In e, the spring has returned to its relaxed length and the system contains only kinetic energy associated with the moving block.

Internal Energy

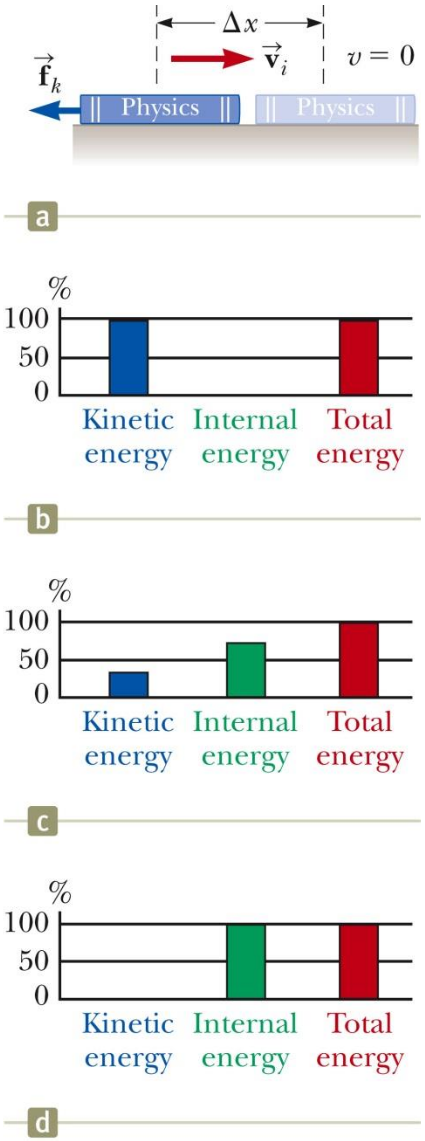
The energy associated with an object's temperature is called its *internal energy*, E_{int}.

In this example, the surface is the system.

The friction does work and increases the internal energy of the surface.

When the book stops, all of its kinetic energy has been transformed to internal energy.

The total energy remains the same.



Conservative Forces

The work done by a conservative force on a particle moving between any two points is independent of the path taken by the particle.

The work done by a conservative force on a particle moving through any closed path is zero.

- A closed path is one in which the beginning and ending points are the same.

Examples of conservative forces:

- Gravity
- Spring force

Conservative Forces, cont

We can associate a potential energy for a system with any conservative force acting between members of the system.

- This can be done only for conservative forces.
- In general: $W_{int} = -\Delta U$
  - W_{int} is used as a reminder that the work is done by one member of the system on another member and is internal to the system.
- Positive work done by an outside agent on a system causes an increase in the potential energy of the system.
- Work done on a component of a system by a conservative force internal to an isolated system causes a decrease in the potential energy of the system.

Non-conservative Forces

A non-conservative force does not satisfy the conditions of conservative forces.

Non-conservative forces acting in a system cause a *change* in the mechanical energy of the system.

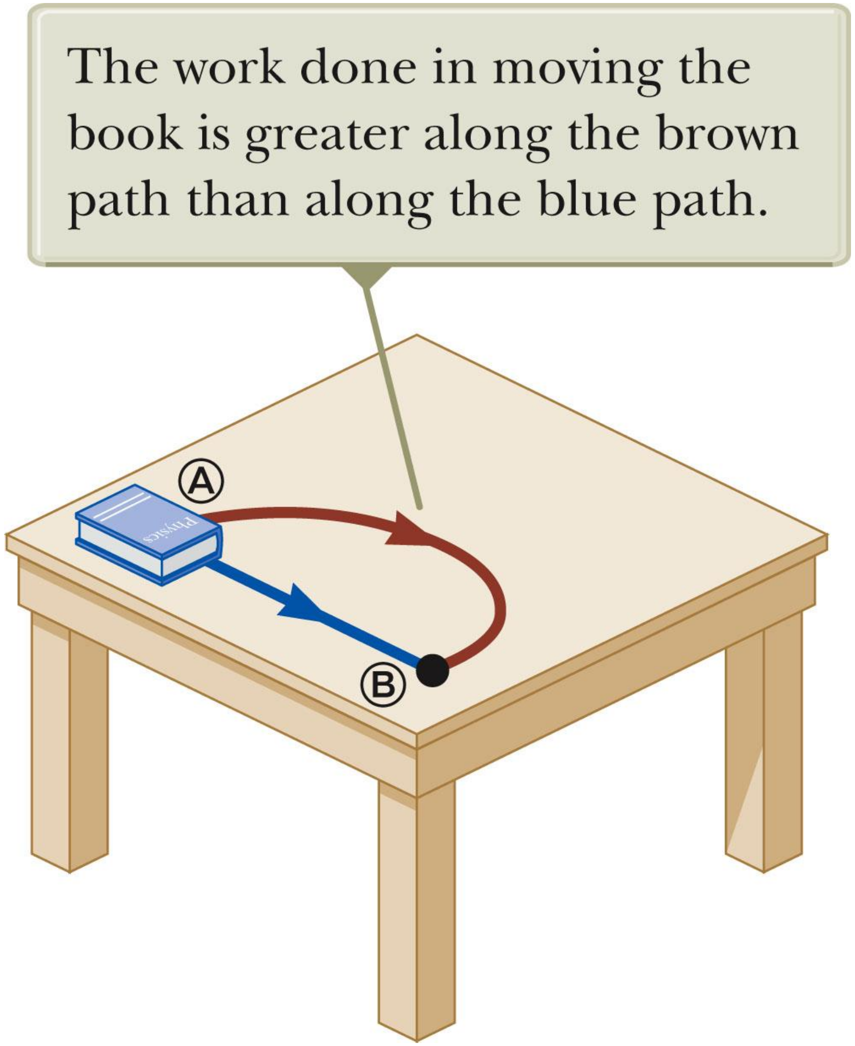
$E_{mech} = K + U$

- K includes the kinetic energy of all moving members of the system.
- U includes all types of potential energy in the system.

Non-conservative Forces, cont.

The work done against friction is greater along the brown path than along the blue path.

Because the work done depends on the path, friction is a non-conservative force.



Conservative Forces and Potential Energy

Define a potential energy function, *U*, such that the work done by a conservative force equals the decrease in the potential energy of the system.

The work done by such a force, *F*, is

$$W_{\text{int}} = \int_{x_i}^{x_f} F_x \, dx = -\Delta U$$

- ΔU is negative when *F* and *x* are in the same direction□

Conservative Forces and Potential Energy

The conservative force is related to the potential energy function through.

$$F_x = -\frac{dU}{dx}$$

The *x* component of a conservative force acting on an object within a system equals the negative of the potential energy of the system with respect to *x.*

- Can be extended to three dimensions

Conservative Forces and Potential Energy – Check

Look at the case of a deformed spring:

$$F_s = -\frac{dU_s}{dx} = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx$$

- This is Hooke's Law and confirms the equation for U

U is an important function because a conservative force can be derived from it.