**Chapter- 2**

**The General Law of Simple Interest**

**Introduction:**

* Whether you are in a position to invest money or to borrow money, it is important for both consumers and business managers to understand *interest*.
* This chapter will emphasize the method of computing *simple interest*. It first introduces the basic concepts and operations of simple interest and simple discount.
* These basic topics are very important to understand *bank discount, compound interest, annuities,* and *life insurance premium*. This chapter also illustrates the general applications of the simple interests and simple discount methods for debt payments.

**Concept and Computation:**

* A person who borrows money usually pays *interest* as a fee for the use of the money.
* The money borrowed is called the *principal*.
* The sum of the *principal* and the *interest* due is called the *amount*.
* The *rate of interest* is usually expressed as a percent of the principal for a specified period of time, which is generally one year. Interest paid only on the principal borrowed is called simple interest.
* When the interest for each period is added to the principal in computing the interest for the next period, it is called *compound interest*.
* Simple interest is usually charged for short- term borrowing, whereas compound interest is commonly employed in long- term obligations.
* Simple interest is the product of the principal, rate of interest and the time.
* The basic formula for calculating simple interest is:

*Interest = Principal × (Interest rate per period) × (No. of interest periods or Time)*

***S.I. = Prt***

Where;

 S.I. = Simple interest;

P = Principal;

r = annual rate of interest;

t = time in years.

***Example 1: What is the simple interest on SR 1000 at 10% (a) for 3 years and (b) for 2 months?***

**Solution:** (a) P = SR 1000; r = 10% = 0.1 (per year); t = 3 (years)

 Substituting these values in simple interest formula such as-

 *S.I. = Prt = 1000 × 0.1 × 3 = SR 300*

 (b) t = 2/12, since 2 months = 2/12 years

 Substituting these values in simple interest formula such as-

 *S.I. = Prt = 1000 × 0.1 × 2/12 = SR 16.67*

***Example 2: To buy furniture for a new apartment, Mohammad borrowed SR 10,000 at 5% simple interest for 10 months. How much interest he will pay?***

**Solution:** Given values are-

P = SR 10,000; r = 5% = 0.05; and t = 10/12 (in a year).

 The total interest he will pay is-

*S.I. = Prt = 10,000 × 0.05 × 10/12 = SR 416.67*

**Determining the Number of Days:**

* Since time may be expressed in days, the number of days in a year must be determined before computing interest.
* There are two methods used in determining the number of days in a year- the exact method and the approximate method.
* Under the exact method, each year has 365 days except leap years, which have 366 days.
* When approximate method is used, it is assumed that each of the 12 months in a year has 30 days, and therefore there are 360 days in a year.
* In finding the number of days between two given dates, count either the beginning date or the ending date, but not both. Here, we will follow the ending date for counting.

***Example 3: Find (a) the exact time and (b) the approximate time from June 24, 1998, to September 27, 1998.***

**Solution:** *(a) The exact time:*

|  |  |
| --- | --- |
| June | 6 days (Remainder, or 30 - 24 = 6) |
| July | 31 days |
| Aug | 31 days |
| Sept | 27 days |
| **Total days** | **95 days** |

 *Or,*

 According to Table- 1, which shows the exact number of days, June 24 is the 175th day of the year and September 27 is the 270th day. So, total days = 270 – 175 = 95 days.

*(b) The approximate time:*

First, write the two given dates in the following form, arranging the months to the left of the days, and then subtract:

|  |  |
| --- | --- |
| **Month** | **Day** |
| 9 (September) | 27 |
| 6 (June) | 24 |
| **3 months** | **3 days** |

 Since there are 30 days in each month by the approximate method, the number of days between the two given dates is 3 × 30 + 3 = 93 days.

***Example 4: Find (a) the exact time and (b) the approximate time from November 14, 1996 to April 24, 1997.***

**Solution:** *(a) The exact time:*

|  |  |  |
| --- | --- | --- |
| 1996 | November | 16 days (Remainder, or 30 – 14 = 16) |
|  | December | 31 days |
| 1997 | January | 31 days |
|  | February | 28 days |
|  | March | 31 days |
|  | April | 24 days |
|  | **Total days** | **161 days** |

*Or,* According to table 1, which shows the exact number of days, November 14 is the (318 + 1)th or 319th day of year 1996 (1996 is a leap year). The number of days remaining in 1996 is 366- 319 = 47 days. April 24 is the 114th day of the year 1997. Thus, the total number of days between the two given dates is 47 + 114 = 161 days.

*(b) The approximate time:*

 Arrange the two given dates in the following form by the order of year, month, and day, and then subtract:

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Year** | **Month** | **Day** |
| Ending date | 199~~7~~6 | ~~4~~16 | 24 |
| Beginning date | 1996 | 11 | 14 |
|  | 0000 | 05 | 10 |

 In the month column, since 11 months is larger than 4 months, borrow 1 year or 12 months from the year column to make a total of 16 months before subtracting.

The approximate time = 5 × 30 + 10 = 160 days.

***Note:*** *Leap years are those years evenly divisible by 4, such as 1988, 1992, and 1996, in which the month of February has 29 days instead of 28 as in other years. But the last year of any century, although it is divisible by 4, is not a leap year unless it is divisible by 400. Thus, 1700, 1800, and 1900 were not leap years, but 2000 is a leap year.*

**Ordinary and Exact Interest:**

* The value of interest computed by using 360 as the divisor in the time factor (exact time/360 or approximate time/360) is called *ordinary interest*.
* When ordinary interest is computed by exact time divided by 360 (exact time/360), it is called *Banker’s Rule*. Banker’s Rule method of computing the ordinary interest is much more commonly used in commercial practice than the second one (approximate time/360). So, we will follow Banker’s rule for computing ordinary interest.
* But, when value of interest computed by using 365 as the divisor in the time factor (exact time/365 or approximate time/365) is called *exact interest*.
* Exact interest (exact time/365) is usually used in calculating interest payments on government obligation, in foreign trade, and in rediscounting notes for member banks by central bank.

|  |  |
| --- | --- |
| **For computing ordinary interest** | **For computing exact interest** |
| 1. exact time/360 (*Banker’s Rule*)
 | 1. exact time/365
 |
| 1. approximate time/360
 | 1. approximate time/365
 |

***Example 5: Express the time from June 24, 1998 to September 27, 1998, in years for computing (a) ordinary interest and (b) exact interest.***

Solution: The exact time between the two dates is 95 days. Thus,

1. Time by using ordinary interest method is- $\frac{95}{360}$
2. Time by using exact interest method is- $\frac{95}{365}$

**Calculating Ordinary Interest by Formula:**

When the number of days is given, the days should be expressed as a fraction of a year. Let *t* is denoted for time in exact number of days. In computing ordinary simple interest (SI) by the Banker’s Rule, formula becomes:

*Io = Pr(t/360) =* $\frac{Prt}{360}$

***Example 6: Find the ordinary interest on SR 500 at 10% for 30 days.***

**Solution:** P = SR 500; r = 10% = 0.1; t = 30 days =$(\frac{30}{360}$) year

 *Io = Pr(t/360) =* $\frac{Prt}{360}$ *= 500×0.1×*$\frac{30}{360}$ *= SR 4.17*

**Calculating Exact Interest by Formula:**

Let Ie denote the exact simple interest. When the number of days is given, formula of simple interest is written in the following way:

*Ie = Pr(*$\frac{t}{365}$*) =* $\frac{Prt}{365}$

**The Relationship between *Io* and *Ie*:**

Use of the ordinary interest method always gives a larger interest value than does the exact interest method.

$\frac{Io }{Ie}$ *=* $\frac{\frac{Prt}{360}}{\frac{Prt}{365}}$ *=* $\frac{365}{360}$ *=* $\frac{73}{72}$

*Thus, Ie is (72/73=0.9863) less than Io.*

*OR, Io = 1.014Ie.*

*Or, Io is 1.014 time greater than Ie*

|  |  |
| --- | --- |
| *Io = 1.014Ie* | *Ie = 0.9863Io* |

***Example 7: What would be the ordinary interest (Io) be if the exact interest (Ie) is SR 27.70?***

**Solution:** *Io = 1.014Ie = 1.014×27.70 = SR 28.08*

***Example 8: Obtain the exact interest corresponding to an ordinary interest of SR 502.66.***

**Solution:** *Ie = 0.9863Io = 0.9863×502.66 = SR 495.77*

**Amount, Rate of Interest and Time:**

**Finding the Amount:**

The amount (A) is the sum of the principal and the rate of interest, or

*Amount (A) = Principal (P) + Interest (I)*

*Or, A = P + SI*

*Or, A = P + Prt*

*A = P(1 + rt)*

***Example 9: Muhammad borrows SR 5000 for 10 months at 5%. How much must he repay?***

**Solution:** Principal = SR 5000; r = 5% and t = 10 months = (10/12) year

 A = P(1 + rt) = 5000(1+0.05×$\frac{10}{12}$) = 5208.33

 He must repay SR 5208.33.

***Example 10: On May 24, 1999, Ahmad borrowed SR 10,000 and agreed to repay the loan together with interest at 10% in 90 days. What amount must he repay? On what date?***

**Solution:** P = SR 10,000; r = 10%; t = 90 days = (90/360) year (Banker’s Rule)

 A = P(1 + rt) = 10,000(1+0.1×$\frac{90}{360}$) = SR 10,250

 He must repay SR 10,250.

|  |  |
| --- | --- |
| May | 07 days (31- 24 = 7) |
| June | 30 |
| July | 31 |
|  | **68 days** |
| August | 22 days |
| **Total days** | **90** |

The amount, SR 10,250, must be repaid on August 22, 1999.

**Finding Rate of Interest:**

*Rate of Interest (r) =* $\frac{Simple Interest (SI) }{Principal (P) ×Time (t)}$

***Example 11: At what interest rate will SR 500 yield SR 200 in 5 years?***

**Solution:** Here the value of P = SR 500; SI = SR 200; t = 5 years then r =?

 Rate of Interest (r) = $\frac{Simple Interest (SI) }{Principal (P) ×Time (t)}$

 = $\frac{200 }{500 ×5}$ = 0.08 = 8%

***Example 12: A payment of SR 1567.50 was made for discharging a four- month loan of SR 1500. What was the interest rate charged?***

**Solution:** Here, A = SR 1567.50; P = SR 1500; t = 4 months = (4/12 = 1/3) year then r =?

 Simple Interest (SI) = A – P = 1567.50 – 1500.00 = SR 67.50

 Hence, Rate of Interest (r) = $\frac{Simple Interest (SI) }{Principal (P) ×Time (t)}$ = $\frac{67.50 }{1500 ×1/3}$ = 0.135 = 13.5%

**Finding the Time:**

*Time (t) =* $\frac{Simple Interest (SI) }{Principal \left(P\right)×Rate of Interest (r)}$

***Example 13: How long will it take SR 1000 to yield SR 100 interest at 5%?***

**Solution:** Here, P = SR 1000; SI = SR 100 and r = 5% then t =?

 Substituting these values in the following formula,

 Time (t) = $\frac{Simple Interest (SI) }{Principal \left(P\right)×Rate of Interest (r)}$= $\frac{100 }{1000 × 0.05}$= 2 years.

**Principal, Present Value and Simple Discount:**

**Finding the Principal:**

*Principal (P) =* $\frac{Simple Interest (SI)}{Rate of Interest \left(r\right)×Time (t)}$

***Example 14: Muhammad receives SR 500 interest in 3 months from an investment that pays 10% interest. What is the principal that he has invested?***

**Solution:** Here, SI = SR 500; t = 3 months = 3/12 = (¼) year and r = 10% = 0.1

 So, *Principal (P) =* $\frac{Simple Interest (SI)}{Rate of Interest \left(r\right)×Time (t)}$ *=* $\frac{500}{0.1×1/4}$*= SR 20,000*

***Example 15: How much money must Ahmad invest today at 10% simple interest if he is to receive SR 15,000 the amount, in*** $1\frac{1}{2}$ ***years?***

**Solution:** Here, values are given as r = 10% = 0.1; A = SR 15,000; t = $1\frac{1}{2}$years = 3/2 = 1.5 years; then P = ?

 Substituting these values in the formula as-

 A = P(1 + rt)

 P = $\frac{A}{(1 + rt)}$ = $\frac{15,000}{(1 + 0.1×1.5)}$ = 13,043.478

 So, Ahmad must invest SR 13,043.48 today to receive SR 15,000 after $1\frac{1}{2}$ years at 10% simple rate of interest.

**Present Value- PV (Current Value, CV):**

Present value is the value at the time of investment, such as the principal, or at any time before the maturity date (due date). The above example 13 indicates that Ahmad invests SR 13,043.48 today at 10% simple interest, he will have SR 15,000 in $1\frac{1}{2}$ years. In other words, the present value of SR 15,000 that is due in $1\frac{1}{2}$ years and includes 10% interest is SR 13,043.48. Thus, the method of finding the present value of a given amount that is due in the future is the same as the method used in the above example 13 for finding the principal.

**Simple Discount:**

The process of finding the present value of a given amount that is due on a future date and includes a simple interest is called discounting at simple interest, or commonly, the simple discount method. In other words, to discount an amount by the simple interest process is to find its present value.

When interest is involved, the amount must be larger than its present value. The difference between the amount and its present value is called the simple discount. Thus, the simple discount on the amount is the same as the simple interest on the principal or the present value.

*Discounting a Non- Interest- Bearing Debt:*

***Example 16: What is the present value of SR 3,248 that is due at the end of the 2 months if the interest rate is 9%? What is the simple discount?***

**Solution:** Here, the amount at maturity (due in two months) is SR 3,248 which bears no interest, as stated in the problem. The 9% interest is used for discounting the maturity amount. Thus,

A = SR 3,248; r = 9%, t = 2/12 = (1/6) year, the discount period.

Substituting the values in A = P(1 + rt)

Or, P = $\frac{A}{(1 + rt)}$ = $\frac{3248}{(1 + 0.09×1/6)}$ = SR 3,200 (Present value)

SI = A – P = 3,248 – 3,200 = SR 48 (simple discount)

***Check:*** *According to the answer, the amount due at the end of 2 months should be:*

A = P(1 + rt) = 3200(1+0.09×1/6) = SR 3,248.

***Note:*** *The simple discount on the amount SR 3248 is the same as the simple interest on the present value, SR 3200. In other words, SI has a twofold meaning: It is the simple interest on the principal or the present value; it is also the simple discount on the amount.*

***Example 17: A debt of SR 875.50 is due in six months. If the debt is settled now and the simple interest rate of 6% is allowed, what is the present value and the simple discount?***

**Solution:** Here, the values are given as: A = SR 875.50; t = 6/12 = ½ year; r = 6% = 0.06

Substituting these values in the formula-

 P = $\frac{A}{(1 + rt)}$ = $\frac{875.50}{(1 + 0.06×1/2)}$ = SR 850 (present value)

 The simple discount is:

 SI = A – P = 875.50 – 850.00 = SR 25.50

*Discounting an Interest- Bearing Debt:*

To find the present value of an interest- bearing debt (or to discount the amount by the simple discount method), take the following steps:

* *Step I:* Find the maturity value (the amount, A) according the original interest rate and the time stipulated for the debt. Use the formula, A = P(1 + rt), where A = Amount, is the maturity value and P is the original debt.
* *Step II:* Find the present value (the value on the date of discount) of the maturity value according to the interest rate for discounting and the discount period. The discount period is the period from the date of discount to the maturity date. Use the formula in the form P = $\frac{A}{(1 + rt)}$, where P is the present value and A is the maturity value. However, the values of *r* and *t* in this step are often different from the values of *r* and *t* in *Step I.*

***Example 18: A man borrowed SR 1,000 on May 1, 1999 and agreed to repay the money plus 8% interest in six months. Two months after the money was borrowed, the creditor agreed to settle the debt by discounting it at the simple interest rate of 9%. How much did the creditor receive when he discounted the debt?***

**Solution:***Step I:*Find the maturity value of the debt according to the original stipulation of the debt. P = 1,000, r = 8% = 0.08, t = 6/12 = ½ year.

Substituting these values in the formula,

A = P(1 + rt) = 1,000(1+0.08×1/2) = 1,040 (amount on November1, 1999)

*Step II:*Find the present value of the maturity value according to the discounting terms. A = 1,040, r = 9% = 0.09, t = 4/12 = 1/3 year, or 6-2 = 4 months.

Substituting these values in the formula,

P = $\frac{A}{(1 + rt)}= \frac{1,040}{(1 + 0.09×1/3)}$ = 1,009.71 (value on July 1, 1999)

**Focal Date and Equation of Value:**

The focal date is the date at which various funds are chosen to be evaluated. Most often, a focal date is at the present time, with values of funds maturing at different times all pulled back to the current time. In other words, this process is to evaluate future funds as if they are to be cashed in today. However, a focal date can be at a future date.

***Example 19: Muhammad received a loan that he was to pay off in three installments: SR 500 in a year, SR 1200 in 20 months, and SR 1500 in 2 years. What would be the amount of loan received if the annual simple interest is 9***$\frac{1}{2}$***%?***

**Solution:** We have three future values, which need individually to be brought back to the present- to a focal date that is today.

*CV =* $\frac{FV1}{1+rt}$ *+* $\frac{FV2}{1+rt}+ \frac{FV3}{1+rt}$

*=* $\frac{500}{1+9.5×1}$ *+* $\frac{1200}{1+9.5×1\frac{2}{3}}+ \frac{1500}{1+9.5×2}$

= 456.62 + 1036 + 1260.50 = SR 2753.12

**Partial Payments:**

* If partial payments are made on a debt before it is a debt before it is due, there should be an agreement between the creditor and the borrower regarding the interest on each partial payment. Some creditors may agree to reduce the interest when partial payments on the debt are made.
* In general, the two methods used by a creditors in reducing the interest are- the *Merchants’ Rule* and the *United States Rule*.
* The *Merchants’ Rule* is simpler and is preferred by most business people in computing the interest on a short- term debt.
* The *United States Rule*, on the other hand, is well known in the academic world. This rule applies consistently to both short- term and long- term debts.

**Merchants’ Rule (A Simple Interest Method):**

Under the Merchants’ Rule, the principal and all partial payments are treated as if they earn interest from the time they are made to the date of final settlement. The following steps may be employed:

*Step I:* Find the sum of the principal and its interest for the period from the date of borrowing to the date of final settlement

*Step II:* Find the sum of the partial payments and interest on each partial payment from the date of payment to the date of final settlement. This sum is the debtor’s credit against the sum in Set I.

*Step III:* Subtract the result in Step II from the result in Step I. The difference is the balance to be discharged on the date of final settlement.

**In other words,**

*In Merchants’ rule, the focal date is the final due date, and therefore each partial payment earns interest from the time it is made to the focal date. The balance due is therefore the difference between the amount of the debt and the sum of the partial payments made.*

***Example 20: On July 1, a man borrowed SR 2,000 at 6%. He paid SR 500 on August 30 and SR 600 on September 29. Find the balance on October 29 of the same year, by the Merchants’ Rule.***

**Solution: The Merchants’ Rule:**

|  |  |  |  |
| --- | --- | --- | --- |
| Original debt (as on July 1) |  |  | SR 2,000 |
| *Add:* interest on SR 2,000 for 120 days (July 1 to October 29) |  |  | SR 40 |
|  **Amount on October 29** |  |  | **SR 2,040** |
| Deduct: partial payments and their interest |  |  |  |
| First payment (August 30) | SR 500 |  |  |
| *Add:* interest on SR 500 for 60 days (Aug 30 to Oct 29) | SR 5 | SR 505 |  |
| Second payment (Sept 29) | SR 600 |  |  |
| *Add:* interest on SR 600 for 30 days (Sept 29 to Oct 29) | SR 3 | SR 603 |  |
| Total partial payments and their interest  |  |  | SR 1,108 |
|  **Balance on Oct 29**  |  |  | **SR 932** |

**United States Rule (A Compound Interest Method):**

Under the United States Rule, each partial payment must first be applied to the accumulated interest up to the date of the payment. Any remainder is then credited as a deduction from the principal. Therefore, when the United States Rule is applied, the successive interest is computed from a declining balance each time a payment is made. A debtor may thus know the actual amount of unpaid balance immediately after each payment. The following steps may be used:

*Step I:* Find the interest on the principal for the period from the date of borrowing to the date of the first partial payment.

*Step II:* Subtract the interest from the first payment. If there is a remainder, subtract the remainder from the principal to obtain the unpaid balance. If the partial payment is not sufficient to cover the interest due, the partial payment is then held and is included in the next payment.

*Step III:* If there are further partial payments, the processes in Step I and II are repeated, but the interest is computed on the declining unpaid balance. Each payment must be first applied to the interest that has accumulated up to the date of each payment. The final balance is the sum of the unpaid principal and the accumulated interest up to the date of the final settlement.

**In other words,**

*In U. S. rule, the outstanding principal would be adjusted each time a partial payment is made. Any partial payment exceeding the interest would be discounted from the outstanding principal, and any partial payment that less than the interest would be held without interest until another partial payment is made and until the combined payment exceeds the interest and results in reduction of the principal.*

***Example 21: On July 1, a man borrowed SR 2,000 at 6%. He paid SR 500 on August 30 and SR 600 on September 29. Find the balance on October 29 of the same year, by the United States Rule.***

**Solution: The United States Rule:**

|  |  |  |
| --- | --- | --- |
| Original debt (as on July 1) |  | SR 2,000.00 |
| Deduct: |  |  |
|  First payment (Aug 30) | SR 500.00 |  |
|  Deduct: interest on SR 2,000 for 60 days (Aug 30 to Oct 29) | SR 20.00 |  |
|  Remainder applied to principal |  | SR 480.00 |
| Balance on Aug 30 |  | SR 1,520.00 |
| Deduct: |  |  |
|  Second payment (Sept 29) | SR 600.00 |  |
|  Deduct: interest on SR 1,520 for 30 days (Aug 30 to Sept 29) | SR 7.60 |  |
|  Remainder applied to principal |  | SR 592.40 |
| *Balance on Sept 29* |  | *SR 927.60* |
| *Add: interest on SR 927.60 for 30 days (Sept 29 to Oct 29)* |  | *SR 4.64* |
| **Balance on Oct 29** |  | **SR 932.24** |

**Note:** *The balance on the date of final settlement as calculated by the United States Rule is slightly greater than that by the Merchants’ Rule, because compound interest is involved in the United States Rule method. Hence, it is better for a debtor to use the Merchants’ Rule in reducing a debt by partial payments.*

***Example 22: On July 1, a man borrowed SR 1,000 at 12%. He paid SR 300 on July 31, SR 6 on September 29, and SR 400 on October 14. Find the balance due November 13 of the same year by the United States Rule.***

**Solution: The United States Rule:**

|  |  |  |
| --- | --- | --- |
| Original debt (July 1) |  | SR 1,000.00 |
| Deduct: |  |  |
|  First payment (July 31) | SR 300.00 |  |
|  Deduct: interest on SR 1,000 for 30 days (July 1 to July 31) | SR 10.00 |  |
|  Remainder applied to principal |  | SR 290.00 |
| Balance on July 31 |  | SR 710.00 |
| Deduct: |  |  |
|  Second payment (Sept 29) | SR 6.00\* |  |
|  Third payment (Oct 14) | SR 400.00 |  |
|  Total payment as of Oct 14 | SR 406.00 |  |
|  Deduct: interest on SR 710 for 75 days (July 31 to Oct 14) | SR 17.75 |  |
|  Remainder applied to principal  |  | SR 388.25 |
| Balance on Oct 14  |  | SR 321.75 |
| Add: interest on SR 321.75 for 30 days (Oct 14 to Nov 13) |  | SR 3.22 |
| **Balance on Nov 13** |  | **SR 324.97** |

\**Interest on SR 710 for 60 days (July 31 to Sept 29) is SR 14.20, which is larger than the partial payment, SR 6. Thus, the payment is held and is included in the third payment on Oct 14.*

***Example 23: A loan of SR 1300 with 7% interest is due in a year. The borrower made a SR 300 payment after 3 months and SR 500 after 8 months. How much would the final balance be at the end of the year? Use both the merchants’ rule and the U. S. rule.***

**Solution: By Merchants’ Rule:**

FV1 = CV(1+rt)

 = 1300(1+0.07×1) = SR 1391

FV2 = 300(1+0.07×9/12) = SR 315.75

FV3 = 500(1+0.07×4/12) = SR 511.66

*Hence, the balance due = 1391- (315.75+511.66) = SR 563.39*

**By the U. S. Rule:**

FV1 = CV(1+rt)

 = 1300(1+0.07×1/12) = SR 1022.75

FV2 = 1022.75 (1+0.07×5/12) = SR 1052.58

FV3 = 552.58(1+0.07×4/12) = SR 565.47 (final balance)

**Equivalent Values involving Simple Interest:**

Occasionally there arises the need to replace a single debt or a set of debts by another single debt or another set of debts due at different times. In order to satisfy both the creditor and the debtor, the values of the new debts should be equivalent to the values of the original ones. For example, if a debt of SR 100 due now is to be replaced by a new debt due in one year and the money is worth 6%, the new debt is computed as follows:

A = P(1+rt) = 100(1+6%×1) = SR 106

The computation indicates that SR 100 due now is equivalent to SR 106 due in one year if the money is worth 6%. Thus, the creditor may allow the debtor to repay SR 100 now or SR 106 in a year. On the other hand, if a debt of SR 212 due in a year is to be replaced by a new debt due now, and the interest rate agreed upon by the creditor and the debtor is 6%, the new debt is computed as follows:

P = $\frac{A}{(1 + rt)}= \frac{212}{(1 + 6\%×1)}$ = SR 200

The computation indicates that SR 212 due in a year is equivalent to SR 200 due now if the rate of interest is 6%. Thus, the creditor and the debtor may agree to settle the debt now by the debtor’s payment of only SR 200.

When interest is involved, a sum of money has different values at various times. For convenience, a comparison date, also called a focal date, should first be chosen in comparing the values of old debts with the values of new debts. An equation of value, which gives the equivalent values of original debts and new debts on the comparison date at the specified interest rate, should then be arranged for obtaining the required equivalent values. The answer for a required equivalent value may vary slightly in simple interest problems, depending on the selection of the comparison date, but it does not vary in compound interest problems.

***Example 24: A man has two loans:***

1. ***SR 1500 that is due 2 months from now with 7% annual simple interest;***
2. ***SR 750 that is due 5 months from now.***

***If he wants to mix them in a single payment 10 months from now, how much would he pay given that the interest rate is 5%?***

***Solution:* First we calculate the first debt, as it is due in 2 months with 7% annual interest.**

FV = CV(1 + rt)

 = 1500(1 + 0.07 × 2/12)

 = SR 1517.50

***And if this debt is pushed to be paid in 8 months at 5% interest, then***

FV = CV(1 + rt)

 = 1517.50(1 + 0.05 × 8/12)

 = SR 1568.08

**The second debt (SR 750) is going to be pushed to be paid in 5 months at 5% interest:**

FV = CV(1 + rt)

 = 750(1 + 0.05× 5/12)

 = SR 765.62

**The single payment 10 months from now would be-**

SR 1568.08 + 765.62 = SR 2333.70 (which is equivalent value of the two debts).

***Example 25: A debt of SR 200 is due in six months. If the rate of interest is 15%, what is the value of the debt if it is paid (a) two months hence? (b) six months hence? (c) nine months hence?***

**Solution:** According to this question, SR 200 is the maturity value or the amount (A) due at the end of six months, and the interest rate agreed upon by both creditor and debtor for settlement of the debt is 15%.

1. If the SR 200 debt is paid two months hence, which is 4 months before the original due date, the required equivalent value is less than SR 200. Thus, the present value formula P = $\frac{A}{(1 + rt)}$ should be used to compute the required value. In other words, the value is obtained by discounting the maturity value at a simple interest rate for the advance time of the payment.

A = SR 200, r = 15% = 0.15, t = 4/12 = 1/3 year

Substituting the values in the formula,

P = $\frac{A}{(1 + rt)}= \frac{200}{(1 + 0.15×1/3)}$ = SR 190.48

If the debt is paid two months hence, the payment is SR 190.48.

1. If the debt is paid in 6 months, at which time the debt is due, the payment is SR 200, unchanged.
2. If the debt of SR 200 is paid 9 months from now, which is three months after the due date, the required equivalent value is more than SR 200. Thus, the amount (A) formula A = P(1+rt) should be used to compute the required value. In other words, the value is obtained by accumulating the original debt, SR 200, for the extended time.

P = SR 200, r = 15% = 0.15, t = 3/12 = ¼ year

Substituting the values in the formula,

A = P(1+rt) = 200(1+0.15×1/4) = SR 207.50

When the debt is paid at the end of 9 months, the payment is SR 207.50.

***Example 26: A man owes (1) SR 100, due in 2 months, and (2) SR 400, due in 8 months. His creditor has agreed to settle the debts by two equal payments in 4 months and 10 months, respectively. Find the size of each payment if the rate of interest is 6% and the comparison date is 4 months hence.***

**Solution:** Let x represent each equal payment. The values as of the comparison date are computed as follows:

1. The value of the old debt of SR 100 becomes SR 101 on the comparison date. The value is computed as follows:

P = SR 100, r = 6% = 0.06, t = 2/12 = 1/6 year, since the comparison date is 2 months after the due date. So,

A = P(1+rt) = 100(1+0.06×1/6) = SR 101.

1. The value of the old debt of SR 400 becomes SR 392.16 on the comparison date. The value is computed as follows:

A = SR 400, r = 6% = 0.06, t = 4/12 = 1/3 year, since the comparison date is 4 months before the due date. So,

P = $\frac{A}{(1 + rt)}= \frac{400}{(1 + 0.06×1/3)}$ = SR 392.16

1. The value of the first new debt, which is due in 4 months, does not change and is x, since the comparison date is also in 4 months.
2. The value of the second new debt, which is due in 10 months, becomes (x/1.03) on the comparison date. It is computed as follows:

A = x, r = 6% = 0.06, t = 6/12 = ½ year, since the comparison date is 6 months before the due date. So,

P = $\frac{x}{(1 + 0.06×1/2)}= \frac{x}{(1.03)}$

***Example 27: A man owes (1) SR 100, due in 2 months, and (2) SR 400, due in 8 months. His creditor has agreed to settle the debts by two equal payments in 4 months and 10 months, respectively. Find the size of each payment if the rate of interest is 6% and the comparison date is 10 months hence.***

**Solution:** Let x represent each equal payment. The values as of the comparison date are computed as follows:

1. The value of the old debt of SR 100 becomes SR 104 on the comparison date. The value is computed as follows:

P = SR 100, r = 6% = 0.06, t = 8/12 = 2/3 year, since the comparison date is 8 months after the due date. So,

A = P(1+rt) = 100(1+0.06×2/3) = SR 104 (8 months after due date)

1. The value of the old debt of SR 400 becomes SR 404 on the comparison date. The value is computed as follows:

P = SR 400, r = 6% = 0.06, t = 2/12 = 1/6 year, So,

A = P(1+rt) = 400(1+0.06×1/6) = SR 404. (2 months after due date)

1. The value of the first new debt, which is due in 4 months, becomes:

A = x(1+0.06×6/12) = 1.03x (6months after due date)

1. The value of the second new debt, which is due in 10 months, does not change and is x, since the comparison date is also in 10 months.

**Equivalent Time: Finding an average due date**

In this section, the unknown in an equation of value is the value of each new debt. The value of each new debt in the equation is known, but the equivalent time at which the new debt is due is unknown.

If several obligations are due on different maturity dates, and if there is a desire to pay them all off with interest in a single payment, a new date when that single payment will be due has to be found. The date on which the single payment would discharge all debts is called the *average due date* or *equated date*. It is the corresponding date of the last day of the average term, which can be obtained as the weighted average of all maturity terms of the various obligations. It is called the equivalent time (T)

T = $\frac{ƩPiTi}{ƩPi}$ = $\frac{P1T1+P2T2+…PkTk}{P1+P2+…+Pk}$

Where *i* = 1, 2, 3,…k; P1, P2, P3,…Pk are payments; and T1, T2, T3,…Tk are the due dates of those payments, respectively.

***Example 28: A small store owner has to pay his supplier three payments: SR 200 in 30 days, SR 400 in 60 days, and SR 600 in 90 days. If the interest is 8%, what single payment would discharge all three payments? What would be the equated date?***

**Solution:** Here, P1 = SR 200, P2 = SR 400, P3= SR 600, and T1 = 30 days, T2 = 60 days and T3 = 90 days.

Now, putting these values in the formula of equivalent time,

T = $\frac{ƩPiTi}{ƩPi}$ = $\frac{P1T1+P2T2+…PkTk}{P1+P2+…+Pk}$

T = $\frac{200×30+400×60+600×90}{200+400+600}$ = $\frac{6000+12000+36000}{1200}$ = $\frac{54000}{1200}$ = 45 days.

***Example 29: When will a single payment of SR 1010 discharge the debts of (a) SR 400, (b) SR 500, and (c) SR 100, due in 30 days, 60 days, and 90 days, respectively? Assume that the rate of interest is 6%.***

**Solution:** Here, P1 = SR 400, P2 = SR 500, P3= SR 100, and T1 = 30 days, T2 = 60 days and T3 = 90 days.

Now, putting these values in the formula of equivalent time,

T = $\frac{ƩPiTi}{ƩPi}$ = $\frac{P1T1+P2T2+…PkTk}{P1+P2+…+Pk}$

T= $\frac{400×30+500×60+100×90}{400+500+100}$ = $\frac{12000+30000+9000}{1000}$ = $\frac{51000}{1000}$ = 51 days.

**Summary of Simple Interest Formula**

|  |  |
| --- | --- |
| **Application** | **Formula** |
| Simple Interest (SI) | SI = Prt |
|  Ordinary Interest (Io) | *Io = Pr(t/360) =* $\frac{Prt}{360}$ |
|  Exact Interest (Ie) | *Ie = Pr(t/365) =* $\frac{Prt}{365}$ |
| Amount (A) | A= P + SI = P + Prt = P(1 + rt) |
| Rate of Interest (r) | r = $\frac{Simple Interest (SI) }{Principal (P) ×Time (t)}$ |
| Time (t) | t = $\frac{Simple Interest (SI) }{Principal \left(P\right)×Rate of Interest (r)}$ |
| Principal (P) | *P =* $\frac{Simple Interest (SI)}{Rate of Interest \left(r\right)×Time (t)}$= P = $\frac{A}{(1 + rt)}$ |
| Equivalent Time (T) | T = $\frac{ƩPiTi}{ƩPi}$ = $\frac{P1T1+P2T2+…PkTk}{P1+P2+…+Pk}$ |