Chapter 30

Sources of the Magnetic Field

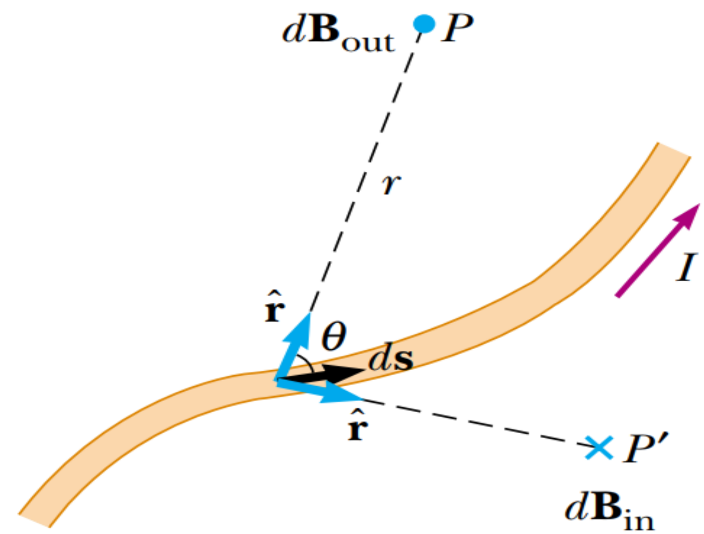
CHAPTER OUTLINE

30.1 The Biot–Savart Law

❖ Introduction

- We learned from last chapter, Oersted discovered a compass needle is deflected by a current-carrying conductor in 1819.
- After that, Jean-Baptiste Biot (1774–1862) and Félix Savart (1791–1841) performed quantitative experiments on the force exerted by an electric current on a nearby magnet.
- From their experimental results, Biot and Savart arrived at a mathematical expression that gives the magnetic field at some point in space in terms of the current that produces the field. That expression is based on the following experimental observations for the magnetic field $d\mathbf{B}$ at a point P associated with a length element $d\mathbf{s}$ of a wire carrying a steady current I

30.1 The Biot–Savart Law



- The vector $d\mathbf{B}$ is perpendicular both to $d\mathbf{s}$ (which points in the direction of the current) and to the unit vector $\hat{\mathbf{r}}$ directed from $d\mathbf{s}$ toward P.
- The magnitude of $d\mathbf{B}$ is inversely proportional to r^2, where r is the distance from $d\mathbf{s}$ to P.
- The magnitude of $d\mathbf{B}$ is proportional to the current and to the magnitude ds of the length element $d\mathbf{s}$.
- The magnitude of $d\mathbf{B}$ is proportional to $\sin\theta$, where θ is the angle between the vectors $d\mathbf{s}$ and $\hat{\mathbf{r}}$.

These observations are summarized in the mathematical expression known today as the **Biot–Savart law:**

Biot–Savart law

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,d\mathbf{s}\times\hat{\mathbf{r}}}{r^2} \tag{30.1}$$

30.1 The Biot–Savart Law

where μ_0 is a constant called the **permeability of free space:**

Permeability of free space $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$

- The field $d\mathbf{B}$ in

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I\, d\mathbf{s} \times \hat{\mathbf{r}}}{r^2}$$

is the field created by the current in only a small length element ***ds*** of the conductor.

- To find the ***total* magnetic field B** created at some point by a current of finite size, we must sum up contributions from all current elements *I d***s** that make up the current.

$$\mathbf{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\mathbf{s} \times \hat{\mathbf{r}}}{r^2}$$

where the integral is taken over the entire current distribution.

30.1 The Biot–Savart Law

Quick Quiz 30.1 Consider the current in the length of wire shown in Figure 30.2. Rank the points *A*, *B*, and *C*, in terms of magnitude of the magnetic field due to the current in the length element shown, from greatest to least.

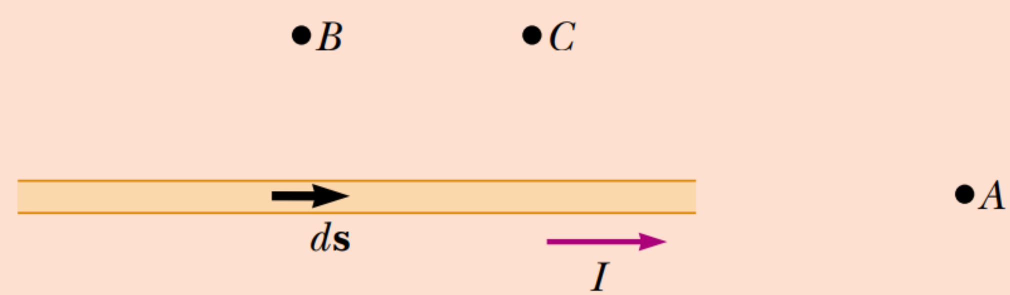


Figure 30.2 (Quick Quiz 30.1) Where is the magnetic field the greatest?

30.1 The Biot–Savart Law

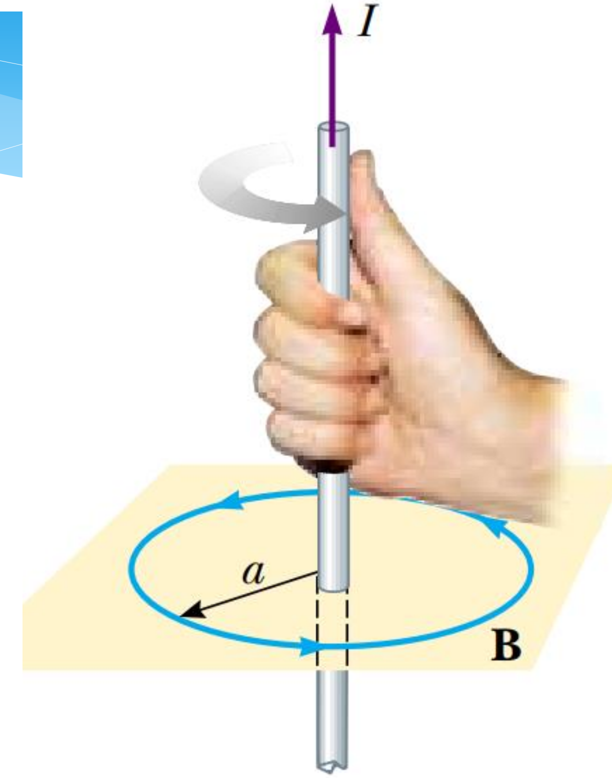
- The right-hand rule for determining the direction of the magnetic field surrounding a long, straight wire carrying a current.
  Note that the magnetic field lines form circles around the wire.

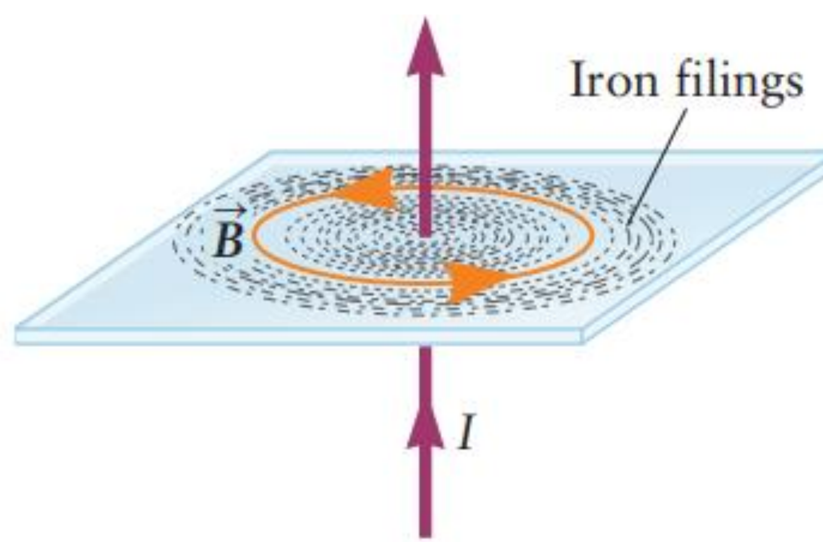
A convenient rule for determining the direction of **B** is to

1- grasp the wire with the right hand,

2-positioning the thumb along the direction of the current.

3- The four fingers wrap in the direction of the magnetic field



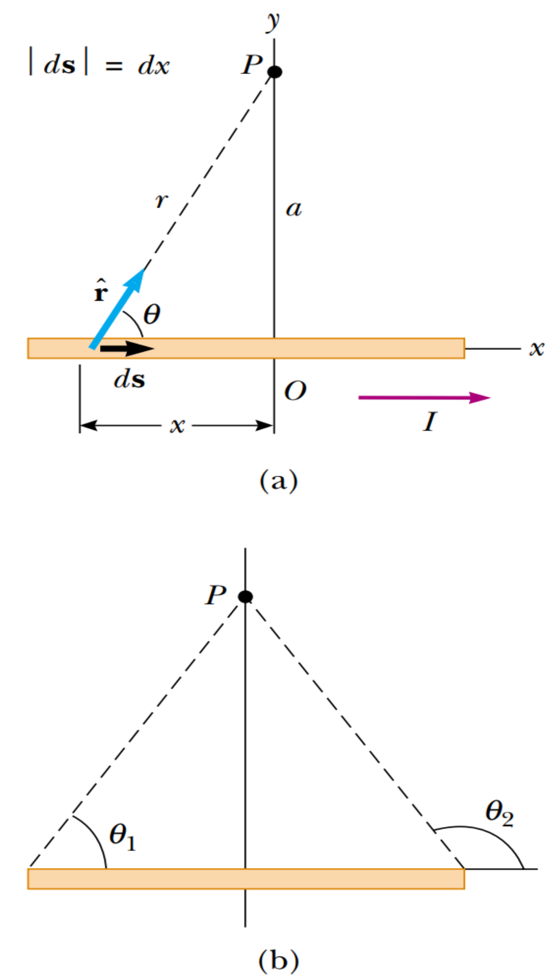


30.1 The Biot–Savart Law

Example 30.1 Magnetic Field Surrounding a Thin, Straight Conductor

❖ Consider a thin, straight wire carrying a constant current *I* and placed along the *x* axis as shown in Figure 30.3. Determine the magnitude and direction of the magnetic field at point *P* due to this current

$$B = \frac{\mu_0 I}{2\pi a}$$



30.1 The Biot–Savart Law

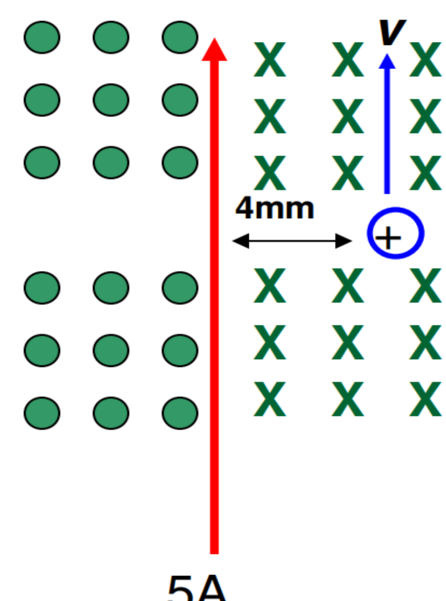
4. Calculate the magnitude of the magnetic field at a point 100 cm from a long, thin conductor carrying a current of 1.00 A.

A long straight wire carries a current of 5.00 A. At one instant a proton, 4.00 mm from the wire, travels at 1.50×10^{5} m/s parallel to the wire and in the same direction as the current.
Find (a) the magnitude and direction of the magnetic force that is acting on the proton because of the magnetic field produced by the wire.

$$F = qvB \qquad B = B = \frac{\mu_0 I}{2\pi r} = \left(4\pi\, x\, 10^{-7}\, \frac{T\cdot m}{A}\right)\left(5\, A\right)\left(\frac{1}{2\pi}\right)\left(\frac{1}{4\, x\, 10^{-3}\, m}\right)$$

$$B = 2.5\, x\, 10^{-4}\, T \qquad F = \left(1.6\, x\, 10^{-19}\, C\right)\left(1.5\, x\, 10^{5}\, \frac{m}{s}\right)\left(2.5\, x\, 10^{-4}\, T\right)$$

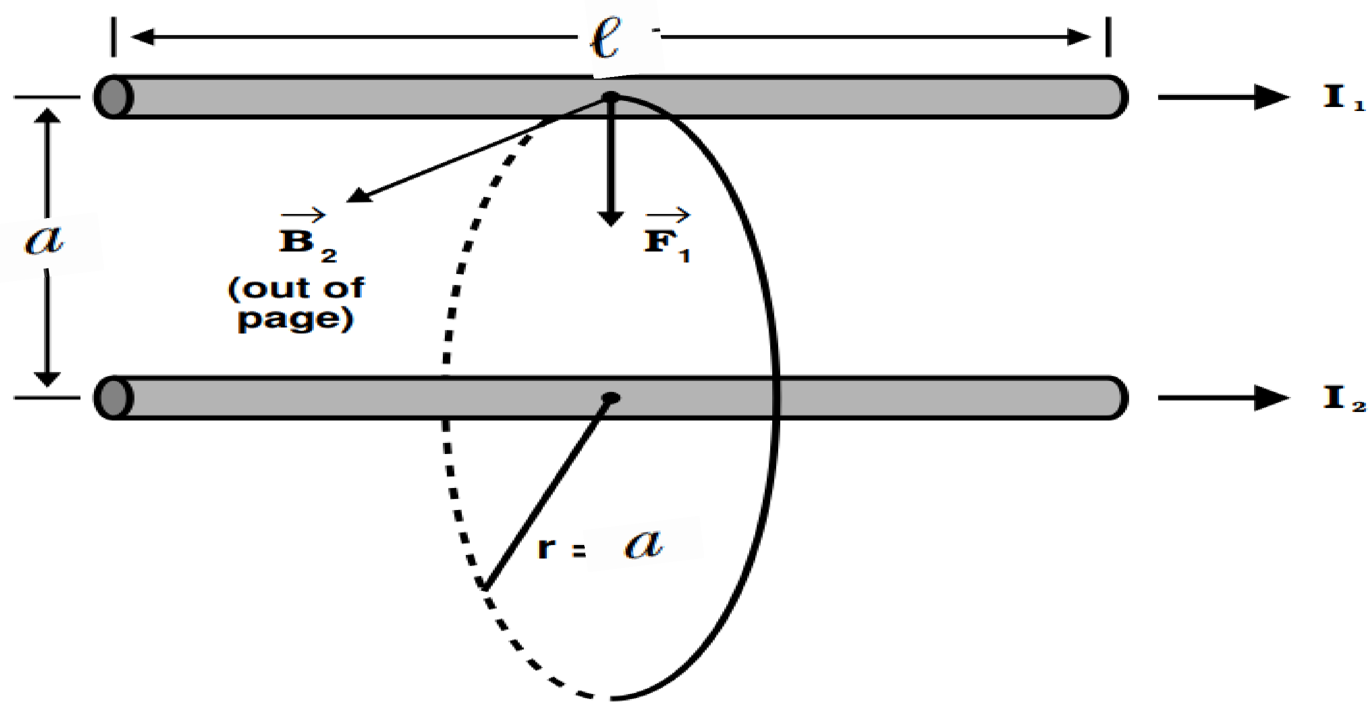
$$F = \boxed{6.00\, x\, 10^{-18}\, N}$$



30.2 The Magnetic Force Between Two Parallel Conductors

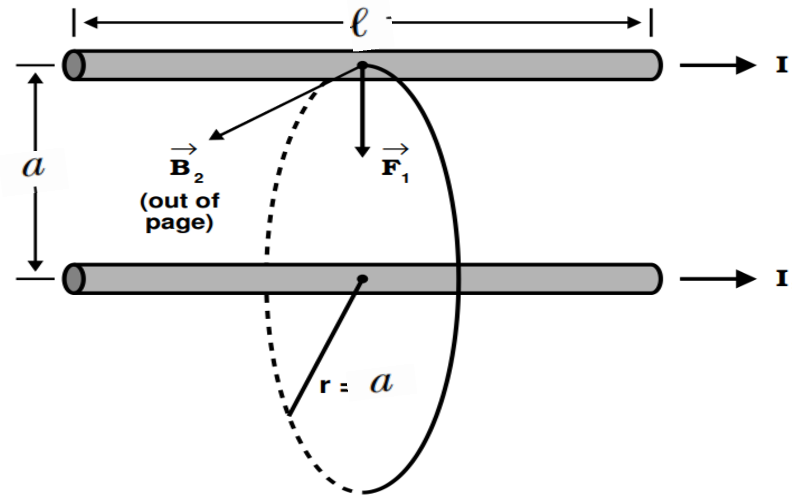
❖ Because a current in a conductor **sets up its own magnetic field**, it is easy to understand that two current-carrying conductors exert magnetic forces on each other.

❖ Two long, straight, parallel wires separated by a distance a and carrying currents I_1 and I_2 in the same direction



❖ We can determine the force exerted on one wire due to the magnetic field set up by the other wire.

30.2 The Magnetic Force Between Two Parallel Conductors



❖ Wire 2, which carries a current I_2 and is identified arbitrarily as the source wire, creates a magnetic field $\vec{B}_2$ at the location of wire1, the test wire. The direction of $\vec{B}_2$ is perpendicular to wire1,

❖ The magnetic force on a length l of wire1 is $F_1 = I_1\, l \times B_2$. Because l is perpendicular to $\mathbf{B}_2$ in this situation, the magnitude of $\mathbf{F}_1$ is

$$F_1 = I_1\, l\, B_2 = I_1\, l\left(\frac{\mu_0 I_2}{2\pi a}\right) = \frac{\mu_0 I_1 I_2}{2\pi a}\, l$$

❖ If the field set up at wire 2 by wire1 is calculated, the force $\mathbf{F}_2$ acting on wire 2 is found to be equal in magnitude and opposite in direction to $\mathbf{F}_1$.

❖ When the currents are in opposite directions (that is, when one of the currents is reversed), the forces are reversed and the wires repel each other.

❖ **Parallel conductors carrying currents in the same direction attract each other, and parallel conductors carrying currents in opposite directions repel each other.**

30.2 The Magnetic Force Between Two Parallel Conductors

❖ Because **the magnitudes of the forces** are the same on both wires, we denote the magnitude of the magnetic force between the wires as simply F_B.

$$\frac{F_B}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi a}$$

❖ The force between two parallel wires is used to define the **ampere**

Definition of the ampere

When the magnitude of the force per unit length between two long parallel wires that carry identical currents and are separated by 1 m is 2×10^{-7} N/m, the current in each wire is defined to be 1 A.

30.2 The Magnetic Force Between Two Parallel Conductors

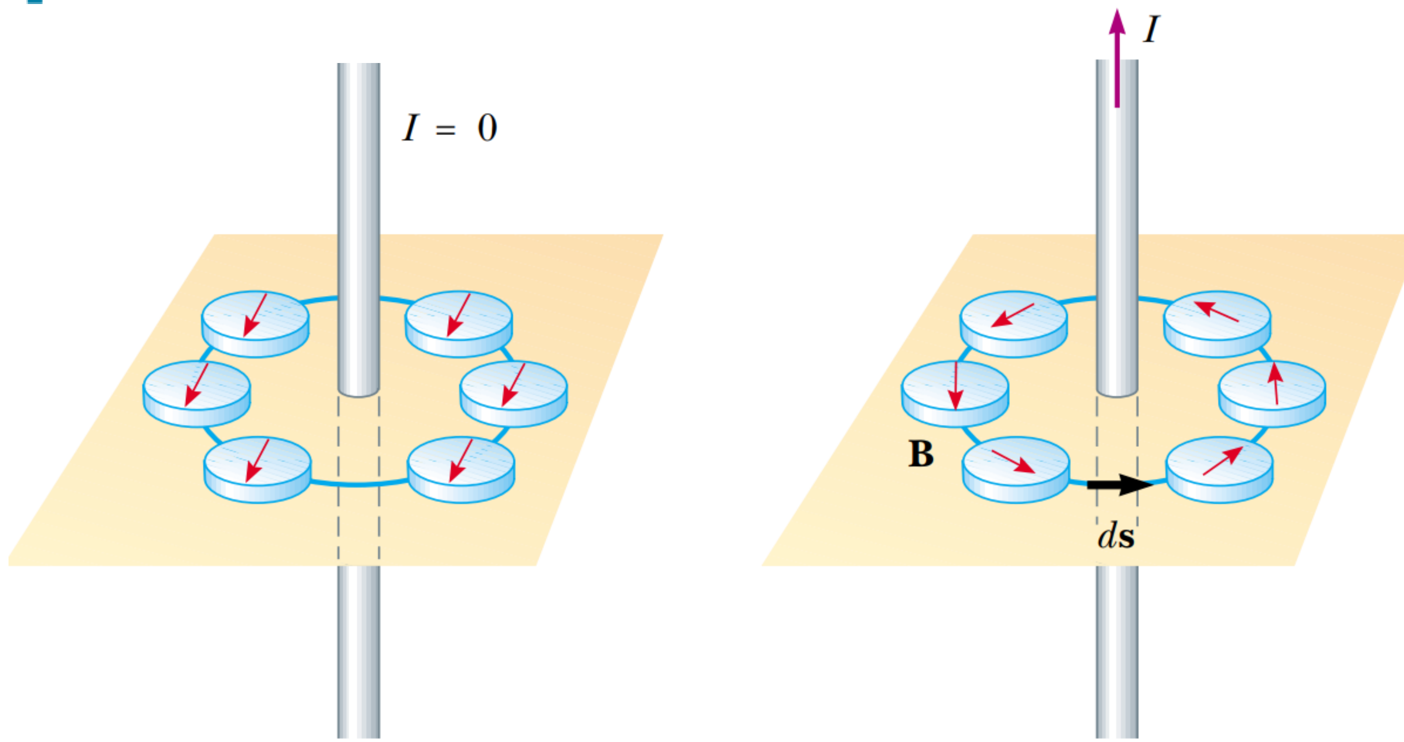
❖ The SI unit of charge, the **coulomb,** is defined in terms of the ampere

When a conductor carries a steady current of 1 A, the quantity of charge that flows through a cross section of the conductor in 1 s is 1 C.

Quick Quiz 30.2 For $I_1 = 2$ A and $I_2 = 6$ A in Figure 30.8, which is true: (a) $F_1 = 3F_2$, (b) $F_1 = F_2/3$, (c) $F_1 = F_2$?

Quick Quiz 30.3 A loose spiral spring carrying no current is hung from the ceiling. When a switch is thrown so that a current exists in the spring, do the coils move (a) closer together, (b) farther apart, or (c) do they not move at all?

30.3 Ampère's Law



(a) (b)

❑ Several compass needles are placed in a horizontal plane near a long vertical wire.

(a) When no current is present in the wire, all the needles point in the same direction (that of the Earth's magnetic field), as expected.

(b) When the wire carries a strong, steady current, the needles all deflect in a direction tangent to the circle which is the direction of the magnetic field created by the current.

❖ These observations is consistent with the right-hand rule described. When the current is reversed, the needles also reverse.

30.3 Ampère's Law

Ampère's law

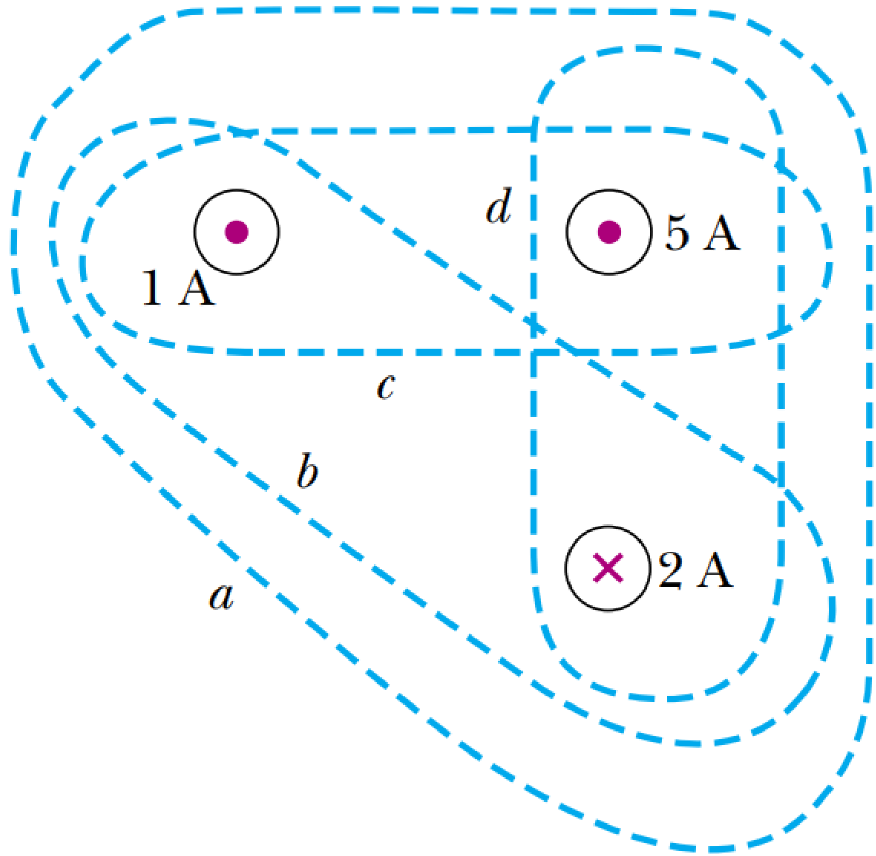
$$\oint \mathbf{B} \cdot d\mathbf{s} = B \oint ds = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I$$

The line integral of $\mathbf{B} \cdot d\mathbf{s}$ around any closed path equals $\mu_0 I$, where I is the total steady current passing through any surface bounded by the closed path.

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I \quad (30.13)$$

Ampère's law describes the creation of magnetic fields by all continuous current configurations, but at our mathematical level it is useful only for calculating the magnetic field of current configurations having a high degree of symmetry.

30.3 Ampère's Law

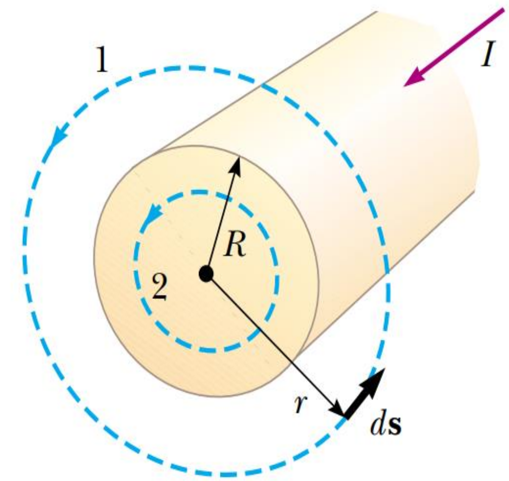


Quick Quiz 30.4 Rank the magnitudes of $\oint \mathbf{B} \cdot d\mathbf{s}$ for the closed paths in Figure 30.10, from least to greatest.

30.3 Ampère's Law

Example 30.4 The Magnetic Field Created by a Long Current-Carrying Wire

A long, straight wire of radius R carries a steady current I that is uniformly distributed through the cross section of the wire (Fig. 30.12). Calculate the magnetic field a distance r from the center of the wire in the regions $r \geq R$ and $r < R$.



- Because the wire has a high degree of symmetry, we categorize this as an Ampère's law problem.

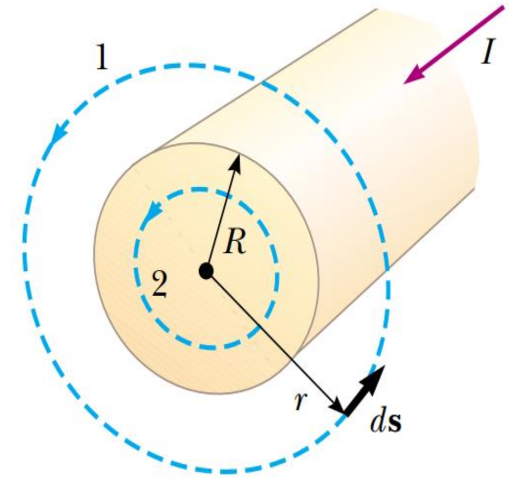
- From symmetry, **B** must be constant in magnitude and parallel to $d\mathbf{s}$ at every point on this circle. Because the total current passing through the plane of the circle is I, Ampère's law gives

$$\oint \mathbf{B} \cdot d\mathbf{s} = B \oint ds = B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r} \qquad (\text{for } r \geq R)$$

The magnetic field at any point can be calculated from Ampère's law using a circular path of radius r, concentric with the wire.

30.3 Ampère's Law



Now consider the interior of the wire, where $r < R$. Here the current I' passing through the plane of circle 2 is less than the total current I. Because the current is uniform over the cross section of the wire, the fraction of the current enclosed by circle 2 must equal the ratio of the area πr^2 enclosed by circle 2 to the cross-sectional area πR^2 of the wire :

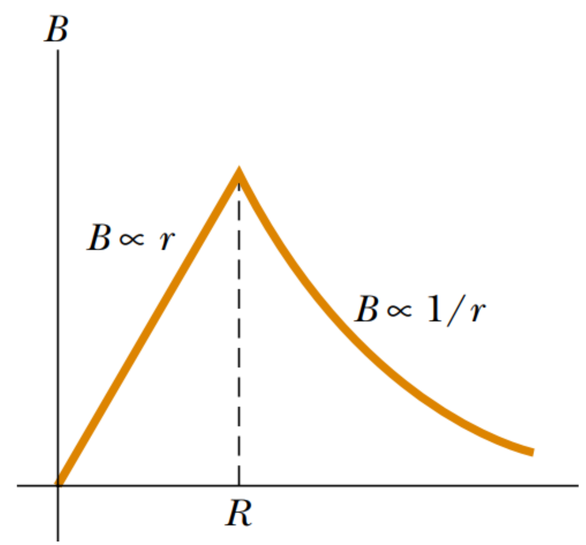
$$\frac{I'}{I} = \frac{\pi r^2}{\pi R^2}$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = B(2\pi r) = \mu_0 I' = \mu_0 \left(\frac{r^2}{R^2} I \right)$$

$$I' = \frac{r^2}{R^2} I$$

$$B = \left(\frac{\mu_0 I}{2\pi R^2} \right) r \qquad \text{(for } r < R\text{)}$$

$$B = \frac{\mu_0 I}{2\pi r} \qquad \text{(for } r \geq R\text{)}$$



They give the same value of the magnetic field at $r = R$, demonstrating that the magnetic field is continuous at the surface of the wire.

30.2 The Magnetic Force Between Two Parallel Conductors

16. Two long, parallel conductors, separated by 10.0 cm, carry currents in the same direction. The first wire carries current $I_1 = 5.00$ A and the second carries $I_2 = 8.00$ A. (a) What is the magnitude of the magnetic field created by I_1 at the location of I_2? (b) What is the force per unit length exerted by I_1 on I_2? (c) What is the magnitude of the magnetic field created by I_2 at the location of I_1? (d) What is the force per length exerted by I_2 on I_1?

17. In Figure P30.17, the current in the long, straight wire is $I_1 = 5.00$ A and the wire lies in the plane of the rectangular loop, which carries the current $I_2 = 10.0$ A. The dimensions are $c = 0.100$ m, $a = 0.150$ m, and $\ell = 0.450$ m. Find the magnitude and direction of the net force exerted on the loop by the magnetic field created by the wire.

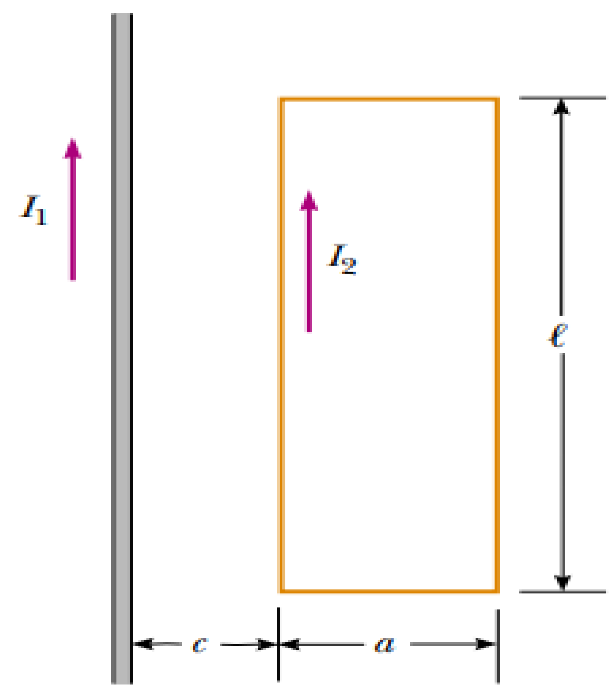
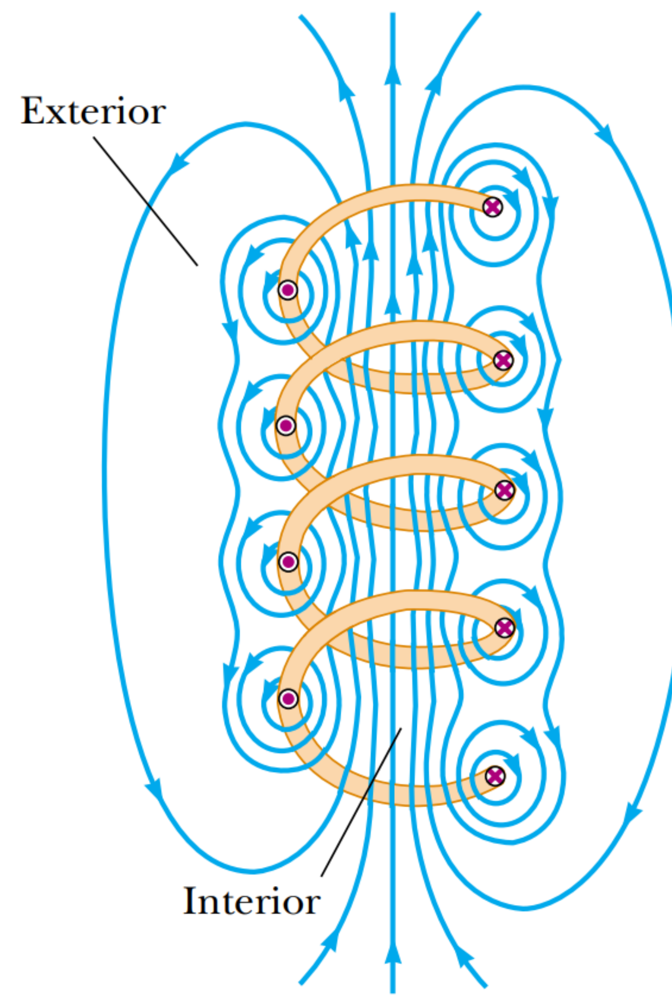


Figure P30.17

30.4 The Magnetic Field of a Solenoid

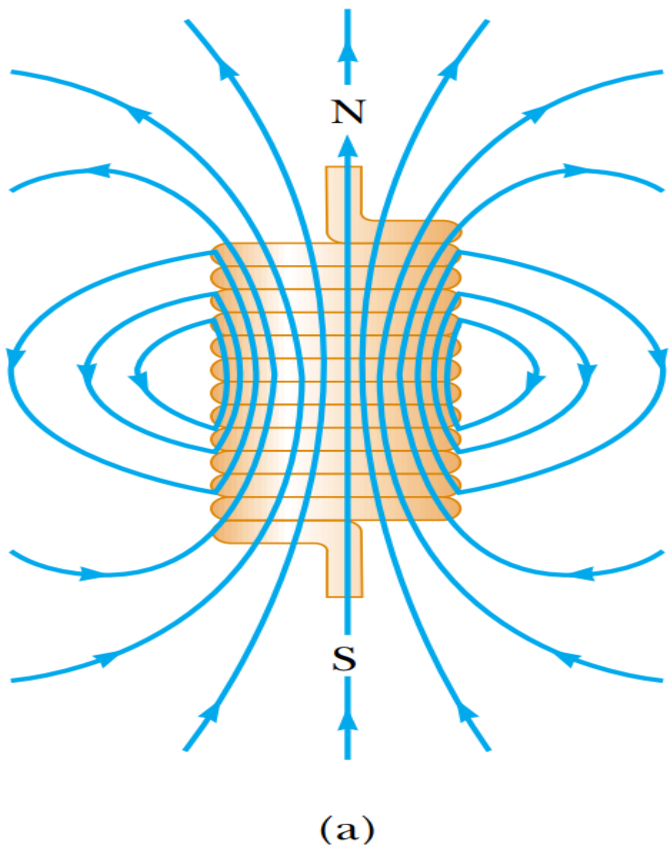
A **solenoid** is a long wire wound in the form of a helix. With this configuration, a reasonably uniform magnetic field can be produced in the space surrounded by the turns of wire—which we shall call the *interior* of the solenoid—when the solenoid carries a current.

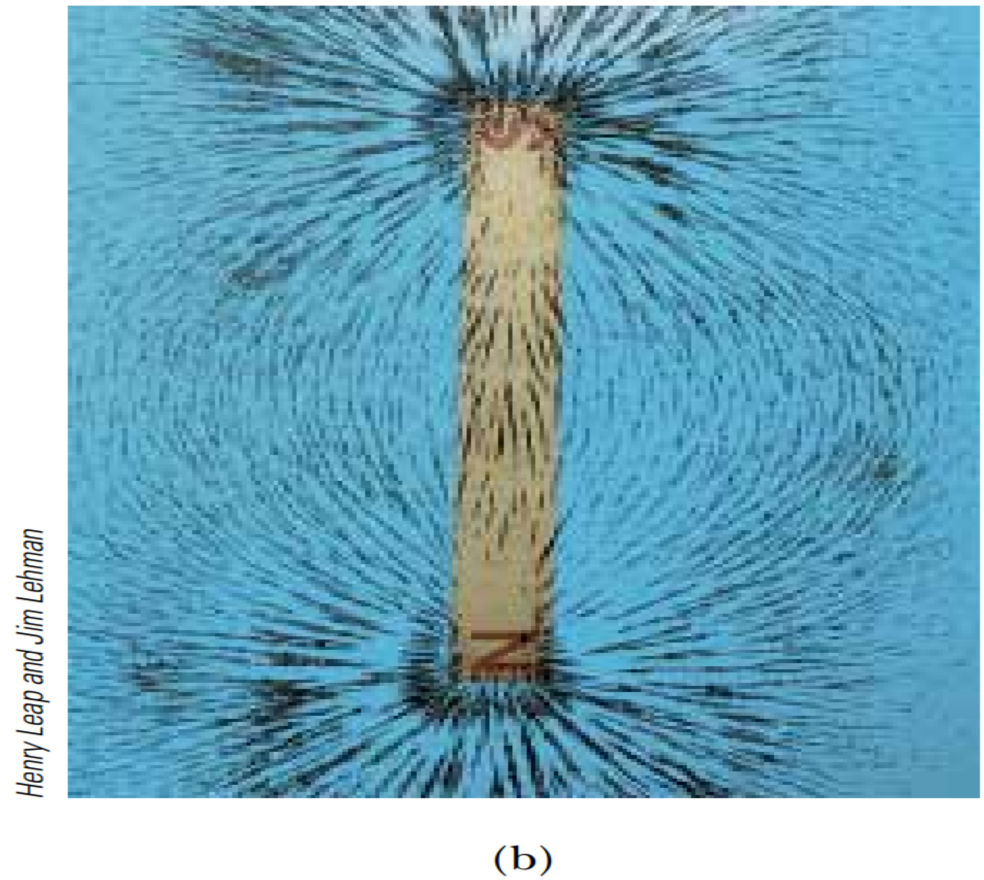


❖ The turns can be approximated as a circular loop, and the net magnetic field is the vector sum of the fields resulting from all the turns.

30.4 The Magnetic Field of a Solenoid

This field line distribution is similar to that surrounding a bar magnet as one end of the solenoid behaves like the north pole of a magnet, and the opposite end behaves like the south pole.





(a) Magnetic field lines for a tightly wound solenoid of finite length, carrying a steady current. The field in the interior space is strong and nearly uniform. Note that the field lines resemble those of a bar magnet, meaning that the solenoid effectively has north and south poles. (b) The magnetic field pattern of a bar magnet.

30.4 The Magnetic Field of a Solenoid

- As the length of the solenoid increases, the interior field becomes more uniform and the exterior field becomes weaker.

- An *ideal solenoid* is approached when the turns are closely spaced and the length is much greater than the radius of the turns.

- In this case, the external field is close to zero, and the interior field is uniform over a great volume.

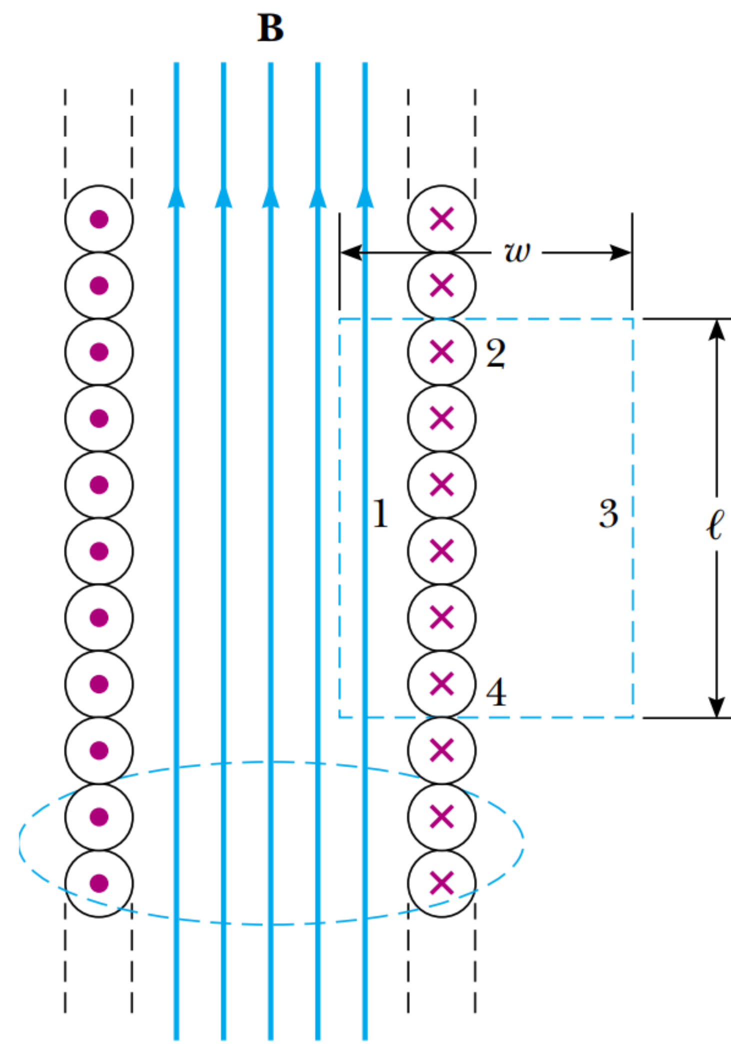
30.4 The Magnetic Field of a Solenoid

❖ We can use Ampère's law to obtain a quantitative expression for the interior magnetic field in an ideal solenoid.

❖ Consider the rectangular path of length l and width w . We can apply Ampère's law to this path by evaluating the integral of $\vec{B}$ ($d\mathbf{s}$ over each side of the rectangle.

$$\oint \vec{B}.\vec{ds} = \int_1 \vec{B}.\vec{ds} + \int_2 \vec{B}.\vec{ds} + \int_3 \vec{B}.\vec{ds} + \int_4 \vec{B}.\vec{ds}$$

$$\oint \vec{B}.\vec{ds} = Bl + 0 + 0 + 0 = \mu_0 I_{enclosed}$$



Cross-sectional view of an ideal solenoid, where the interior magnetic field is uniform and the exterior field is close to zero.

30.4 The Magnetic Field of a Solenoid

Magnetic field inside a solenoid

$$\oint \mathbf{B} \cdot d\mathbf{s} = B\ell = \mu_0 NI$$

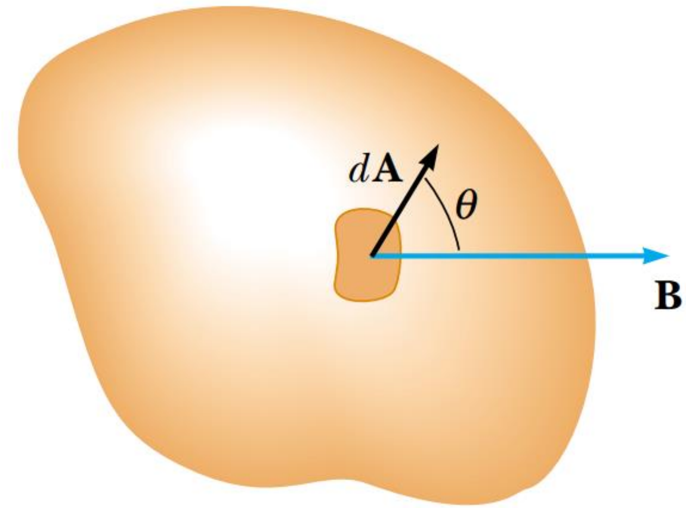
$$B = \mu_0 \frac{N}{\ell} I = \mu_0 nI$$

where $n = N/\ell$ is the number of turns per unit length.

30.4 The Magnetic Field of a Solenoid

31. What current is required in the windings of a long solenoid that has 1 000 turns uniformly distributed over a length of 0.400 m, to produce at the center of the solenoid a magnetic field of magnitude 1.00×10^{-4} T?

30.5 Magnetic Flux



- Consider an element of area *dA* on an arbitrarily shaped surface

- If the magnetic field at this element is $\vec{B}$, the magnetic flux through the element is $\vec{B}$ **.** (*d***A**, where *d***A** is a vector that is perpendicular to the surface and has a magnitude equal to the area *dA*.

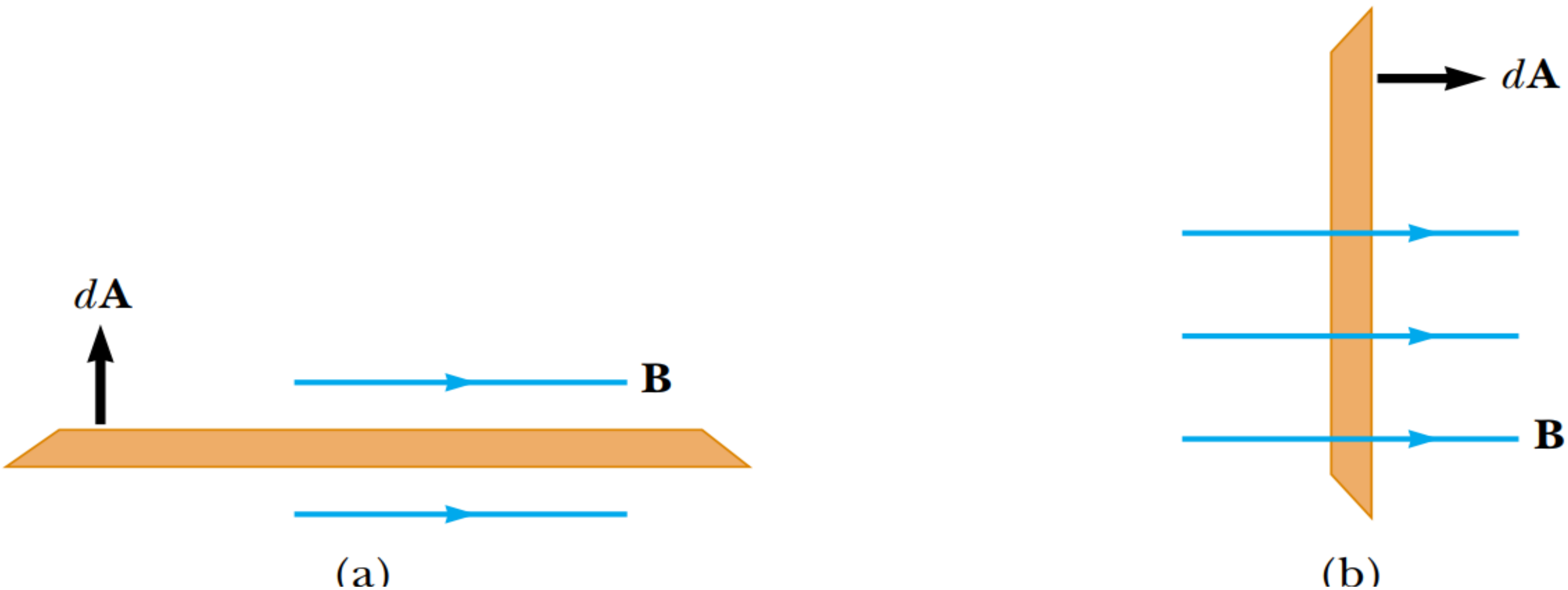
- The total magnetic flux $\emptyset_B$ through the surface is

$$\Phi_B = \int \mathbf{B} \cdot d\mathbf{A}$$

30.5 Magnetic Flux

Consider the special case of a plane of area A in a uniform field **B** that makes an angle θ with $d\mathbf{A}$. The magnetic flux through the plane in this case is

$$\Phi_B = BA\cos\theta$$



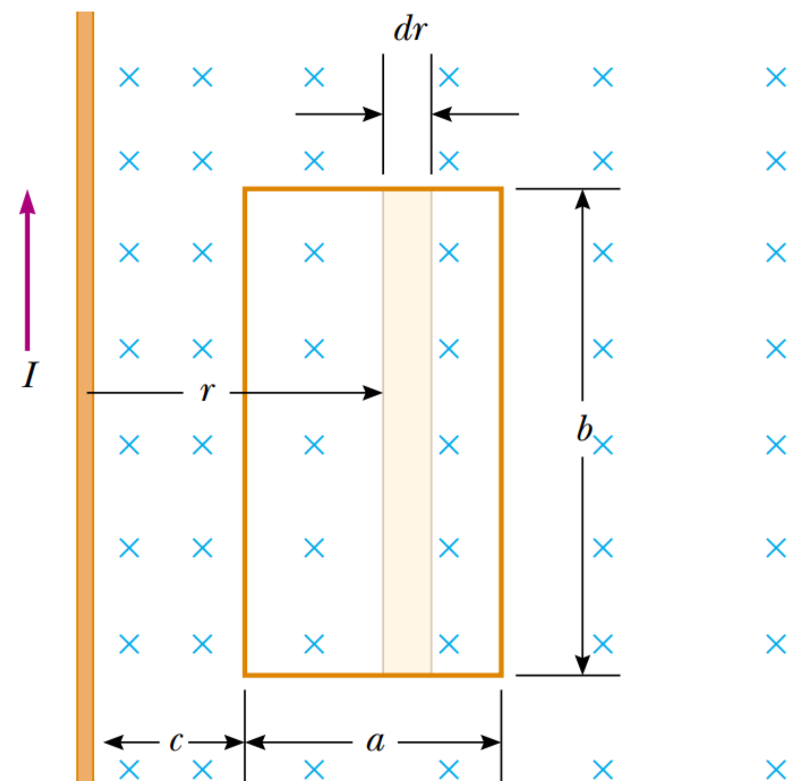
The unit of magnetic flux is $T \cdot m^2$, which is defined as a *weber* (Wb); $1\ Wb = 1\ T \cdot m^2$.

30.5 Magnetic Flux

Example 30.8 Magnetic Flux Through a Rectangular Loop

Find the total magnetic flux through the loop due to the current in the wire.

$$B = \frac{\mu_0 I}{2\pi r}$$



30.6 Gauss's Law in Magnetism

Gauss's law in magnetism states that

the net magnetic flux through any closed surface is always zero:

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

- ❖ for any closed surface, the number of lines entering the surface equals the number leaving the surface; thus, the net magnetic flux is zero.

30.6 Gauss's Law in Magnetism

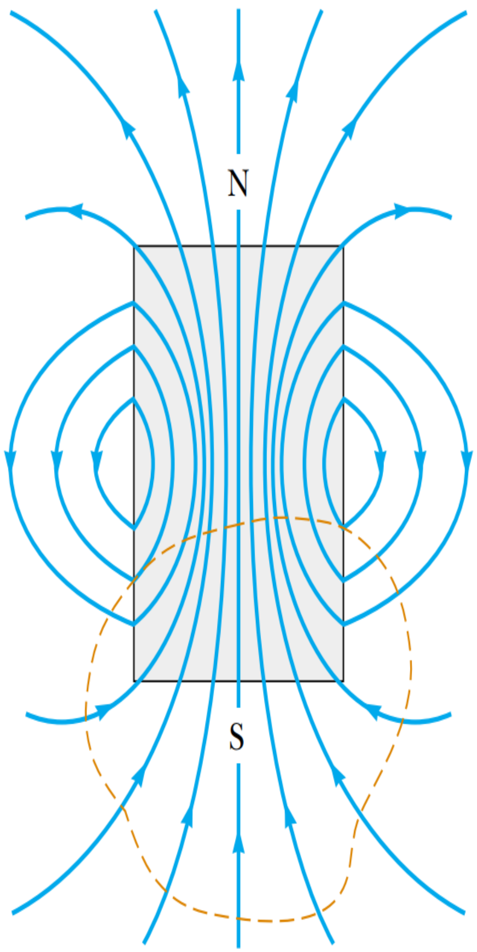


Figure 30.23 The magnetic field lines of a bar magnet form closed loops. Note that the net magnetic flux through a closed surface surrounding one of the poles (or any other closed surface) is zero. (The dashed line represents the intersection of the surface with the page.)

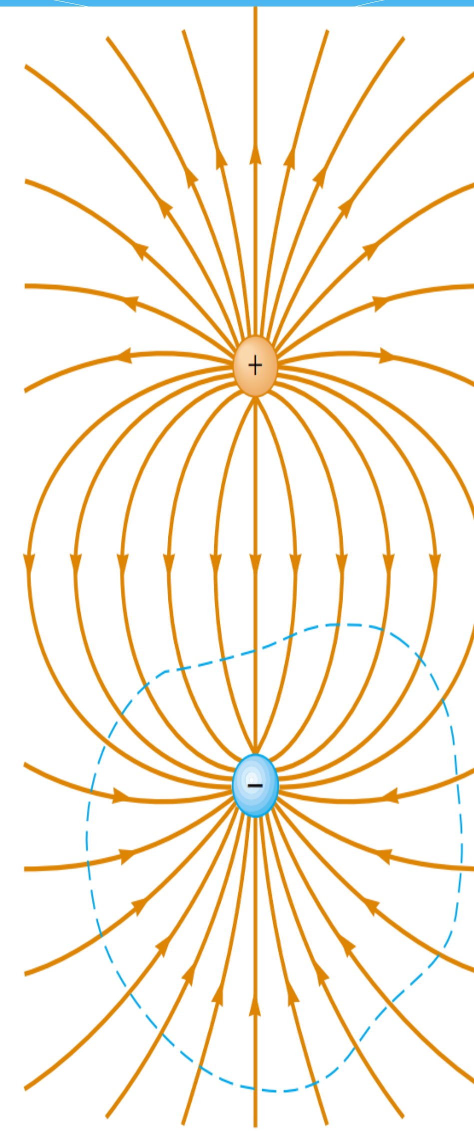


Figure 30.24 The electric field lines surrounding an electric dipole begin on the positive charge and terminate on the negative charge. The electric flux through a closed surface surrounding one of the charges is not zero.

35. A cube of edge length $\ell = 2.50$ cm is positioned as shown in Figure P30.35. A uniform magnetic field given by $\mathbf{B} = (5\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$T exists throughout the region. (a) Calculate the flux through the shaded face. (b) What is the total flux through the six faces?

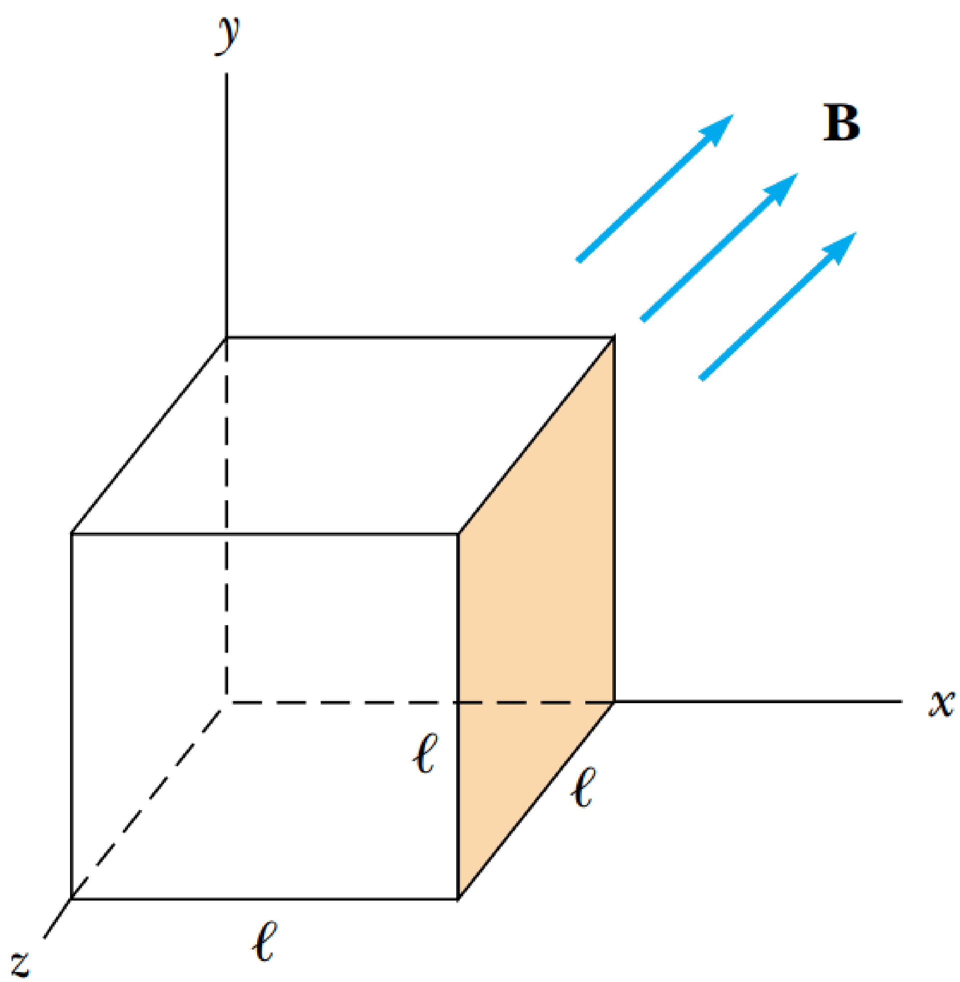


Figure P30.35

Chapter # 30 Problems

Selected Old exam questions

Q20. A current 3A is passing a wire and if the resulted magnetic field was 2T.The diameter of this field will be:

20. يمر تيار كهربائي 3A في سلك لينتج مجال مغناطيسي قدره 2T، فإن قطر مقطع هذا المجال هو:

A) 100 nm B) 600 nm C) 400 nm D) 150 nm E) 20 nm

Q21. Two long, straight, parallel wires separated by a distance of 10 cm; and both are carrying in the same direction currents 1 A and 2 A respectively. The magnetic force per unit length is:

21. سلكان طويلان متوازيان منفصلان عن بعضهما مسافة 10 cm ، وكليهما يحملان تياراً كهربائياً الأول 1 A والثاني 2 A على الترتيب. إن القوة المغناطيسية الناتجة في وحدة الطول هي:

A) 2μN/m B) 8μN/m C) 10μN/m D) 4μN/m E) 6μN/m

Q22. Two cables have the same length and producing the magnetic force. If the first cable is carrying 20A and the second is carrying 100A. The ratio B_2/B_1 is:

22. بفرض سلكين (كابلين) لهما نفس الطول وينتجان نفس القوة المغناطيسية، الأول يحمل تياراً 20A فقط والثاني يحمل 100A، فإن النسبة B_2/B_1 هي:

A) 6 B) 2 C) 5 D) 50 E) 8

Q23. A solenoid has 100 turns per unit length. If the current was 10 A, then the magnetic field **B** is:

23. يحتوي سولونيد (ملف حلزوني) على 100 لفة في وحدة الطول ، ويمر فيه تيارا قيمته 10 A، فإن المجال المغناطيسي **B** هو:

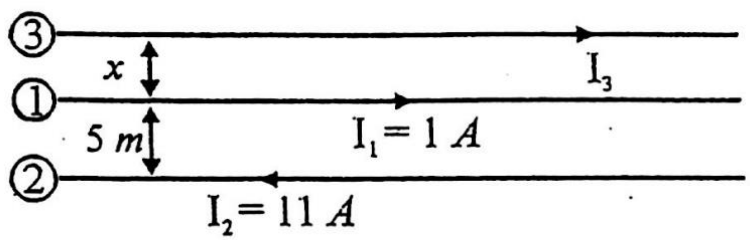
A) 3.06×10^{-3}T B) 3.18×10^{-3}T C) 5.45×10^{-3}T D) 1.26×10^{-3}T E) 9.35×10^{-3}T

Chapter # 30 Problems

Selected Old exam questions

س٢٢- عندما تكون القوة المؤثرة على السلك رقم 3 تساوي الصفر فان المسافة x تساوي:

Q22- When the magnetic force exerted on wire 3 is zero, then the distance x equals:



A) 7 B) 4.8 C) 0.83 D) 0.5

س٢٣- مقدار التكامل $\oint \mathbf{B}.d\mathbf{s}$ على مسار مغلق يمر من خلاله تيار كهربي قدره I يساوي:

Q23- The magnitude of integrating $\oint \mathbf{B}.d\mathbf{s}$ over a closed path through which electric current I is passing equals:

A) μ_o I B) μ_o/I C) ε_o I D) I / ε_o

س٢٤- إذا كان التدفق المغناطيسي خلال جزء من سطح مغلق يساوي *Weber* 150- فإن التدفق المغناطيسي خلال باقي السطح المغلق يساوي:

Q24- If the magnetic flux though a portion of a closed surface equals -150 *Weber*, the magnetic flux through the rest of the surface is:

A) -150 B) - 300 C) 150 D) 300