Chapter (4) : Basic Probability (Examples)

Example (1):

A box contain five red balls, a ball is drawn at random, what is the possibility that the ball will be red?

$T = 5 \qquad S = 5$

$P(R) = \frac{T}{S} = \frac{5}{5} = 1$ (Certain event)

Example (2):

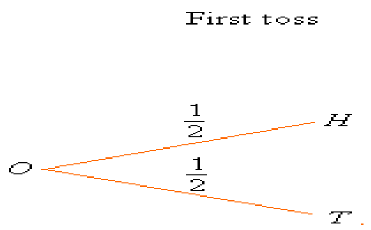
A box contain five red balls, a ball is drawn at random, what is the possibility that the ball will be blue?

$T = 0 \qquad S = 5$

$P(B) = \frac{T}{S} = \frac{0}{5} = 0$ (Impossible event)

Example (3):

An experiment is consisting of tossing (flip) a fair coin once, what is the probability of getting a head?



S={ H,T}

S= 2 n(H)= 1

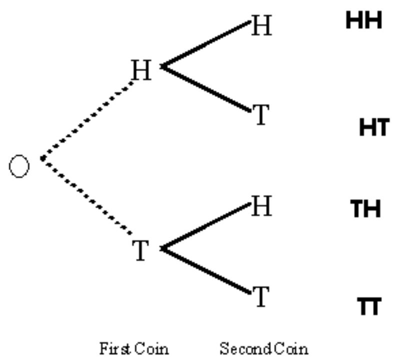
$P(H) = \frac{T}{S} = \frac{1}{2} = 0.50$

Example (4):

If an experiment is consisting of tossing a fair coin twice, find:

1. The Set of all possible outcomes of the experiment.
2. The probability of the event of getting at least one head.
3. The probability of the event of getting exactly one head in the two tosses.
4. The probability of the event of getting two heads.

Solution:



1.
S= {HH, HT, TH, TT}
Where,
And since the coin is fair, then all of the elementary events are equally likely, i.e.
P(HH) = P(HT) = P(TH) = P(TT) = 0.25
2.
Let
E_1 = {HH, HT, TH} be the event of getting at least one head, then
$n(E_1)$=T =3

And hence $P(E_1) = \frac{T}{S} = \frac{3}{4} = 0.75$

3.
E_2 = {HT, TH} be the event of getting exactly one head, then
n (E_2) =2   And hence

$$P(E_2) = \frac{T}{S} = \frac{2}{4} = 0.5$$

4. Let
E_3 = {HH} be the event of getting two heads, then n (E_3) =1

$$P(E_3) = \frac{T}{S} = \frac{1}{4} = 0.25$$

Example (5):
If the experiment is consisting of rolling a fair die once, find:
1. Set of all possible outcomes of the experiment.
2. The probability of the event of getting an even number.
3. The probability of the event of getting an odd number.
4. The probability of the event of getting a four or five.
5. The probability of the event of getting a number less than 5.

Solution:

1.
S={1,2,3,4,5,6 }   n(S)=6
Since the coin is fair, then all events are equally likely, i.e.

$$P(1) = P(2) = \cdots = P(6) = \frac{1}{6}$$

2. Let,
E_1= {2, 4, 6} be the event of getting an even number, then
n (E_1) = 3

$$P(E_1) = \frac{3}{6} = 0.50$$

3.
E_2= {1, 3, 5} be the event of getting an odd number, then
n (E_2) =3

$$P(E_2) = \frac{3}{6} = 0.50$$

4. Let,
E_3= {4, 5} be the event of getting a four or five, then
n (E_3) =2

$$P(E_3) = \frac{2}{6} = 0.33$$

5. Let,
E_4= {1, 2, 3, 4} be the event of getting a number less than 5, then

$$P(E_4) = \frac{4}{6} = 0.67$$

Example (6):

The probabilities of the events A and B are 0.20 and 0.25, respectively. The probability that both A and B occur is 0.10.What is the probability of either A or B occurring.

Solution:

$$P(A\,or\,B)=P(A\cup B)=P(A)+P(B)-P(A\cap B)$$
$$=0.20+0.25-0.10=0.45-0.10=0.35$$

Example (7):

Suppose P (A) =0.3 and P (B) =0.15 .What is the probability of A and B**?**

Solution:

$$P(A\,and\,B)=P(A\cap B)=P(A)\times P(B)=0.30\times 0.15=0.045$$

Example (8): Suppose P (A) =0.45 & P (B\A) =0.12. What is the probability of A and B**?**

Solution:

$$P(A\,and\,B)=P(A\cap B)=P(A)P(B\setminus A)=0.45\times 0.12=0.054$$

Example (9):

Suppose that P (A) =0.7and P (A ∩ B) =0.21, find:

1. The value of P (B/A)
2. If P (B) = 0.3 are events A and B independent?

Solution:

1. $P(B/A)=\dfrac{P(A\cap B)}{P(A)}=\dfrac{0.21}{0.7}=0.30$

2. $\because P(B/A)=0.30\ and P(B)=0.3$
$\therefore A\ and\ B\ independent$

Example (10)

A box contains eight red balls and five white balls, two balls are drawn at random, find:

1. The probability of getting both the balls white, when the first ball drawn is replace.
2. The probability of getting both the balls red, when the first ball drawn is replace
3. The probability of getting one of the balls red, when the first drawn ball is replaced back.

Solution:

Let W_1 be the event that the in the first draw is white and W_2. In a similar way define R_1 and R_2. Since the result of the first draw has no effects on the result of the second draw, it follows that W_1 and W_2 are independent and similarly R_1 and R_2 are independent.

1.

$$P(W_1 \cap W_2) = P(W_1)\,P(W_2) = \left(\frac{5}{13}\right)\left(\frac{5}{13}\right) = \frac{25}{169}$$

2.

$$P(R_1 \cap R_2) = P(R_1)\,P(R_2) = \left(\frac{8}{13}\right)\left(\frac{8}{13}\right) = \frac{64}{169}$$

3. Since the first drawn ball is replaced back, then the result of the first draw has no effect on the result of the second draw. Let E be the event that one of the ball is red, then:

$$P(E) = P(R_1)\,P(W_2) + P(W_1)\,P(R_2) = \left(\frac{8}{13}\right)\left(\frac{5}{13}\right) + \left(\frac{5}{13}\right)\left(\frac{8}{13}\right) = \frac{80}{169}$$

Example (11)

A box contains seven blue balls and five red balls, two balls are drawn at random without replacement, find:

1. The probability that both balls are blue.
2. The probability that both balls are red.
3. The probability that one of the balls is blue.
4. The probability that at least one of the balls is blue.
5. The probability that at most one of the balls is blue.

Solution:

Let B_1 denote the event that the ball in the first draw is blue and B_2 denote the event that the ball in the second draw is blue. In a similar way define R_1 and R_2.

1.

$$P(B_1 \text{ and } B_2) = P(B_1 \cap B_2) = P(B_1)\,P(B_2|B_1)$$
$$= \left(\tfrac{7}{12}\right)\left(\tfrac{6}{11}\right) = \tfrac{42}{132} = 0.32$$

2.

$$P(R_1 \text{ and } R_2) = P(R_1 \cap R_2) = P(R_1)\,P(R_2|R_1)$$
$$= \left(\tfrac{5}{12}\right)\left(\tfrac{4}{11}\right) = \tfrac{20}{132} = 0.15$$

3.

$$P(\text{one ball is blue}) =$$
$$P((B_1 \text{and } R_2) \text{ or } (R_1 \text{ and } B_2)) = P(B_1)\,P(R_2|B_1) + P(R_1)P(B_2|R_1)$$
$$= \left(\tfrac{7}{12}\right)\left(\tfrac{5}{11}\right) + \left(\tfrac{5}{12}\right)\left(\tfrac{7}{11}\right)$$
$$= \tfrac{35}{132} + \tfrac{35}{132} = \tfrac{70}{132} = 0.53$$

4. That at least one of the balls is blue

$$P(At\ least\ one\ ball\ is\ blue) = P((B_1\ and\ B_2)\ or\ (B_1 and\ R_2)\ or\ (R_1\ and\ B_2))$$

$$= P(B_1)\,P(B_2|B_1) + P(B_1)\,P(R_2|B_1) + P(R_1)\,P(B_2|R_1)$$

$$= \left(\tfrac{7}{12}\right)\left(\tfrac{6}{11}\right) + \left(\tfrac{7}{12}\right)\left(\tfrac{5}{11}\right) + \left(\tfrac{5}{12}\right)\left(\tfrac{7}{11}\right)$$

$$= \tfrac{42}{132} + \tfrac{35}{132} + \tfrac{35}{132} = \tfrac{112}{132} = 0.85$$

Another solution:

$$P(at\ least\ one\ blue\ is\ ball) = 1 - P(zero\ blue\ ball)$$

$$= 1 - P(R_1 and\ R_2)$$

$$= 1 - \left[\left(\tfrac{5}{12}\right)\left(\tfrac{4}{11}\right)\right]$$

$$= 1 - \tfrac{20}{132} = 1 - 0.15 = 0.85$$

5.

$$P(at\ most\ one\ ball\ is\ blue)$$

$$= ((B_1 and\ R_2)\ or\ (R_1 and\ B_2)\ or\ (R_1 and\ R_2))$$

$$= P(B_1)\,P(R_2|B_1) + P(R_1)P(B_2|R_1) + P(R_1)P(R_2|R_1)$$

$$= \left(\tfrac{7}{12}\right)\left(\tfrac{5}{11}\right) + \left(\tfrac{5}{12}\right)\left(\tfrac{7}{11}\right) + \left(\tfrac{5}{12}\right)\left(\tfrac{4}{11}\right)$$

$$= \tfrac{35}{132} + \tfrac{35}{132} + \tfrac{20}{132} = \tfrac{90}{132} = 0.68$$

Another solution:

$$P(at\ most\ one\ blue\ ball) = 1 - P(two\ blue\ balls)$$

$$= 1 - P(B_1 and\ B_2)$$

$$= 1 - \left[\left(\tfrac{7}{12}\right)\left(\tfrac{6}{11}\right)\right]$$

$$= 1 - \tfrac{42}{132} = 1 - 0.32 = 0.68$$

Example (12):

In a math class of 30 students, 17 are boys and 13 are girls. On a unit test, 4 boys and 5 girls made an A grade. If a student is chosen at random from the class, what is the probability of choosing a girl or an A student?

P(girl or A) = P(girl) + P(A) + P(girl and A)

= (13/30) +(9/30) – (5/30) =17/30

Example (13):
On New Year's Eve, the probability of a person having a car accident is 0.09. The probability of a person driving while talking mobile is 0.32 and probability of a person having a car accident while Driving while talking is 0.15. What is the probability of a person driving while talking mobile or having a car accident?

P(talking mobile or accident) = P(talking mobile)+ P(accident) - P(talking mobile and accident)

= 0.32+0.09 -0.15 = 0.26

Example (14):
A school survey found that 9 out of 10 students like pizza. If three students are chosen at random **with replacement**, what is the probability that all three students like pizza?

P(student 1 likes pizza) = 9/10
P(student 2 likes pizza)= 9/10
P(student 3 likes pizza) = 9/10
P(student 1 and student 2 and student 3 like pizza)
=(9/10)*(9/10)*(9/10)= 729/1000

Example (15):
A committee consists of four women and three men. The committee will randomly select two people to attend a conference in Hawaii. Find the probability that both are women.
Solution:
Let *A* be the event that first person selected is woman and *B* be the event that second person selected is woman.

Now we selected a woman as the first person to attend the conference, we cannot select her as a second person to attend the conference.

So now there are 6 people left to select from and only 3 of them are women. So to find the probability of selecting both women is

P(*A* and *B*) = P(*A*) * P(*B* / *A*) = (4/7) * (3/6) = 12/42 = 0.2857