24.3 Applications of Gauss' Law

One of the approaches used to determine the electric filed is Gauss's law.

What conditions should the Gauss's surface satisfy when calculating electric filed due to charge distributions?

- The value of the electric field can be argued from symmetry to be constant over the surface.
- The dot product of $\vec{\boldsymbol{E}}.\ \overrightarrow{\Delta \boldsymbol{A}}$can be expressed as a simple algebraic product EdA because $\vec{E}$and $\overrightarrow{\Delta A}$ are parallel.
- The dot product is 0 because $\vec{E}$and $\overrightarrow{\Delta A}$ are perpendicular.
- The field is constant over the portion of the surface.

I- Sphere of Uniform Charge

$$\rho = \frac{Q}{V}$$

$$a)\ E = k_e \frac{Q}{r^2}, \qquad r \succ a$$

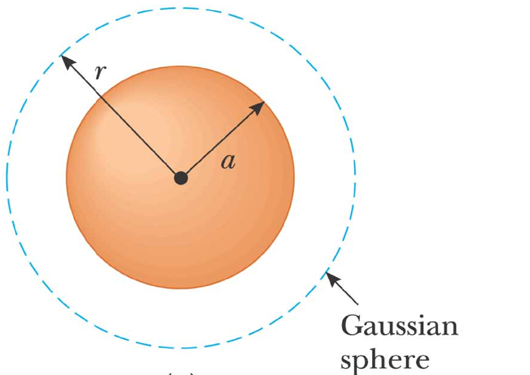
$$b)\ q_{in} = \rho V' = \rho\left(\frac{4}{3}\pi r^3\right), \qquad r \prec a$$

$$\phi = \frac{q_{in}}{\varepsilon_0}$$

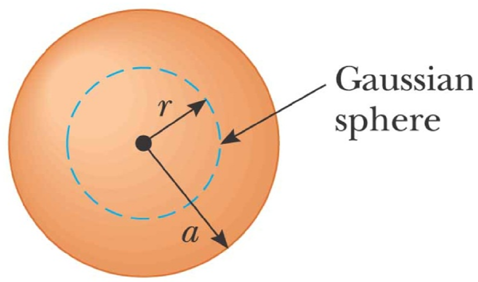
$$E = \frac{q_{in}}{4\pi\varepsilon_0 r^2} = \frac{\rho \frac{4}{3}\pi r^3}{4\pi\varepsilon_0 r^2} = \frac{\rho}{3\varepsilon_0} r$$

$$\rho = \frac{Q}{\frac{4}{3}\pi a^3}$$

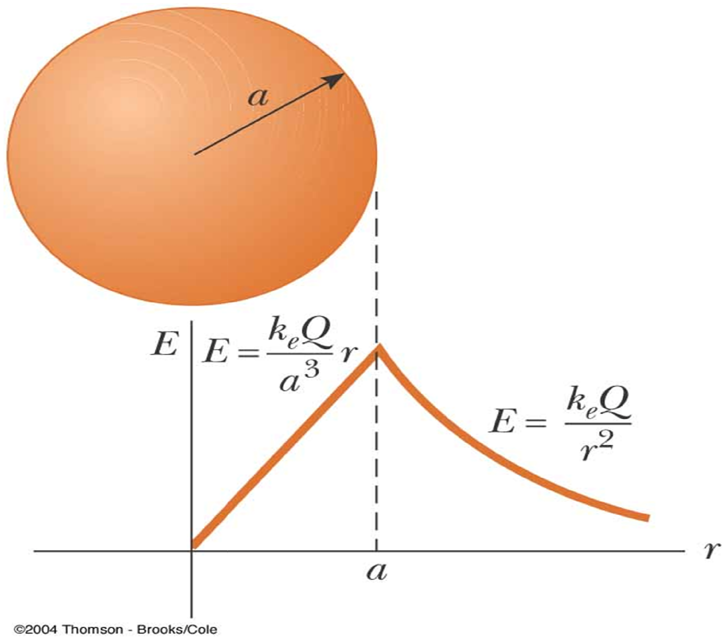
$$E = \frac{Q r}{4\pi\varepsilon_0 a^3} = k_e \frac{Q}{a^3} r \quad \Rightarrow E \to 0 \ as\ r \to o$$



(a)



(b)

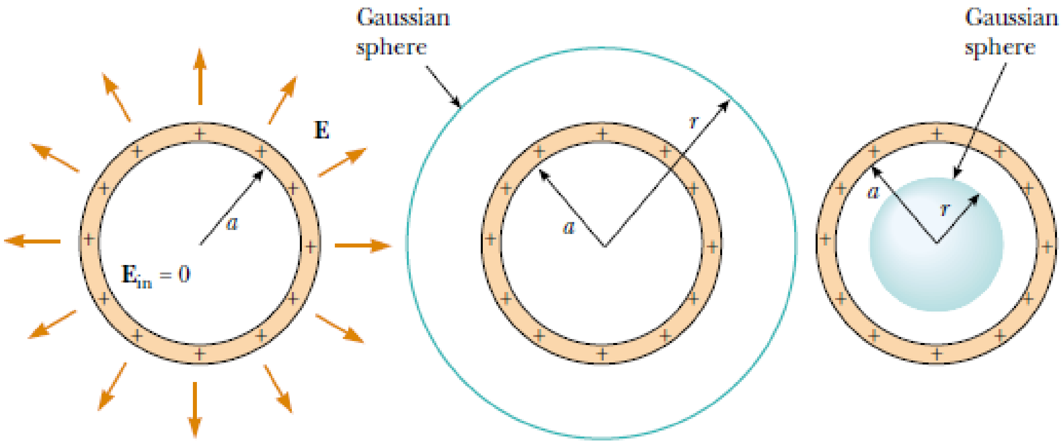


Comments:

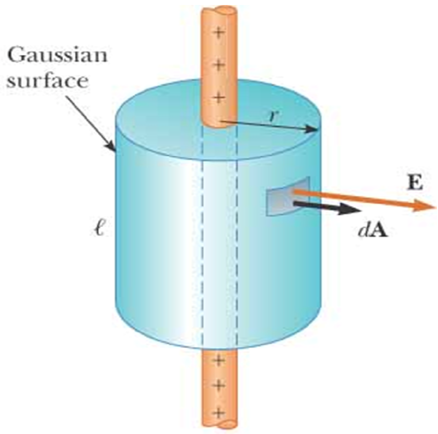
Exercise:

The Electric Field Due to a Thin Spherical Shell

A thin spherical shell of radius a has a total charge Q distributed uniformly over its surface (Fig. 24.13a). Find the electric field at points (A) outside and (B) inside the shell.



II- Electric Field of Line Charge



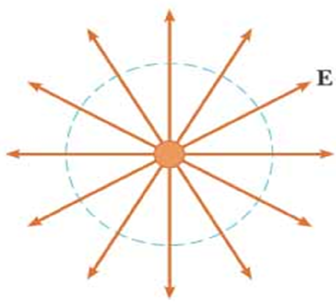
(a)

$$\lambda = Const.\ ,\quad Q=\lambda\ell,\quad dA \,\|\, E$$

$$E\oint dA = \frac{q_{in}}{\varepsilon_0}$$

$$E(2\pi r\ell) = \frac{\lambda\ell}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} = 2k_e\frac{\lambda}{r}$$



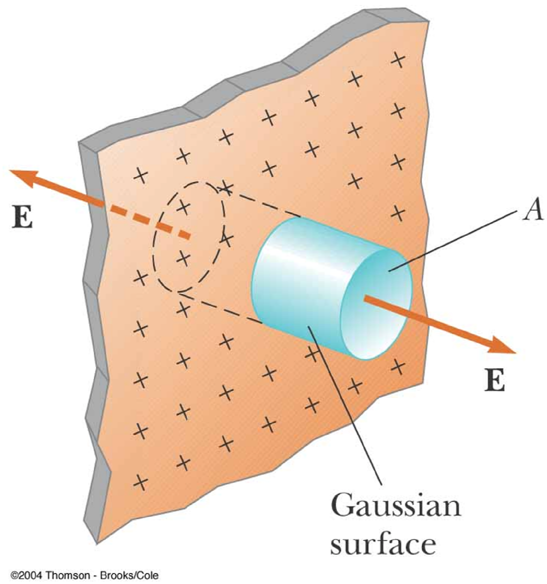
(b)

Comments:

Is it possible to use Gauss's law to determine an electric filed produced by charged distributed over a finite wire? Why?

III A plan of charge

The electric field due to a non-conducting, infinite plane of positive charge with uniform surface charge density σ.



$$\sigma = \frac{q_{in}}{A}$$

$$\phi_c = E \oint dA = \frac{q_{in}}{\varepsilon_0}$$

$$2EA = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

In the figure below, can you calculate the electric field in the three regions?

I II III