Chapter 12
Inventory Management

Outline

- ☑ Global Company Profile: Amazon.Com
- ☑ Functions Of Inventory
  - ☑ Types of Inventory
- ☑ Inventory Management
  - ☑ ABC Analysis
  - ☑ Record Accuracy
  - ☑ Cycle Counting
  - ☑ Control of Service Inventories

Outline – Continued

- ☑ ***Inventory Models***
  - ☑ ***Independent versus Dependent Demand***
  - ☑ ***Holding, Ordering, and Setup Costs***

Outline – Continued

- ☑ ***Inventory Models For Independent Demand***
  - ☑ ***Basic Economic Order Quantity (EOQ) Model***
  - ☑ ***Minimizing Costs***
  - ☑ ***Reorder Points***
  - ☑ ***Production Order Quantity Model***
  - ☑ ***Quantity Discount Models***

Outline – Continued

- ☑ *Probabilistic Models and Safety Stock*
  - ☑ *Other Probabilistic Models*
- ☑ *Fixed-Period (P) Systems*

Learning Objectives

When you complete this chapter, you should be able to:

Identify or Define:

- ☑ *ABC analysis*
- ☑ *Record accuracy*
- ☑ *Cycle counting*
- ☑ *Independent and dependent demand*
- ☑ *Holding, ordering, and setup costs*

Learning Objectives

When you complete this chapter, you should be able to:

Describe or Explain:

- ☑ *The functions of inventory and basic inventory models*

Amazon.com

- ☑ *Amazon.com started as a "virtual" retailer – no inventory, no warehouses, no overhead; just computers taking orders to be filled by others*
- ☑ *Growth has forced Amazon.com to become a world leader in warehousing and inventory management*

Amazon.com

1. Each order is assigned by computer to the closest distribution center that has the product(s)
2. A “flow meister” at each distribution center assigns work crews
3. Lights indicate products that are to be picked and the light is reset
4. Items are placed in crates on a conveyor. Bar code scanners scan each item 15 times to virtually eliminate errors.

Amazon.com

5. Crates arrive at central point where items are boxed and labeled with new bar code
6. Gift wrapping is done by hand at 30 packages per hour
7. Completed boxes are packed, taped, weighed and labeled before leaving warehouse in a truck
8. Order arrives at customer within a week

Inventory

- ☑ One of the most expensive assets of many companies representing as much as 50% of total invested capital
- ☑ Operations managers must balance inventory investment and customer service

Functions of Inventory

1. To decouple or separate various parts of the production process
2. To decouple the firm from fluctuations in demand and provide a stock of goods that will provide a selection for customers
3. To take advantage of quantity discounts
4. To hedge against inflation

Types of Inventory

- ☑ Raw material
  - ☑ Purchased but not processed
- ☑ Work-in-process
  - ☑ Undergone some change but not completed
  - ☑ A function of cycle time for a product
- ☑ Maintenance/repair/operating (MRO)
  - ☑ Necessary to keep machinery and processes productive
- ☑ Finished goods
  - ☑ Completed product awaiting shipment

The Material Flow Cycle

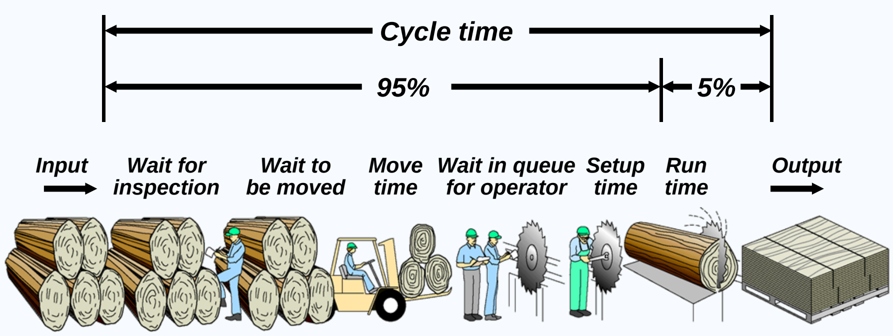


Figure 12.1

Inventory Management

- ☑ How inventory items can be classified
- ☑ How accurate inventory records can be maintained

ABC Analysis

- ☑ Divides inventory into three classes based on annual dollar volume
  - ☑ Class A - high annual dollar volume
  - ☑ Class B - medium annual dollar volume
  - ☑ Class C - low annual dollar volume
- ☑ Used to establish policies that focus on the few critical parts and not the many trivial ones

ABC Analysis

Item Stock Number	Percent of Number of Items Stocked	Annual Volume (units)	x	Unit Cost	=	Annual Dollar Volume	Percent of Annual Dollar Volume		Class
#10286	20%	1,000		$ 90.00		$ 90,000	38.8%	72%	A
#11526		500		154.00		77,000	33.2%		A
#12760		1,550		17.00		26,350	11.3%		B
#10867	30%	350		42.86		15,001	6.4%	23%	B
#10500		1,000		12.50		12,500	5.4%		B

ABC Analysis

Item Stock Number	Percent of Number of Items Stocked	Annual Volume (units)	x	Unit Cost	=	Annual Dollar Volume	Percent of Annual Dollar Volume		Class
#12572		600		$ 14.17		$ 8,502	3.7%		C
#14075		2,000		.60		1,200	.5%		C
#01036	50%	100		8.50		850	.4%	5%	C
#01307		1,200		.42		504	.2%		C
#10572		250		.60		150	.1%		C

ABC Analysis

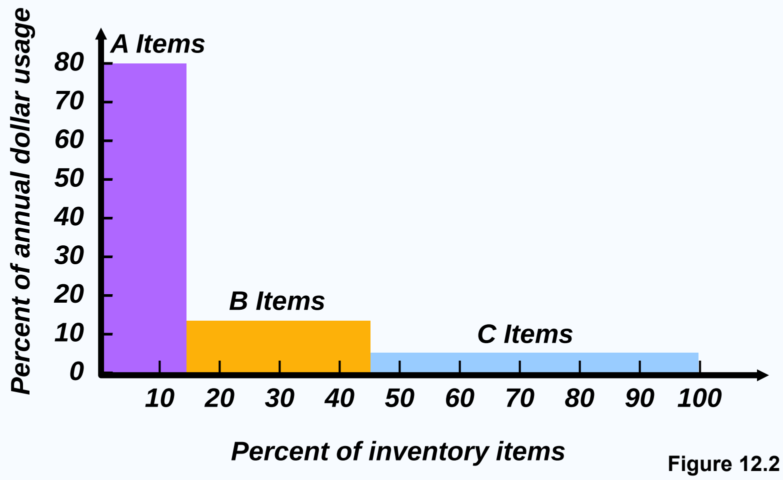


Figure 12.2

ABC Analysis

- ☑ *Other criteria than annual dollar volume may be used*
  - ☑ *Anticipated engineering changes*
  - ☑ *Delivery problems*
  - ☑ *Quality problems*
  - ☑ *High unit cost*

ABC Analysis

- ☑ *Policies employed may include*
  - ☑ *More emphasis on supplier development for A items*
  - ☑ *Tighter physical inventory control for A items*
  - ☑ *More care in forecasting A items*

Record Accuracy

- ☑ *Accurate records are a critical ingredient in production and inventory systems*
- ☑ *Allows organization to focus on what is needed*
- ☑ *Necessary to make precise decisions about ordering, scheduling, and shipping*
- ☑ *Incoming and outgoing record keeping must be accurate*
- ☑ *Stockrooms should be secure*

Cycle Counting

- ☑ Items are counted and records updated on a periodic basis
- ☑ Often used with ABC analysis to determine cycle
- ☑ Has several advantages
  - ☑ Eliminates shutdowns and interruptions
  - ☑ Eliminates annual inventory adjustment
  - ☑ Trained personnel audit inventory accuracy
  - ☑ Allows causes of errors to be identified and corrected
  - ☑ Maintains accurate inventory records

Cycle Counting Example

5,000 items in inventory, 500 A items, 1,750 B items, 2,750 C items

Policy is to count A items every month (20 working days), B items every quarter (60 days), and C items every six months (120 days)

Item Class	Quantity	Cycle Counting Policy	Number of Items Counted per Day
A	500	Each month	500/20 = 25/day
B	1,750	Each quarter	1,750/60 = 29/day
C	2,750	Every 6 months	2,750/120 = 23/day
			77/day

Control of Service Inventories

- ☑ Can be a critical component of profitability
- ☑ Losses may come from shrinkage or pilferage
- ☑ Applicable techniques include
  1. Good personnel selection, training, and discipline
  2. Tight control on incoming shipments
  3. Effective control on all goods leaving facility

Independent Versus Dependent Demand

- ☑ Independent demand - the demand for item is independent of the demand for any other item in inventory
- ☑ Dependent demand - the demand for item is dependent upon the demand for some other item in the inventory

Holding, Ordering, and Setup Costs

- ☑ *Holding costs - the costs of holding or "carrying" inventory over time*
- ☑ *Ordering costs - the costs of placing an order and receiving goods*
- ☑ *Setup costs - cost to prepare a machine or process for manufacturing an order*

Holding Costs

Category	Cost (and Range) as a Percent of Inventory Value
Housing costs (including rent or depreciation, operating costs, taxes, insurance)	6% (3 - 10%)
Material handling costs (equipment lease or depreciation, power, operating cost)	3% (1 - 3.5%)
Labor cost	3% (3 - 5%)
Investment costs (borrowing costs, taxes, and insurance on inventory)	11% (6 - 24%)
Pilferage, space, and obsolescence	3% (2 - 5%)
Overall carrying cost	26%

Table 12.1

Inventory Models for Independent Demand

Need to determine when and how much to order

- ☑ Basic economic order quantity
- ☑ Production order quantity
- ☑ Quantity discount model

Basic EOQ Model

Important assumptions

1. Demand is known, constant, and independent
2. Lead time is known and constant
3. Receipt of inventory is instantaneous and complete
4. Quantity discounts are not possible
5. Only variable costs are setup and holding
6. Stockouts can be completely avoided

Inventory Usage Over Time

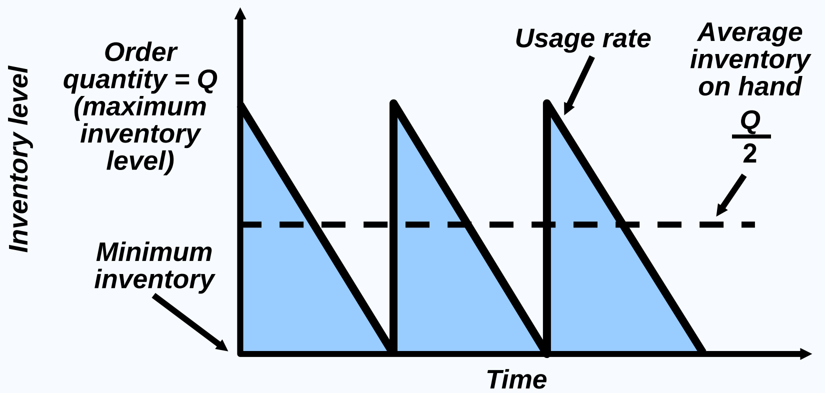


Figure 12.3

Minimizing Costs

Objective is to minimize total costs

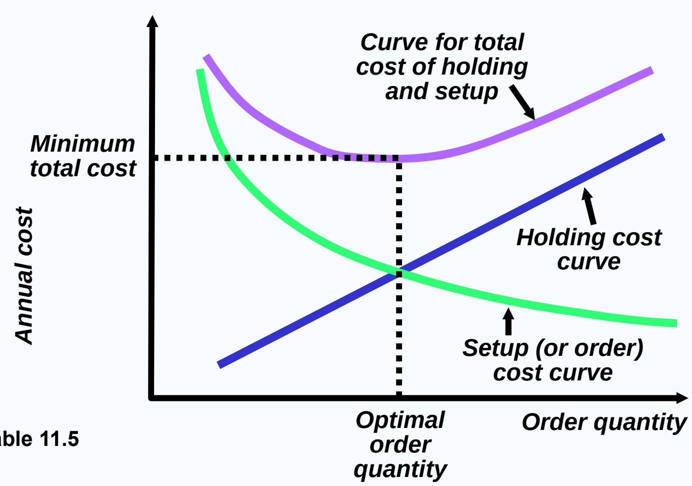


Table 11.5

The EOQ M

Annual setup cost = $\frac{D}{Q}S$

Q = Number of pieces per order
*Q** = Optimal number of pieces per order (EOQ)
D = Annual demand in units for the Inventory item
S = Setup or ordering cost for each order
H = Holding or carrying cost per unit per year

Annual setup cost = (*Number of orders placed per year*)
x (*Setup or order cost per order*)

$$= \left(\frac{\text{Annual demand}}{\text{Number of units in each order}}\right)\left(\text{Setup or order cost per order}\right)$$

$$= \left(\frac{D}{Q}\right)(S)$$

The EOQ M

Annual setup cost = $\frac{D}{Q}S$

Annual holding cost = $\frac{Q}{2}H$

Q = Number of pieces per order
*Q** = Optimal number of pieces per order (EOQ)
D = Annual demand in units for the Inventory item
S = Setup or ordering cost for each order
H = Holding or carrying cost per unit per year

Annual holding cost = (*Average inventory level*)
x (*Holding cost per unit per year*)

$$= \left(\frac{\text{Order quantity}}{2}\right)(\text{Holding cost per unit per year})$$

$$= \left(\frac{Q}{2}\right)(H)$$

The EOQ Model

Annual setup cost = $\frac{D}{Q}S$

Annual holding cost = $\frac{Q}{2}H$

Q = *Number of pieces per order*
*Q** = *Optimal number of pieces per order (EOQ)*
D = *Annual demand in units for the Inventory item*
S = *Setup or ordering cost for each order*
H = *Holding or carrying cost per unit per year*

Optimal order quantity is found when annual setup cost equals annual holding cost

$$\frac{D}{Q}S = \frac{Q}{2}H$$

*Solving for Q**

$$2DS = Q^2H$$

$$Q^2 = 2DS/H$$

$$Q^* = \sqrt{2DS/H}$$

An EOQ Example

Determine optimal number of needles to order
D = 1,000 *units*
S = $10 *per order*
H = $.50 *per unit per year*

$$Q^* = \sqrt{\frac{2DS}{H}}$$

$$Q^* = \sqrt{\frac{2(1{,}000)(10)}{0.50}} = \sqrt{40{,}000} = 200 \text{ units}$$

An EOQ Example

Determine optimal number of needles to order

D = 1,000 *units* Q^* = 200 *units*

S = $10 *per order*

H = $.50 *per unit per year*

$$\text{Expected number of orders} = N = \frac{\text{Demand}}{\text{Order quantity}} = \frac{D}{Q^*}$$

$$N = \frac{1{,}000}{200} = 5 \text{ orders per year}$$

An EOQ Example

Determine optimal number of needles to order

D = 1,000 *units* Q^* = 200 *units*

S = $10 *per order* N = 5 *orders per year*

H = $.50 *per unit per year*

$$\text{Expected time between orders} = T = \frac{\text{Number of working days per year}}{N}$$

$$T = \frac{250}{5} = 50 \text{ days between orders}$$

An EOQ Example

Determine optimal number of needles to order

D = 1,000 *units* — *Q** = 200 *units*

S = $10 *per order* — *N* = 5 *orders per year*

H = $.50 *per unit per year* — *T* = 50 *days*

Total annual cost = Setup cost + Holding cost

$$TC = \frac{D}{Q}S + \frac{Q}{2}H$$

$$TC = \frac{1{,}000}{200}(\$10) + \frac{200}{2}(\$.50)$$

$$TC = (5)(\$10) + (100)(\$.50) = \$50 + \$50 = \$100$$

Robust Model

- ☑ *The EOQ model is robust*
- ☑ *It works even if all parameters and assumptions are not met*
- ☑ *The total cost curve is relatively flat in the area of the EOQ*

An EOQ Example

Management underestimated demand by 50%

D = ~~1,000 units~~ 1,500 *units* *Q** = 200 *units*
S = $10 *per order* *N* = 5 *orders per year*
H = $.50 *per unit per year* *T* = 50 *days*

$$TC = \frac{D}{Q}S + \frac{Q}{2}H$$

$$TC = \frac{1{,}500}{200}(\$10) + \frac{200}{2}(\$.50) = \$75 + \$50 = \$125$$

Total annual cost increases by only 25%

An EOQ Example

Actual EOQ for new demand is 244.9 units

D = ~~1,000 units~~ 1,500 *units* *Q** = 244.9 *units*
S = $10 *per order* *N* = 5 *orders per year*
H = $.50 *per unit per year* *T* = 50 *days*

$$TC = \frac{D}{Q}S + \frac{Q}{2}H$$

$$TC = \frac{1{,}500}{244.9}(\$10) + \frac{244.9}{2}(\$.50)$$

$$TC = \$61.24 + \$61.24 = \$122.48$$

Only 2% less than the total cost of $125 when the order quantity was 200

Reorder Points

- ☑ EOQ answers the “how much” question
- ☑ The reorder point (ROP) tells when to order

$$ROP = \begin{pmatrix}\text{Demand}\\ \text{per day}\end{pmatrix}\begin{pmatrix}\text{Lead time for a}\\ \text{new order in days}\end{pmatrix}$$

$$= d \times L$$

$$d = \frac{D}{\text{Number of working days in a year}}$$

Reorder Point Curve

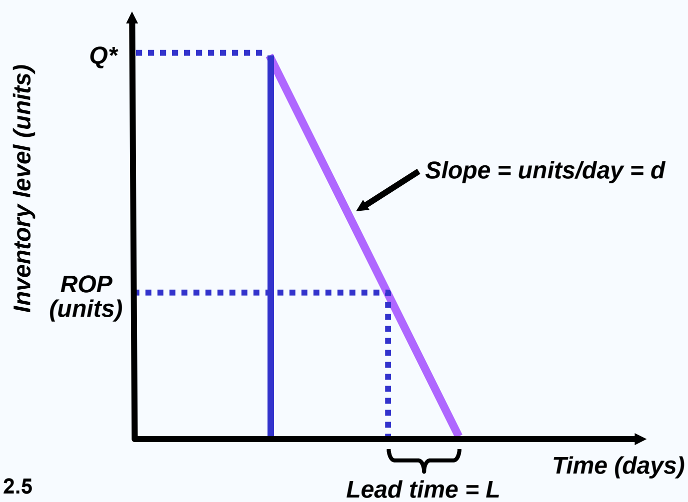


Figure 12.5

Reorder Point Example

Demand = 8,000 *DVDs per year*
250 *working day year*
Lead time for orders is 3 *working days*

$$d = \frac{D}{\text{Number of working days in a year}}$$

= 8,000/250 = 32 *units*

ROP = *d x L*

= 32 *units per day x* 3 *days* = 96 *units*

Production Order Quantity Model

- ☑ *Used when inventory builds up over a period of time after an order is placed*
- ☑ *Used when units are produced and sold simultaneously*

Production Order Quantity Model

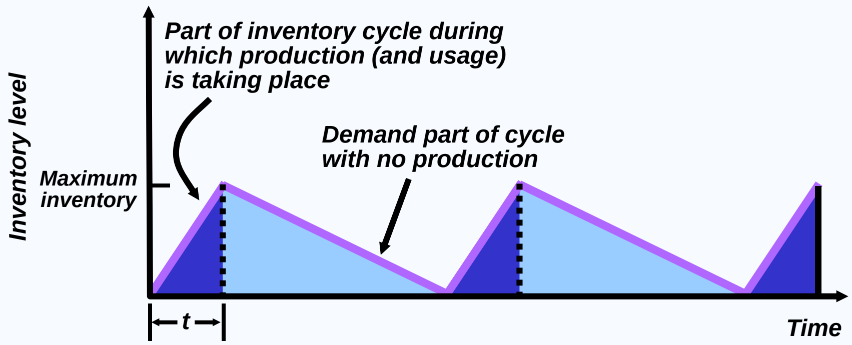


Figure 12.6

Production Order Quantity Model

Q = *Number of pieces per order* — *p* = *Daily production rate*
H = *Holding cost per unit per year* — *d* = *Daily demand/usage rate*
t = *Length of the production run in days*

$$\left(\text{Annual inventory holding cost}\right) = (\text{Average inventory level}) \times \left(\text{Holding cost per unit per year}\right)$$

$$\left(\text{Average inventory level}\right) = (\text{Maximum inventory level})/2$$

$$\left(\text{Maximum inventory level}\right) = \left(\text{Total produced during the production run}\right) - \left(\text{Total used during the production run}\right)$$

$$= pt - dt$$

Production Order Quantity Model

Q = Number of pieces per order
H = Holding cost per unit per year
t = Length of the production run in days
p = Daily production rate
d = Daily demand/usage rate

$$\begin{pmatrix}\text{Maximum}\\ \text{inventory level}\end{pmatrix} = \begin{pmatrix}\text{Total produced during}\\ \text{the production run}\end{pmatrix} - \begin{pmatrix}\text{Total used during}\\ \text{the production run}\end{pmatrix}$$

$$= pt - dt$$

However, Q = total produced = pt ; thus $t = Q/p$

$$\begin{pmatrix}\text{Maximum}\\ \text{inventory level}\end{pmatrix} = p\left(\frac{Q}{p}\right) - d\left(\frac{Q}{p}\right) = Q\left(1 - \frac{d}{p}\right)$$

$$\text{Holding cost} = \frac{\text{Maximum inventory level}}{2}(H) = \frac{Q}{2}\left[1 - \left(\frac{d}{p}\right)\right]H$$

Production Order Quantity Model

Q = Number of pieces per order
H = Holding cost per unit per year
D = Annual demand
p = Daily production rate
d = Daily demand/usage rate

$$\text{Setup cost} = (D/Q)S$$
$$\text{Holding cost} = 1/2\, HQ[1 - (d/p)]$$

$$(D/Q)S = 1/2\, HQ[1 - (d/p)]$$

$$Q^2 = \frac{2DS}{H[1 - (d/p)]}$$

$$Q^* = \sqrt{\frac{2DS}{H[1 - (d/p)]}}$$

Production Order Quantity Example

D = 1,000 *units*
S = $10
H = $0.50 *per unit per year*
p = 8 *units per day*
d = 4 *units per day*

$$Q^* = \sqrt{\frac{2DS}{H[1 - (d/p)]}}$$

$$Q^* = \sqrt{\frac{2(1{,}000)(10)}{0.50[1 - (4/8)]}} = \sqrt{80{,}000}$$

= 282.8 *or* 283 *hubcaps*

Production Order Quantity Model

When annual data are used the equation becomes

$$Q^* = \sqrt{\frac{2DS}{H\left(1 - \dfrac{\text{annual demand rate}}{\text{annual production rate}}\right)}}$$

Quantity Discount Models

- ☑ Reduced prices are often available when larger quantities are purchased
- ☑ Trade-off is between reduced product cost and increased holding cost

Total cost = Setup cost + Holding cost + Product cost

$$TC = \frac{D}{Q}S + \frac{QH}{2} + PD$$

Quantity Discount Models

A typical quantity discount schedule

Discount Number	Discount Quantity	Discount (%)	Discount Price (P)
1	0 to 999	no discount	$5.00
2	1,000 to 1,999	4	$4.80
3	2,000 and over	5	$4.75

Table 12.2

Quantity Discount Models

Steps in analyzing a quantity discount

1. For each discount, calculate Q*
2. If Q* for a discount doesn't qualify, choose the smallest possible order size to get the discount
3. Compute the total cost for each Q* or adjusted value from Step 2
4. Select the Q* that gives the lowest total cost

Quantity Discount Models

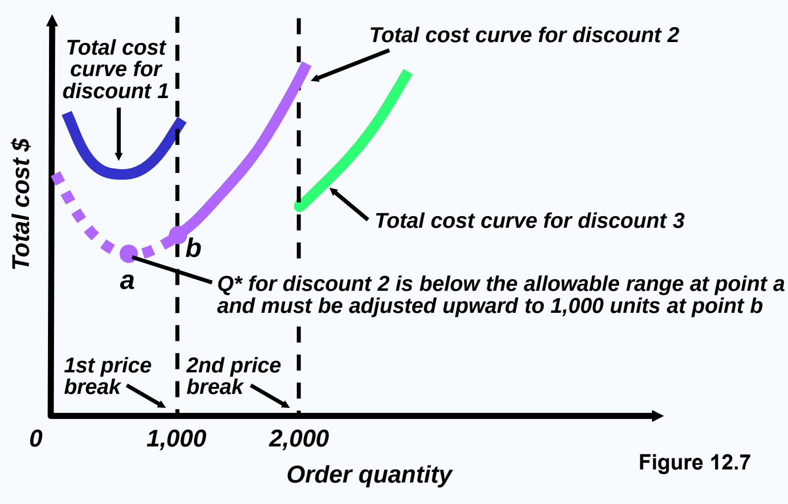


Figure 12.7

Quantity Discount Example

Calculate Q for every discount*

$$Q^* = \sqrt{\frac{2DS}{IP}}$$

$$Q_1^* = \sqrt{\frac{2(5{,}000)(49)}{(.2)(5.00)}} = 700 \text{ cars order}$$

$$Q_2^* = \sqrt{\frac{2(5{,}000)(49)}{(.2)(4.80)}} = 714 \text{ cars order}$$

$$Q_3^* = \sqrt{\frac{2(5{,}000)(49)}{(.2)(4.75)}} = 718 \text{ cars order}$$

Quantity Discount Example

Calculate Q for every discount*

$$Q^* = \sqrt{\frac{2DS}{IP}}$$

$$Q_1^* = \sqrt{\frac{2(5{,}000)(49)}{(.2)(5.00)}} = 700 \text{ cars order}$$

$$Q_2^* = \sqrt{\frac{2(5{,}000)(49)}{(.2)(4.80)}} = \sout{714} \text{ cars order}$$

1,000 — *adjusted*

$$Q_3^* = \sqrt{\frac{2(5{,}000)(49)}{(.2)(4.75)}} = \sout{718} \text{ cars order}$$

2,000 — *adjusted*

Quantity Discount Example

Discount Number	Unit Price	Order Quantity	Annual Product Cost	Annual Ordering Cost	Annual Holding Cost	Total
1	$5.00	700	$25,000	$350	$350	$25,700
2	$4.80	1,000	$24,000	$245	$480	$24,725
3	$4.75	2,000	$23.750	$122.50	$950	$24,822.50

Table 12.3

Choose the price and quantity that gives the lowest total cost

Buy 1,000 *units at* $4.80 *per unit*

Probabilistic Models and Safety Stock

- ☑ *Used when demand is not constant or certain*
- ☑ *Use safety stock to achieve a desired service level and avoid stockouts*

ROP = d x L + ss

Annual stockout costs = the sum of the units short x the probability x the stockout cost/unit x the number of orders per year

Safety Stock Example

ROP = 50 *units*
Orders per year = 6

Stockout cost = $40 *per frame*
Carrying cost = $5 *per frame per year*

	Number of Units	*Probability*
	30	.2
	40	.2
ROP →	50	.3
	60	.2
	70	.1
		1.0

Safety Stock Example

ROP = 50 *units*
Orders per year = 6

Stockout cost = $40 *per frame*
Carrying cost = $5 *per frame per year*

Safety Stock	*Additional Holding Cost*	*Stockout Cost*		*Total Cost*
20	(20)($5) = $100		$0	$100
10	(10)($5) = $50	(10)(.1)($40)(6)	= $240	$290
0	$0	(10)(.2)($40)(6) + (20)(.1)($40)(6)	= $960	$960

A safety stock of 20 *frames gives the lowest total cost*

ROP = 50 + 20 = 70 *frames*

Probabilistic Demand

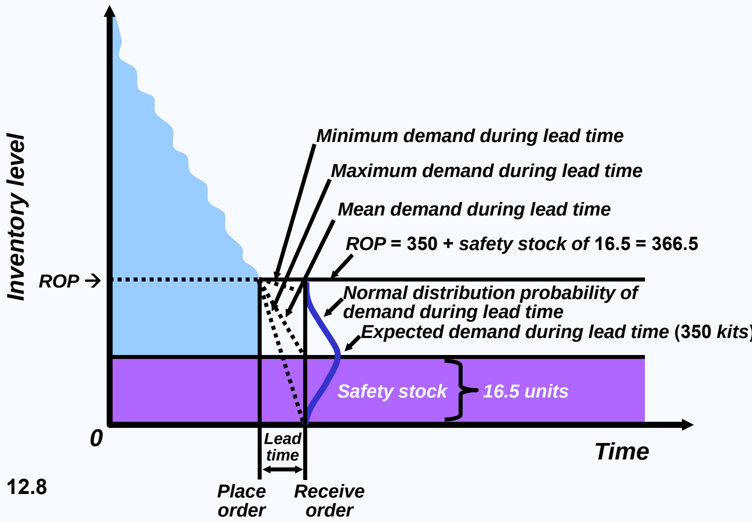
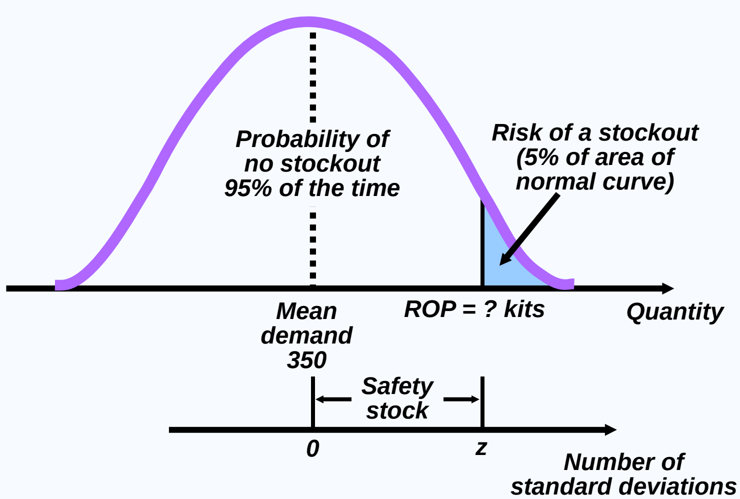


Figure 12.8

Probabilistic Demand



Probabilistic Demand

Use prescribed service levels to set safety stock when the cost of stockouts cannot be determined

$$ROP = \text{demand during lead time} + Z\sigma_{dlt}$$

where Z = *number of standard deviations*

σ_{dlt} = *standard deviation of demand during lead time*

Probabilistic Example

Average demand = μ = 350 *kits*
Standard deviation of demand during lead time = σ_{dlt} = 10 *kits*
5% *stockout policy* (*service level* = 95%)

Using Appendix I, for an area under the curve of 95%, *the* Z = 1.65

Safety stock = $Z\sigma_{dlt}$ = 1.65(10) = 16.5 *kits*

Reorder point = *expected demand during lead time + safety stock*
= 350 *kits* + 16.5 *kits of safety stock*
= 366.5 *or* 367 *kits*

Other Probabilistic Models

When data on demand during lead time is not available, there are other models available

1. *When demand is variable and lead time is constant*
2. *When lead time is variable and demand is constant*
3. *When both demand and lead time are variable*

Other Probabilistic Models

Demand is variable and lead time is constant

ROP = (average daily demand x lead time in days) + $Z\sigma_{dlt}$

where σ_d = standard deviation of demand per day

$\sigma_{dlt} = \sigma_d \sqrt{\text{lead time}}$

Probabilistic Example

Average daily demand (normally distributed) = 15
Standard deviation = 5
Lead time is constant at 2 *days*
90% *service level desired*

Z for 90% = 1.28
From Appendix I

$$ROP = (15 \text{ units} \times 2 \text{ days}) + Z\sigma_{dlt}$$
$$= 30 + 1.28(5)(\sqrt{2})$$
$$= 30 + 8.96 = 38.96 \approx 39$$

Safety stock is about 9 *units*

Other Probabilistic Models

Lead time is variable and demand is constant

$$ROP = (\text{daily demand} \times \text{average lead time in days}) + Z \times (\text{daily demand}) \times \sigma_{lt}$$

where σ_{lt} = *standard deviation of lead time in days*

Probabilistic Example

Z for 98% = 2.055
From Appendix I

Daily demand (*constant*) = 10
Average lead time = 6 *days*
Standard deviation of lead time = σ_{lt} = 3
98% *service level desired*

ROP = (10 *units x* 6 *days*) + 2.055(10 *units*)(3)
= 60 + 61.55 = 121.65

Reorder point is about 122 *units*

Other Probabilistic Models

Both demand and lead time are variable

ROP = (*average daily demand x average lead time*) + $Z\sigma_{dlt}$

where σ_d = *standard deviation of demand per day*

σ_{lt} = *standard deviation of lead time in days*

$\sigma_{dlt} = \sqrt{(\text{average lead time} \times \sigma_d^2) + (\text{average daily demand})^2\sigma_{lt}^2}$

Probabilistic Example

Average daily demand (*normally distributed*) = 150
Standard deviation = σ_d = 16
Average lead time 5 *days* (*normally distributed*)
Standard deviation = σ_{lt} = 1 *day*
95% *service level desired*

Z for 95% = 1.65
From Appendix I

ROP = (150 *packs* x 5 *days*) + $1.65\sigma_{dlt}$
= (150 x 5) + $1.65\sqrt{(5 \text{ days} \times 16^2) + (150^2 \times 1^2)}$
= 750 + 1.65(154) = 1,004 *packs*

Fixed-Period (P) Systems

- ☑ *Orders placed at the end of a fixed period*
- ☑ *Inventory counted only at end of period*
- ☑ *Order brings inventory up to target level*
  - ☑ *Only relevant costs are ordering and holding*
  - ☑ *Lead times are known and constant*
  - ☑ *Items are independent from one another*

Fixed-Period (P) Systems

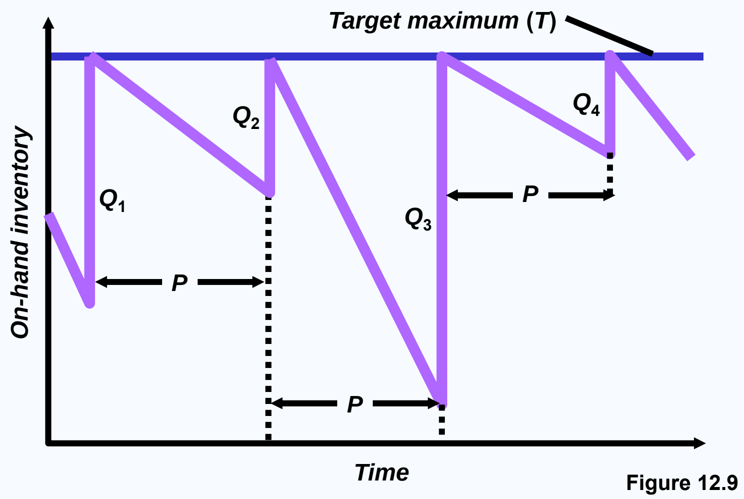


Figure 12.9

Fixed-Period (P) Example

3 jackets are back ordered
It is time to place an order

No jackets are in stock
Target value = 50

Order amount (Q) = Target (T) - On-hand inventory - Earlier orders not yet received + Back orders

$Q = 50 - 0 - 0 + 3 = 53$ *jackets*

Fixed-Period Systems

- ☑ *Inventory is only counted at each review period*
- ☑ *May be scheduled at convenient times*
- ☑ *Appropriate in routine situations*
- ☑ *May result in stockouts between periods*
- ☑ *May require increased safety stock*