King Saud University
College of Engineering
Department of Civil Engineering

FINAL EXAM

CE302 Mechanics of Materials – First Semester 1432-33 (2011-12)

Sunday, 14 Safar 1433 - 8 January 2012
Time allowed: 3 hours

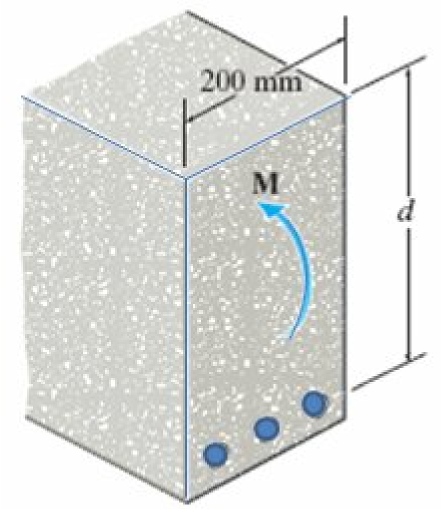
Student Name	
Student Number	
Section (put X please)	☐ 29484 (from 9:00 to 10:00 A.M.) ☐ 33488 (from 10:00 to 11:00 A.M.)
Name of Instructor	**Dr. Ahmet TUKEN**

Questions	Maximum Marks	Marks obtained
Q ≠ 1	6	
Q ≠ 2	6	
Q ≠ 3	9	
Q ≠ 4	9	
Q ≠ 5	10	
Q ≠ 6	10	
	Total marks	____ / 50

Total marks obtained (in words):____

Instructor's Signature

Question ≠ 1 (6 points):



The rectangular concrete beam is reinforced with three 20-mm diameter steel rods as shown. If the allowable compressive stress for concrete is $(\sigma_{all})_{concrete} = 12.5$ MPa and the allowable tensile stress for steel is $(\sigma_{all})_{steel} = 220$ MPa , determine the required dimension d so that both the concrete and the steel achieve their allowable stress simultaneously (i.e. at the same time). The modulus of elasticity for concrete and steel are $E_{concrete} = 25$ GPa and $E_{steel} = 200$ GPa, respectively.

$$n = \frac{E_s}{E_c} = \frac{200}{25} = 8$$

$$\sigma_c = \frac{Mx}{I} \quad and \quad \sigma_s = n.\frac{M(d-x)}{I}$$

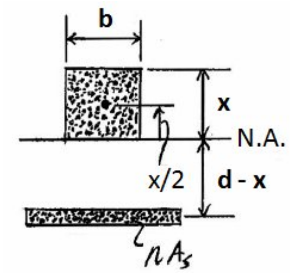
$$\frac{\sigma_s}{\sigma_c} = \frac{\frac{nM(d-x)}{I}}{\frac{Mx}{I}} = \frac{n(d-x)}{x} = n(\frac{d}{x} - 1)$$

$$\frac{d}{x} = \frac{\sigma_s}{n\sigma_c} + 1$$

Substituting the known values for σ_s, σ_c and n

$$\frac{d}{x} = \frac{\sigma_s}{n\sigma_c} + 1 = \frac{220}{8(12.5)} + 1 = 3.2$$

$$\therefore d = 3.2x$$



$$bx.\frac{x}{2} = nA_s(d-x)$$

$$A_s = \frac{bx^2}{2n(d-x)} \quad and \quad A_s = 3\left[\frac{\pi}{4}(20)^2\right] = 942.48mm^2$$

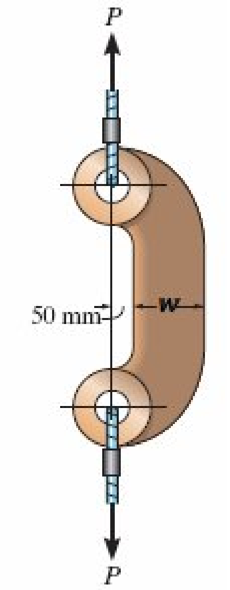
Substituting the known values for A_s, n, b and d=3.2x

$$942.48 = \frac{200x^2}{2(8)(3.2x - x)} = \frac{200x^2}{35.2x} = 5.682x$$

$$\therefore x = 165.87mm$$

$$Therefore \quad d = 3.2x = 3.2(165.87) = 531mm$$

Question ≠ 2 (6 points):



The offset link shown supports the loading of P = 30 kN.

Determine its required width w if the allowable normal stress is $\sigma_{all} = 73$ MPa. The link has a thickness of 40 mm.

σ due to axial force:

$$\sigma_a = \frac{P}{A} = \frac{30(10^3)}{(w)(0.04)} = \frac{750(10^3)}{w}$$

σ due to bending:

$$\sigma_b = \frac{Mc}{I} = \frac{30(10^3)(0.05 + \frac{w}{2})(\frac{w}{2})}{\frac{1}{12}(0.04)(w)^3}$$

$$= \frac{4500\,(10^3)(0.05 + \frac{w}{2})}{w^2}$$

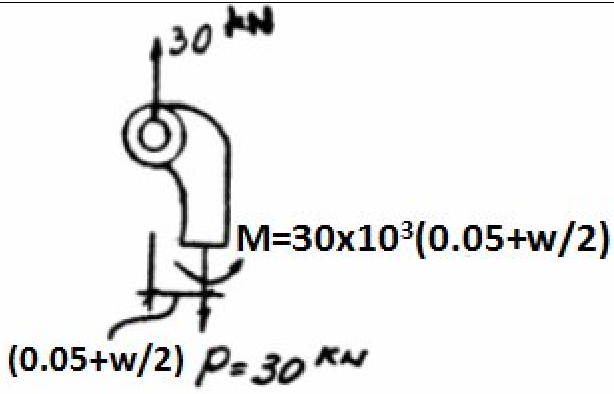
$$\sigma_{\max} = \sigma_{\text{allow}} = \sigma_a + \sigma_b$$

$$73(10^6) = \frac{750(10^3)}{w} + \frac{4500(10^3)(0.05 + \frac{w}{2})}{w^2}$$

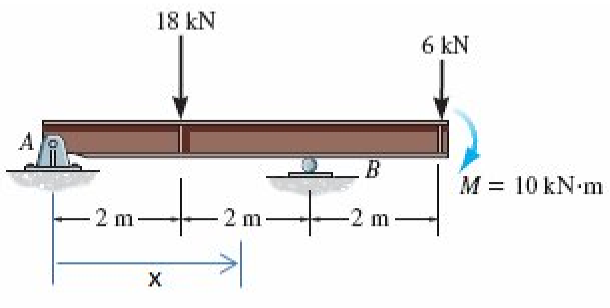
$$73\,w^2 = 0.75\,w + 0.225 + 2.25\,w$$

$$73\,w^2 - 3\,w - 0.225 = 0$$

$$w = 0.0797 \text{ m} = 79.7 \text{ mm}$$



Question ≠ 3 (9 points):



For the beam and loading shown;
a) determine the support reactions
b) draw the shear and bending moment diagram
c) determine the maximum absolute value of the shear and bending moment
d) determine shear and moment as a function of x for the region $2 < x < 4$

a) Support Reactions

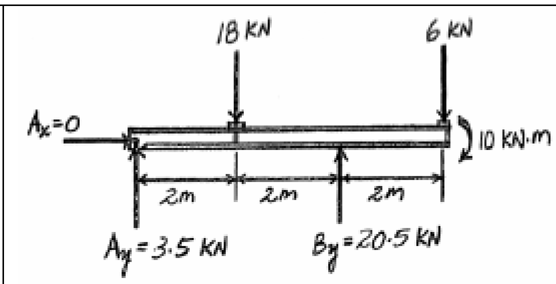
$\sum M_A = 0: \quad 18(2) + 6(6) + 10 - 4B_y = 0$

$\therefore B_y = 20.5N$

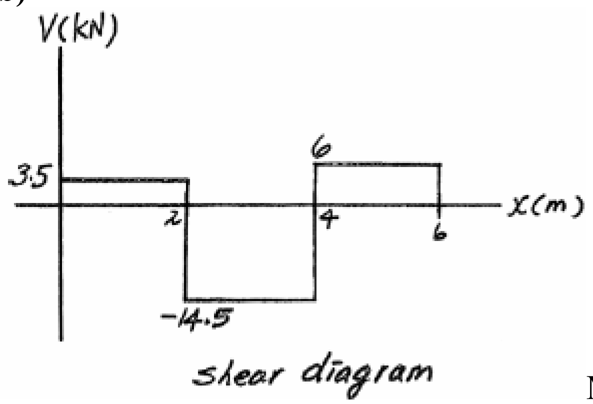
$\sum F_y = 0: \quad A_y + 20.5 - 18 - 6 = 0$

$\therefore A_y = 3.5N$

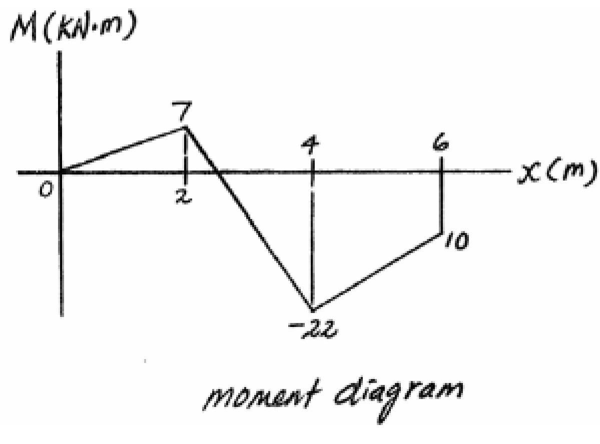
$\sum F_x = 0: \quad A_x = 0$



b)



Maximum absolute value of the shear is 14.5 kN



Maximum absolute value of bending moment is 22 kN.m

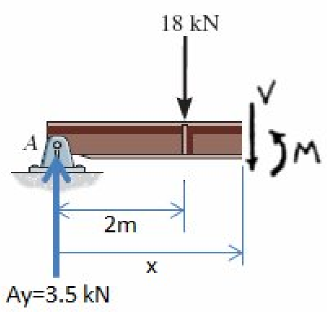
d) shear and moment as a function of x for $2 < x < 4$

$\sum M_{cut} = 0: \quad M + 18(x-2) - 3.5x = 0$

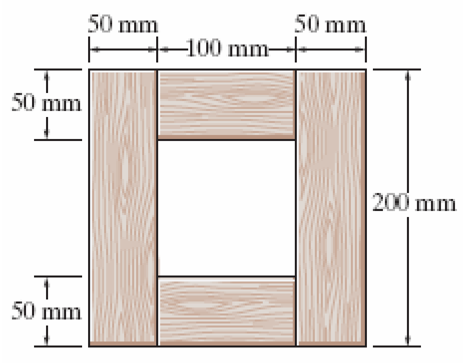
$\therefore M = -14.5x + 36$

$\sum F_y = 0: \quad 3.5 - 18 - V = 0$

$\therefore V = -14.5$



Question ≠ 4 (9 points):



The wood beam given has an allowable shear stress of τ_{all} = 7 MPa.

a) Determine the maximum shear force V that can be applied to the cross section shown.

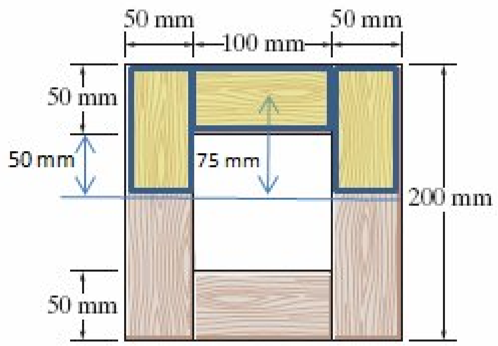
b) Plot the shear-stress variation over the cross section roughly.

$$I = \frac{1}{12}(0.2)(0.2)^3 - \frac{1}{12}(0.1)(0.1)^3 = 125(10^{-6})\ \text{m}^4$$

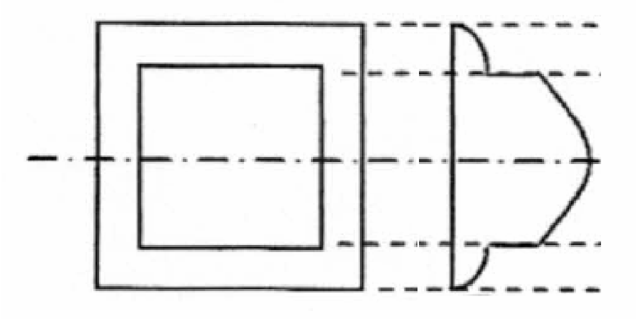
$$\tau_{\text{allow}} = \frac{VQ_{\max}}{It}$$

$$7(10^6) = \frac{V[(0.075)(0.1)(0.05) + 2(0.05)(0.1)(0.05)]}{125(10^{-6})(0.1)}$$

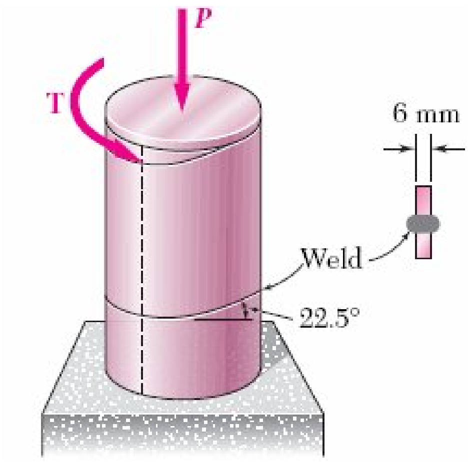
$$V = 100\ \text{kN}$$



The shear-stress variation caused by a vertical shear force over the cross section will be as below:



Question ≠ 5 (10 points):



A steel pipe of 300-mm outer diameter is fabricated from 6-mm-thick plate by welding along a helix which forms an angle of 22.50 with a plane perpendicular to the axis of the pipe. Knowing that a 160-kN axial force P and an 800 N.m torque T, each directed as shown, are applied to the pipe, determine σ and τ in directions respectively normal and tangential to the weld. (Hint: Firstly find the state of stress and then calculate the transformed stresses with $\Theta=22.5^0$).

$d_2 = 0.3\,m$, $c_2 = \frac{1}{2} d_2 = 0.15\,m$, $t = 0.006\,m$

$c_1 = c_2 - t = 0.144$

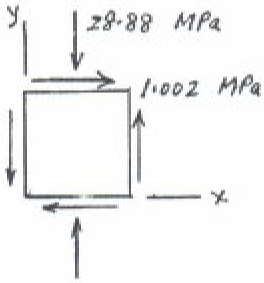
$A = \pi(c_2^2 - c_1^2) = \pi(0.15^2 - 0.144^2) = 5541.8 \times 10^{-6}\,m^2$

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.15^4 - 0.144^4) = 119.8 \times 10^{-6}\,m^4$

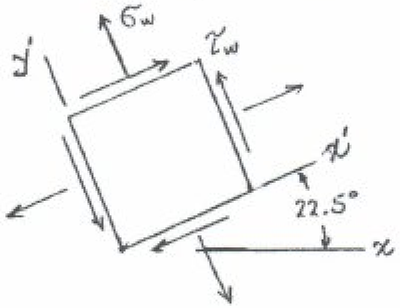
Stresses

$\sigma = -\frac{P}{A} = -\frac{160 \times 10^3}{5541 \times 10^{-6}} = -28.88\ MPa$

$\tau = \frac{Tc_2}{J} = \frac{(800)(0.15)}{119.8 \times 10^{-6}} = 1.002\ MPa$



$\sigma_x = 0$, $\sigma_y = -28.88\ MPa$, $\tau_{xy} = 1.002\ MPa$



Choose the x' and y' axes respectively tangential and normal to the weld.

Then, $\sigma_w = \sigma_{y'}$ and $\tau_w = \tau_{x'y'}$

$\theta = 22.5°$

$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$

$= \frac{(-28.88)}{2} - \frac{[-(-28.88)]}{2}\cos 45° - 1.002 \sin 45°$

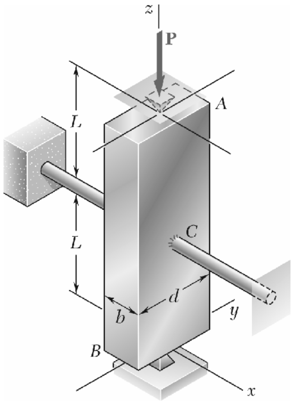
$= -25.36\ MPa$ $\sigma_w = -25.4\ MPa$

$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$

$= -\frac{[-(-28.88)]}{2}\sin 45° + 1.002 \cos 45°$

$= -9.5\ MPa$ $\tau_w = -9.5\ MPa$

Question ≠ 6 (10 points):

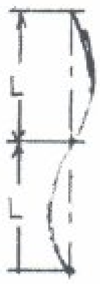


Column AB has a uniform rectangular cross section with b=12 mm and d=22 mm. The column is braced in the xz-plane at its midpoint C and carries a centric load P of magnitude 3.8 kN. Knowing that a factor of safety of 3.2 is required, determine the largest allowable length L. Use E=200 GPa. (Hint: Consider buckling in xz-plane and yz-plane separately).

$$P_{cr} = (F.S.)P = (3.2)(3.8 \times 10^3) = 12.16 \times 10^3 \text{ N}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} \qquad L_e = \pi\sqrt{\frac{EI}{P_{cr}}}$$

Buckling in xz-plane. $L = L_e = \pi\sqrt{\frac{EI}{P_{cr}}}$



$$I = \frac{1}{12} d b^3 = \frac{1}{12}(22)(12)^3 = 3.168 \times 10^3 \text{ mm}^4$$

$$= 3.168 \times 10^{-9} \text{ m}^4$$

$$L = \pi\sqrt{\frac{(200 \times 10^9)(3.168 \times 10^{-9})}{12.16 \times 10^3}} = 0.717 \text{ m}$$

Buckling in yz-plane. $L_e = 2L \qquad L = \frac{L_e}{2} = \frac{\pi}{2}\sqrt{\frac{EI}{P_{cr}}}$

$$I = \frac{1}{12} b d^3 = \frac{1}{12}(12)(22)^3 = 10.648 \times 10^3 \text{ mm}^4 = 10.648 \times 10^{-9} \text{ m}^4$$

$$L = \frac{\pi}{2}\sqrt{\frac{(200 \times 10^9)(10.648 \times 10^{-9})}{12.16 \times 10^3}} = 0.657 \text{ m}$$

The smaller length governs. $L = 0.657 \text{ m} = 657 \text{ mm}$