HEATING THE METAL

$$H = \rho V\{C_s(T_m - T_o) + H_f + C_l(T_p - T_m)\}$$

where H = total heat required to raise the temperature of the metal to the pouring temperature, J (Btu); ρ = density; g/cm3 ; Cs= weight specific heat for the solid metal, J/g; Tm = melting temperature of the metal, ^oC; To = starting temperature—usually ambient, ^oC; Hf = heat of fusion, J/g; Cl=weight specific heat of the liquid metal, J/g ^oC; Tp= pouring temperature, ^oC; and V = volume of metal being heated, cm3.

Example: One cubic meter of a certain eutectic alloy is heated in a crucible from room temperature to 100^oC above its melting point for casting. The alloy's density = 7.5 g/cm3, melting point = 800^0C, specific heat=0.33 J/g^oC in the solid state and 0.29 J/g^oC in the liquid state; and heat of fusion = 160 J/g. How much heat energy must be added to accomplish the heating, assuming no losses?
Solution: We assume ambient temperature in the foundry=25 ^0C and that the density of the liquid and solid states of the metal are the same. Noting that one $m^3 = 10^6\ cm^3$, and substituting the property values into Eq. (10.1), we have

H = (7.5). (10^6).{0.33.(800- 25) + 160 + 0.29.(100)} = 3335 .(10^6) J.

The above equation is of conceptual value, but its computational value is limited, notwithstanding our example calculation. Use of Eq. (10.1) is complicated by the following factors: (1) Specific heat and other thermal properties of a solid metal vary with temperature, especially if the metal undergoes a change of phase during heating. (2) A metal's specific heat may be different in the solid and liquid states. (3) Most casting metals are alloys, and most alloys melt over a temperature range between a solidus and liquidus rather than at a single melting point; thus, the heat of fusion cannot be applied so simply as indicated above. (4) The property values required in the equation for a particular alloy are not readily available in most cases. (5) There are significant heat losses to the environment during heating.

ANSWER THE FOLLOWING

1. A disk 40 cm in diameter and 5 cm thick is to be cast of pure aluminum in an open-mold casting operation. The melting temperature of aluminum = 660^oC, and the pouring temperature will be 800^oC. Assume that the amount of aluminum heated will be 5% more than what is needed to fill the mold cavity. Compute the amount of heat that must be added to the metal to heat it to the pouring temperature, starting from a room temperature of 25^oC. The heat of fusion of aluminum = 389.3 J/g. Other properties can be obtained from Tables 4.1 and 4.2 in the text. Assume the specific heat has the same value for solid and molten aluminum.

2. A sufficient amount of pure copper is to be heated for casting a large plate in an open mold. The plate has dimensions: length = 20 in, width = 10 in, and thickness = 3 in. Compute the amount of heat that must be added to the metal to heat it to a temperature of

2150^oF for pouring. Assume that the amount of metal heated will be 10% more than what is needed to fill the mold cavity. Properties of the metal are: density =0.324 lbm/in^3, melting point = 1981^oF, specific heat of the metal = 0.093 Btu/lbm^oF in the solid state and 0.090 Btu/ lbm ^oF in the liquid state, and heat of fusion = 80 Btu/lbm.

3. The downsprue leading into the runner of a certain mold has a length = 175 mm. The cross-sectional area at the base of the sprue is 400mm^2. The mold cavity has a volume = 0.001m^3. Determine (a) the velocity of the molten metal flowing through the base of the downsprue, (b) the volume rate of flow, and (c) the time required to fill the mold cavity.

4. A mold has a downsprue of length = 6.0 in. The cross-sectional area at the bottom of the sprue is 0.5 in^2. The sprue leads into a horizontal runner which feeds the mold cavity, whose volume = 75 in^3. Determine (a) the velocity of the molten metal flowing through the base of the downsprue, (b) the volume rate of flow, and (c) the time required to fill the mold cavity.

5. The flow rate of liquid metal into the downsprue of a mold = 1 L/s. The cross-sectional area at the top of the sprue = 800 mm2, and its length = 175 mm. What area should be used at the base of the sprue to avoid aspiration of the molten metal?

6. The volume rate of flow of molten metal into the downsprue from the pouring cup is 50 in^3/sec. At the top where the pouring cup leads into the downsprue, the cross-sectional area =1.0 in^2.Determine what the area should be at the bottom of the sprue if its length = 8.0 in. It is desired to maintain a constant flow rate, top and bottom, in order to avoid aspiration of the liquid metal.
7. Molten metal can be poured into the pouring cup of a sand mold at a steady rate of 1000 cm^3/s. The molten metal overflows the pouring cup and flows into the downsprue. The cross-section of the sprue is round, with a diameter at the top = 3.4 cm. If the sprue is 25cm long, determine the proper diameter at its base so as to maintain the same volume flow rate.
8. During pouring into a sand mold, the molten metal can be poured into the downsprue at a constant flow rate during the time it takes to fill the mold. At the end of pouring the sprue is filled and there is negligible metal in the pouring cup. The downsprue is 6.0 in long. Its cross-sectional area at the top =0.8 in2 and at the base =0.6 in^2.The cross-sectional area of the runner leading from the sprue also = 0.6 in^2, and it is 8.0 in long before leading into the mold cavity, whose volume =65 in^3. The volume of the riser located along the runner near the mold cavity = 25 in^3. It takes a total of 3.0 sec to fill the entire mold (including cavity, riser, runner, and sprue. This is more than the theoretical time required, indicating a loss of velocity due to friction in the sprue and runner. Find (a) the theoretical velocity and flow rate at the base of the downsprue; (b) the total volume of the mold; (c) the actual velocity and flow rate at the base of the sprue; and (d) the loss of head in the gating system due to friction.

ENGINEERING ANALYSIS OF POURING

$$h + \frac{p_1}{\rho} + \frac{v_1^2}{2g} + F1 = h + \frac{p_2}{\rho} + \frac{v_2^2}{2g} + F2 = C$$

where h=head, cm , p= pressure on the liquid, N/cm^2 ; ρ = density; g/cm^3 , v = flow velocity; cm/s; g = gravitational acceleration constant= 981 cm/s/s; and F= head losses due to friction, cm. Subscripts 1 and 2 indicate any two locations in the liquid flow. Bernoulli's equation can be simplified in several ways. If **we ignore friction losses** (to be sure, friction will affect the liquid flow through a sand mold), and assume that the system remains at atmospheric pressure throughout, then the equation can be reduced to;

$$h_1 + \frac{v_1^2}{2g} = h_2 + \frac{v_2^2}{2g} = C$$

This can be used to determine the velocity of the molten metal at the base of the sprue. Let us define point 1 at the top of the sprue and point 2 at its base. If point 2 is used as the reference plane, then the head at that point is zero (h2= 0) and h1 is the height (length) of the sprue. When the metal is poured into the pouring cup and overflows down the sprue, its initial velocity at the top is zero (v1 = 0). Hence, further simplifies to;

$$h_1 = \frac{v_2^2}{2g}, \qquad v = \sqrt{2gh}$$

The continuity law $Q = A_1 v_1 = A_2 v_2$

$$\frac{A_1}{A_2} = \sqrt{\frac{h_2}{h_1}}; \quad R_e = \frac{vD\rho}{\eta}$$

Assuming that the runner from the sprue base to the mold cavity is horizontal (and therefore the head **h** is the same as at the sprue base), then the volume rate of flow through the gate and into the mold cavity remains equal to ***Av*** at the base. Accordingly, we can estimate the time T_{MF} required to fill a mold cavity of volume **V** as;

T_{MF} = V/Q

Where **T_{MF}=** mold filling time, s (sec); **V=** volume of mold cavity, cm3; and **Q=** volume flow rate, as before. The mold filling time must be considered a minimum time. This is because the analysis ignores friction losses and possible constriction of flow in the gating system; thus, the mold filling time will however be longer.

Ex. A mold sprue is 20 cm long, and the cross-sectional area at its base is 2.5 cm^2. The sprue feeds a horizontal runner leading into a mold cavity whose volume is 1560 cm^3. Determine:
(a) velocity of the molten metal at the base of the sprue,
(b) volume rate of flow, and (c) time to fill the mold.

Solution: (a) The velocity of the flowing metal at the base of the sprue is given by Eq.

$$\boldsymbol{v = \sqrt{2gh} = \sqrt{2(981)(20)} = 1981.1\ cm/s}$$

(b) The volumetric flow rate is;

$\boldsymbol{Q = A\,v = (2{:}5\ cm^2)\ (198{:}1\ cm/s) = 495\ cm^2/s}$

(c) Time required to fill a mold cavity of 1560 cm^3 at this flow rate is

$\boldsymbol{T_{MF} = V/Q = 1560/495 = 3.2s}$

RISER DESIGN

EX. A cylindrical riser must be designed for a sand-casting mold. The casting itself is a steel rectangular plate with dimensions 7.5 cm x 12.5 cm x 2.0 cm. Previous observations have indicated that the total solidification time (T_{TS}) for this casting = 1.6 min. The cylinder for the riser will have a diameter-to-height ratio=1.0. Determine the dimensions of the riser so that its T_{TS} = 2.0 min.

Solution: First determine the V/A ratio for the plate. Its volume

$$V = 7.5 \times 12.5 \times 2.0 = 187.5 \text{ cm}^3$$

, and its surface area;

$$A = 2 (7.5 \times 12.5 + 7.5 \times 2.0 + 12.5 \times 2) = 267.5 \text{ cm2}.$$

Given that

$$T_{TS} = 1.6 \text{ min},$$

we can determine the mold constant C_m from using a value of $n = 2$ in the equation.

$$C_m = T_{TS} / (V/A)^2 = 1.6/ ((187.5/267.5)^2 = 3.26 \text{ min/cm}^2$$

Next we must design the riser so that its total solidification time is 2.0 min, using the same value of mold constant. The volume of the riser is given by ;

$$V = \pi D^2 h /4$$

and the surface area is given by

$$A = \pi D h + (2\pi D^2/4)$$

Since we are using a D/H ratio = 1.0, then D=H. Substituting D for H in the volume and area formulas, we get;

$$V = \pi D^3/4 \text{ and}$$

$$A = \pi D^2 + 2\pi D^2/4 = 1.5\pi D^2$$

Thus the V/A ratio = D/6. Using this ratio in Chvorinov's equation, we have

$$T_{TS} = 2.0 = 3.26(D/6)^2 = 0.09056\, D^2 \text{ ;}$$

D^2 **= 2.0/0.09056 =22.086 cm^2**
D = 4.7 cm

Since

H = D;

then

H = 4.7 cm also.

ANSWER THE FOLLOWING

1. In the casting of steel under certain mold conditions, the mold constant in Chvorinov's rule is known to be 4.0 min/cm2 , based on previous experience. 'The casting is a flat plate whose length = 30 cm, width = 10em, and thickness = 20 mm. Determine how long it will take for the casting to solidify.
2. Solve for total solidification time in the previous problem only using an exponent value of 1.9 in Chvorinov's rule instead of 2.0. What adjustment must be made in the units of the mold constant?
3. A disk-shaped part is to be cast out of aluminum. The diameter of the disk = 500 mm and its thickness = 20 rom. If the mold constant = 2.0 sec/mm2 in Chvorinov's rule, how long will it take the casting to solidify?
4. In casting experiments performed using a certain alloy and type of sand mold, it took 155 sec for a cube shaped casting to solidify. The cube was 50 mm on a side.
   (a) Determine the value of the mold constant the mold constant in Chvorinov's rule.
   (b) If the same alloy and mold type were used, find the total solidification time for a cylindrical casting in which the diameter = 30 mm and length =50 mm.
5. A steel casting has a cylindrical geometry with 4.0 in diameter and weighs 20 lb. This casting takes 6.0 min to completely solidify. Another cylindrical-shaped casting with the same diameter-to-length ratio weighs 12 lb. This casting is made of the same steel, and the same conditions of mold and pouring were used. Determine:
   (a) the mold constant in Chvorinov's rule,
   (b) the dimensions, and (c) the total solidification time of the lighter casting. The density of steel = 490 lbf/ft3.
6. The total solidification times of three casting shapes are to be compared:
   (1) a sphere with diameter= 10 cm,
   (2) a cylinder with diameter and length both = 10 cm, and
   (3) a cube with each side = 10 cm.
   The same casting alloy is used in the three cases.
   (a) Determine the relative solidification times for each geometry.
   (b) Based on the results of part (a), which geometric element would make the best riser?
   (c) If the mold constant = 3.5 min/cm2 in Chvorinov's rule, compute the total solidification time for each casting.

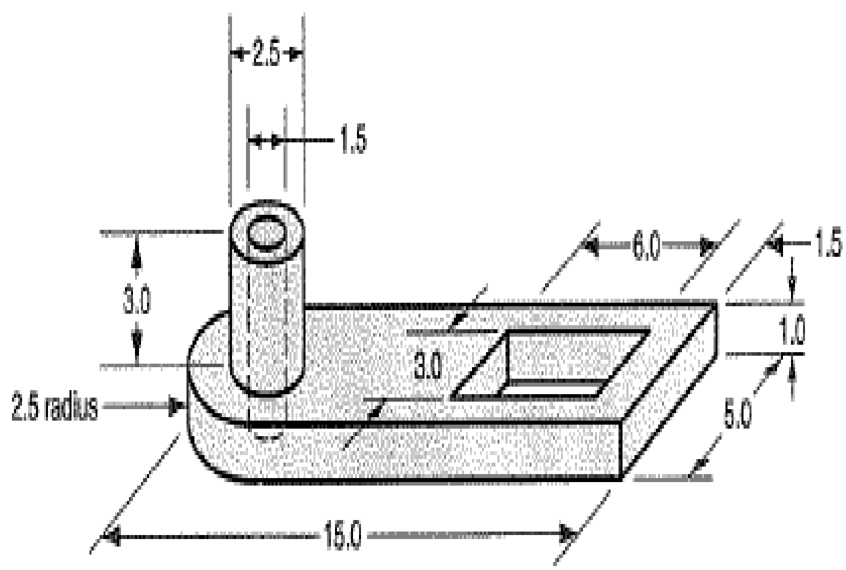
7. The total solidification times of three casting shapes are to be compared:
   (1) a sphere,
   (2) a cylinder, in which the length-to-diameter ratio = 1.0, and
   (3) a cube.

For all three geometries, the volume= 1000 cm3. The same casting alloy is used in the three cases.

   (a) Determine the relative solidification times for each geometry.
   (b) Based on the results of part (a), which geometric element would make the best riser?
   (c) If the mold constant = 3.5 min/cm2 in Chvorinov's rule, compute the total solidification time for each casting.

8. A cylindrical riser is to be used for a sand-casting mold. For a given cylinder volume, determine the diameter to-length ratio that will maximize the time to solidify.
9. A riser in the shape of a sphere is to be designed for a sand casting mold. The casting is a rectangular plate, with length = 200 mm, width= 100 mm, and thickness = 18 mm. If the total solidification time of the casting itself is known to be 3.5 min, determine the diameter of the riser so that it will take 25% longer for the riser to solidify.
10. A cylindrical riser is to be designed for a sand casting mold. The length of the cylinder is to be 1.25 times its diameter. The casting is a square plate, each side = 10 in. and thickness = 0.75 in. If the metal is cast iron, and the mold constant= 16.0 min/in2 in Chvorinov's rule, determine the dimensions of the riser so that it will take 30% longer for the riser to solidify.
11. A cylindrical riser with diameter-to-length ratio = 1.0 is to be designed for a sand casting mold. The casting geometry is illustrated in Figure P10.25, in which the units are inches. If the mold constant= 19.5 min/in2 in Chvorinov's rule, determine the dimensions of the riser so that the riser will take 0.5 min longer to freeze than the casting itself.

FIGURE P10.25 Casting geometry for Problem 10.25 (units are in Inches).



BUOYANCY FORCE:

IF the force tending to lift the core is equal to the weight of the displaced liquid less the weight of the core itself. Expressing the situation in equation form,

$$F_b = W_m - W_c ,$$

where F_b=buoyancy force ,
N(lb); W_m=weight of molten metal displaced,N(lb); and
W_c = weight of the core, N (lb).
Weights are determined as the volume of the core multiplied by the respective densities of the core material (typically sand) and the metal being cast. The density of a sand core is approximately 1.6 g/cm3 (0.058 lb/in3). Densities of several common casting alloys are given in the table below:

TABLE 11.1 Densities of selected casting alloys.

Metal	Density g/cm^3	Density lb/in^3	Metal	Density g/cm^3	Density lb/in^3
Aluminum (99% = pure)	2.70	0.098	Cast iron, gray[a]	7.16	0.260
Aluminum-silicon alloy	2.65	0.096	Copper (99% = pure)	8.73	0.317
Aluminum-copper (92% Al)	2.81	0.102	Lead (pure)	11.30	0.410
Brass[a]	8.62	0.313	Steel	7.82	0.284

Source: [7].
[a]Density depends on composition of alloy; value given is typical.

ANSWER THE FOLLOWING:

1. A 92% aluminum- 8% copper alloy casting is made in a sand mold using a sand core that weighs 20 kg. Determine the buoyancy force in Newtons tending to lift the core during pouring.
2. A sand core located inside a mold cavity has a volume of 157.0 in3. It is used in the casting of cast iron pump housing. Determine the buoyancy force that will tend to lift the core during pouring.
3. Caplets are used to support a sand core inside a sand mold cavity. The design of the caplets and the manner in which they are placed in the mold cavity surface allows each caplet to sustain a force of 10 lb. Several caplets are located beneath the core to support it before pouring; and several other caplets are placed above the core to resist the buoyancy force during pouring. If the volume of the core = 325 in3, and the metal poured is brass, determine the minimum number of caplets that should be placed
   (a) beneath the core, and
   (b) above the core.

4. A sand core used to form the internal surfaces of a steel casting experiences a buoyancy force of 23 kg. The volume of the mold cavity forming the outside surface of the casting = 5000 cm3. What is the weight of the final casting? Ignore considerations of shrinkage.

TRUE CENTRIFUGAL CASTING

Let us consider how fast the mold must rotate in horizontal centrifugal casting for the process to work successfully. Centrifugal force is defined by this physics equation:

$$\mathbf{F = m.v^2/R}$$

where F = force, N (lb); m=mass, kg (lbm); v = velocity, m/s (ft/sec); and R = inside radius of the mold, m (ft).
The force of gravity is its weight W= m g, where W is given in kg (lb), and g = acceleration of gravity, 9.8 m/s^2 (32.2 ft/sec2). The so-called **G-factor GF** is the ratio of centrifugal force divided by weight:

$$\mathbf{GF = m.v^2/(R.mg) = v^2/(R.g)}$$

Velocity v can be expressed as

$$\mathbf{V=2pRN/60 = pRN/30,}$$

where N = rotational speed, rev/min. Substituting this expression we obtain;

$$\mathbf{GF= R(\pi N/30)^2 /g}$$

Rearranging this to solve for rotational **speed N**, and using diameter D rather than radius in the resulting equation, we have

$$\boldsymbol{N = \frac{30}{\pi}\sqrt{\frac{2gGF}{D}}}$$

where D = inside diameter of the mold, m (ft).
Note:

- If the G-factor is too low in centrifugal casting, the liquid metal will not remain forced against the mold wall during the upper half of the circular path but will ‘‘rain’’ inside the cavity.
- Slipping occurs between the molten metal and the mold wall, which means that the rotational speed of the metal is less than that of the mold.
- On an empirical basis, values of GF = 60 to 80 are found to be appropriate for horizontal centrifugal casting , although this depends to some extent on the metal being cast.

ANSWER THE FOLLOWING:

1. A horizontal true centrifugal casting operation will be used to make copper tubing. The lengths will be 1.5 m with outside diameter = 15.0 cm, and inside diameter = 12.5 cm. If the rotational speed of the pipe = 1000 rev/min, determine the Gfactor.

2. A true centrifugal casting operation is to be performed in a horizontal configuration to make cast iron pipe sections. The sections will have a length = 42.0 in, outside diameter = 8.0 in, and wall thickness = 0.50 in. If the rotational speed of the pipe = 500 rev/min, determine the G-factor. Is the operation likely to be successful?

3. A horizontal true centrifugal casting process is used to make brass bushings with the following dimensions: length = 10 cm, outside diameter = 15 cm, and inside diameter = 12 cm. (a) Determine the required rotational speed in order to obtain a G-factor of 70. (b) When operating at this speed, what is the centrifugal force per square meter (Pa) imposed by the molten metal on the inside wall of the mold?

4. True centrifugal casting is performed horizontally to make large diameter copper tube sections. The tubes have a length =1.0 m, diameter =0.25 m, and wall thickness = 15 mm. If the rotational speed of the pipe = 700 rev/min, determine the G-factor on the molten metal. Is the rotational speed sufficient to avoid ‘‘rain?’’ What volume of molten metal must be poured into the mold to make the casting if solidification shrinkage and contraction after solidification are considered? Solidification shrinkage for copper = 4.5%, and solid thermal contraction = 7.5%. If a true centrifugal casting operation were to be performed in a space station circling the Earth, how would weightlessness affect the process?

5. A horizontal true centrifugal casting process is used to make aluminum rings with the following dimensions: length = 5 cm, outside diameter = 65 cm, and inside diameter =60 cm. (a) Determine the rotational speed that will provide a G-factor = 60. (b) Suppose that the ring were made out of steel instead of aluminum. (b) If the rotational speed computed in part (a) were used in the steel casting operation, determine the G-factor and (c) centrifugal force per square meter (Pa) on the mold wall. (d) Would this rotational speed result in a successful operation?

6. A horizontal, true centrifugal casting process is used to make lead pipe for chemical plants. The pipe has length = 0.5 m, outside diameter = 70 mm, and wall thickness = 6.0 mm. Determine the rotational speed that will provide a G-factor = 60.

7. A vertical, true centrifugal casting process is used to make tube sections with length = 10.0 in and outside diameter = 6.0 in. The inside diameter of the tube = 5.5 in at the top and 5.0 in at the bottom. At what speed must the tube be rotated during the operation in order to achieve these specifications?

8. A vertical, true centrifugal casting process is used to produce bushings that are 200 mm long and 200 mm in outside diameter. If the rotational speed during solidification

is 500 rev/min, determine the inside diameter at the top of the bushing if the inside diameter at the bottom is 150 mm.

9. A vertical true centrifugal casting process is used to cast brass tubing that is 15.0 in long and whose outside diameter = 8.0 in. If the speed of rotation during solidification is 1000 rev/min, determine the inside diameters at the top and bottom of the tubing if the total weight of the final casting = 75.0 lbs.

10. The housing for a certain machinery product is made of two components, both aluminum castings. The larger component has the shape of a dish sink, and the second component is a flat cover that is attached to the first component to create an enclosed space for the machinery parts. Sand casting is used to produce the two castings, both of which are plagued by defects in the form of misruns and cold shuts.

11. The foreman complains that the parts are too thin, and that is the reason for the defects. However, it is known that the same components are cast successfully in other foundries. What other explanation can be given for the defects?

12. A large, steel sand casting shows the characteristic signs of penetration defect: a surface consisting of a mixture of sand and metal. (a) What steps can be taken to correct the defect? (b) What other possible defects might result from taking each of these steps?

Shrinkage:

1. Determine the shrink rule to be used by pattern =makers for white cast iron. Using the shrinkage value in Table 10.1, express your answer in terms of decimal fraction inches of elongation per foot of length compared to a standard 1-foot scale.

2. Determine the shrink rule to be used by mold = makers for die casting of zinc. Using the shrinkage value in Table 10.1, express your answer in terms of decimal mm of elongation per 300 mm of length compared to a standard 300-mm scale.

3. A flat plate is to be cast in an open mold whose = bottom has a square shape that is 200mmx200mm. The mold is 40 mm deep. A total of 1,000,000 mm^3 of molten aluminum is poured into the mold. Solidification = shrinkage is known to be 6.0%. Table 10.1 lists the linear shrinkage due to thermal contraction after solidification to be 1.3%. If the availability of molten metal in the mold allows the square shape of the cast plate to maintain its 200 mm x200 mm dimensions until solidification is completed, determine the final dimensions of the plate.

TABLE 10.1 Typical linear shrinkage values for different casting metals due to solid thermal contraction.

Metal	Linear shrinkage	Metal	Linear shrinkage	Metal	Linear shrinkage
Aluminum alloys	1.3%	Magnesium	2.1%	Steel, chrome	2.1%
Brass, yellow	1.3%–1.6%	Magnesium alloy	1.6%	Tin	2.1%
Cast iron, gray	0.8%–1.3%	Nickel	2.1%	Zinc	2.6%
Cast iron, white	2.1%	Steel, carbon	1.6%–2.1%		