CALL OPTIONS

King Saud University
Mathematics Department | ACTU461
Exercise's Lecture (6)
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CALL OPTION

A call option is a financial contract which gives the owner the right, but not the obligation, to buy a specified amount of a given asset at a specified price during a specified period of time.

TO CLARIFY THE CONCEPT OF CALL OPTIONS :

Suppose you are planning to buy a house next month in ABC district and you found a good house for 1 M. so you will pay a small proportion of the price (20k) to hold the house until the agreement sells day.

After Month,

CASE1: Suppose the average prices of the real estate for ABC district decreased by 20% .. You will cancel the agreement and loss only (20k) but you will find better house prices (800k). (0)

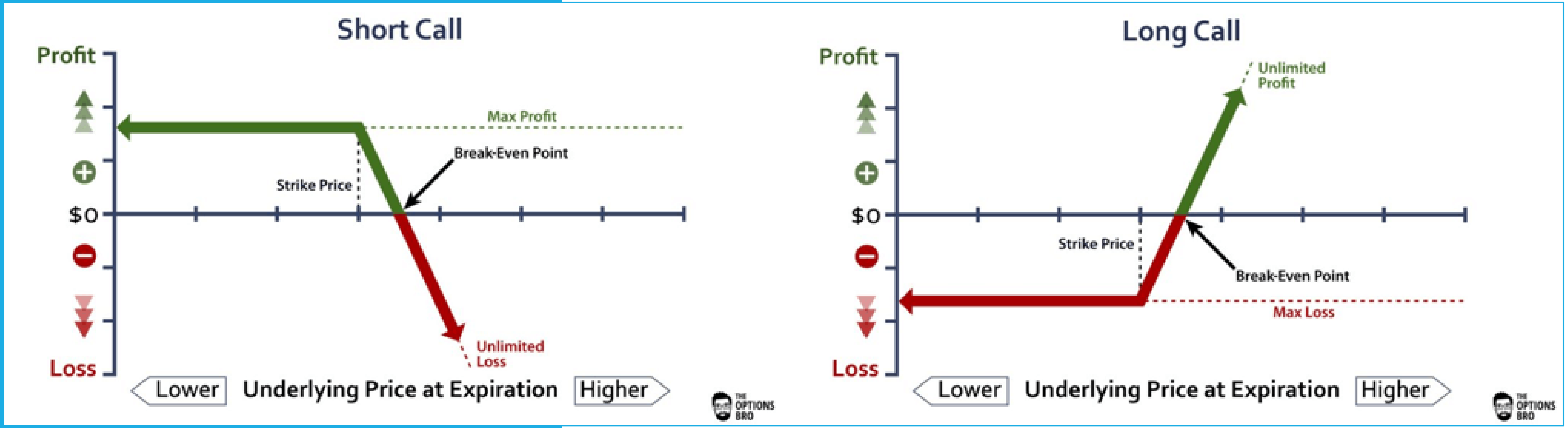
CASE2: Suppose the average prices of the real estate for ABC district increased by 20% .. You will be happy about your agreement that you save 200k.(St-k)

Since, A call–option is a zero–sum game. The seller of a call option or the option call writer has a payoff equals the opposite of the holder's payoff. So, The sum of the two payoffs is zero

The call option holder's profit per unit:

$$\max(S_t - K, 0) - C(K,T)(1+i)^T$$

$$\begin{cases} -\ C(K,T)(1+i)^T & if\ S_T < K \\ S_t - K -\ C(K,T)(1+i)^T & if\ S_T > K \end{cases}$$



1-No Arbitrage

$$max(S_0 - (1+i)^{-T}K, 0) < Call(K,T) < S_0$$

Arbitrage could exist in two main cases:

CASE I

$$(S_0 - (1+i)^{-T}K) > C(K,T)$$

$$\left(S_0 - C(K,T)\right) > \left((1+i)^{-T}K\right)$$

$$\boldsymbol{\left(S_0 - C(K,T)\right)\left((1+i)^{T}\right) > K}$$

CASE II

$$\boldsymbol{C(K,T) > S_0}$$

2-No Arbitrage

$$(1+i)^{-T} max\left(F_{0,T} - K, 0\right) < Call(K,T) < (1+i)^{-T} F_{0,T}$$

Arbitrage could exist in two main cases:

Contracts could appear as prepaid forward contracts. $[\boldsymbol{PV(F_{0,T}) = F^{P}_{0.T}}]$

CASE I

$$(1+i)^{-T}\left(F_{0,T} - K\right) > Call(K,T)$$

$$\boldsymbol{\left(F_{0,T} - K\right) > \left(Call\left(K,T\right)(1+i)^{T}\right)}$$

CASE II

$$\boldsymbol{Call(K,T) > (1+i)^{-T} F_{0,T}}$$

3-No Arbitrage

$$Call(K_2, T) \leq Call(K_1, T) \leq Call(K_2, T) + (K_2 - K_1)e^{-rT}$$

Given, $K_2 > K_1 > 0$

Arbitrage could exist in two main cases:

CASE I

$$\boldsymbol{Call(K_2, T) \geq Call(K_1, T)}$$

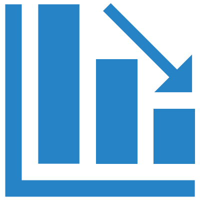
CASE II

$$Call(K_1, T) \geq Call(K_2, T) + (K_2 - K_1)e^{-rT}$$

$$\left(Call(K_1, T) - Call(K_2, T)\right) \geq (K_2 - K_1)e^{-rT}$$

$$\boldsymbol{\left(Call(K_1, T) - Call(K_2, T)\right) e^{rT} \geq (K_2 - K_1)}$$

The purchased of call option is:



Out-the-money

$S_0 < K$

At-the-money

$S_0 = K$

In-the-money

$S_0 > K$

Ethan buys a 35–strike call option for XYZ stock for 4.337 per share. The nominal amount of this call option is 2000 shares. The expiration date of this option is 18 months. The annual effective interest rate is 5.5%.

(i) Calculate Ethan's profit function
(ii) Calculate Ethan's profit for the following spot prices at expiration: 25, 30, 35, 40, 45.
(iii) Calculate Ethan's minimum and maximum profits.
(iv) Find the spot prices at which Ethan's profit is positive.
(v) Calculate the spot price at expiration at which Ethan does not make or lose money on this contract.
(vi) Find the spot price at expiration at which Ethan makes an annual effective yield of 4.75%.
(vii) Find the annual effective rate of return earned by Ethan if the spot price at expiration is 38.

FM SLIDES | Q 7

Hannah sells a 35–strike call option for XYZ stock for 4.337 per share. The nominal amount of this call option is 2000 shares. The expiration date of this option is 18 months. The annual effective interest rate is 5.5%. Hannah invests the proceeds of the sale in a zero–coupon bond.

(i) Calculate Hannah's profit function.
(ii) Calculate Hannah's profit for the following spot prices at expiration: 25, 30, 35, 40, 45.
(iii) Calculate Hannah's minimum and maximum profits.

Consider an European call option on a stock worth S0 =32, with expiration date exactly one year from now, and with strike price \$30. The risk–free annual rate of interest compounded continuously is r = 5%.

(i) If the call is worth \$3, find an arbitrage portfolio.
(ii) If the call is worth \$35, find an arbitrage portfolio.

The current price of a forward contract for 1000 units of an asset with expiration date two years from now is $120000. The risk–free annual rate of interest compounded continuously is 5%. The price of a two–year 100–strike European call option for 1000 units of the asset is $15000. Find an arbitrage portfolio and its minimum profit.

Stock XYZ has the following characteristics:

- The current price is 40.
- The price of a 35-strike 1-year European call option is 9.12,
- The price of a 40-strike 1-year European call option is 6.22,
- The price of a 45-strike 1-year European call option is 4.08.

The annual effective risk-free interest rate is 8%. Let S be the price of the stock price one year from now. All call positions being compared are long.

Determine the range for S such that the 45-strike call produces a higher profit than the 40-strike call, but a lower profit than the 35-strike call.

A) $S<38.13$
B) $38.13<S<40.44$
C) $40.44<S<42.31$
D) $S>42.31$
E) The range is empty

ACTEX VOL. I| Q 1.1.5

The current price of a non-dividend-paying stock is 40 and the continuously compounded annual risk-free rate of return is 8%. You enter into a short position on 3 call options, each with 3 months to maturity, a strike price of 35, and an option premium of 6.13. Simultaneously, you enter into a long position on 5 call options, each with 3 months to maturity, a strike price of 40, and an option premium of 2.78.

All options are held until maturity.
Calculate the maximum possible profit and the maximum possible loss for the entire option portfolio.

	Maximum Profit	Maximum Loss
A)	3.42	4.58
B)	4.58	10.42
C)	Unlimited	10.42
D)	4.58	Unlimited
E)	Unlimited	Unlimited

ACTEX VOL. I| Q 14

In each case identify the arbitrage

b. Now suppose these options have 2 years to expiration and the continuously compounded interest rate is 10%. The 90-strike call costs $10 and the 95-strike call costs $5.25.

DERIVATIVES MARKETS | Q 9.12

Ben sells a 50-strike 1-year call on an index when the market price of the index is also 50.The call price is 4. Assume that the option is for an underlying 100 units of the index and the annual effective interest rate is 2%.

Calculate Ben's profit if the index increases to 52 at expiration.

A) -408
B) -208
C) 0
D) 208
E) 408

ACTEX VOL. II| Q 16

19, The current price of a nondiviaend-pying stock is 40 and the annual effective risk-free interest rate is 8%. You enter into a short position of 5 call options. each with 1 year maturity, a strike price of 35, and a premium of 9.21. Simultaneously, you enter into a long position on 3 call options, each with 1 year to maturity a strike price of 42 premium of 4.29.

All options are held until maturity.

Calculate the maximum possible profit and the maximum possible loss for the entire option portfolio.

	Maximum Profit	Maximum Loss
A)	35.83	Unlimited
B)	35.83	10.13
C)	Unlimited	10.13
D)	2.67	Unlimited
E)	Unlimited	Unlimited

ACTEX VOL. II| Q 19

An investor purchased Option A and Option B for a certain stock today, with strike prices 70 and 80, respectively. Both options are European one-year call options.

Determine which statements Is true about the moneyness of these options. based particular stock price.

A) If Option A is in-the-money, then Option B is in-the-money.
B) If Option A is in-the-money, then Option B is out-of-the-money.
C) If Option A is at-the-money, then Option B is at-the-money.
D) If Option A is out-of-the-money, then Option B is in-the-money.
E) If Option A is out-of-the-money, then Option B is out-of-the-money.

ACTEX VOL. II| Q 13