KING SAUD UNIVERSITY. DEPARTMENT OF PHYSICS

QUANTUM MECHANICS H.W №3

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PROBLEM (1)

Find the eigenvalues of the operator :

$$\hat{A} = \begin{pmatrix} 1 & i & 0 \\ -i & 2 & -i \\ 0 & i & 1 \end{pmatrix}$$

Can we diagonalise it ?.

PROBLEM (2)

Is the operator d/dx acting on the L^2 Hilbert space hermitian ? How about $-id/dx$?

PROBLEM (3)

Show that for a Hermitian operator $\hat{T}$ the operator $\hat{U} \equiv e^{i\hat{T}}$ is unitary.

PROBLEM (4)

Is the function $\cos kx$ an eigenfunction for the operator d/dx ? What about the operator d^2/dx^2 ?

PROBLEM(5)

Show that for an operator $\hat{A}$ commuting with the **Hamiltonian operator** $\hat{H}$ we can write its time evolution as:

$$e^{i\hat{H}t}\hat{A}e^{-i\hat{H}t}$$

Hint: use Hadmard lemma This is known as **Heisenberg equation**

PROBLEM (6)

We define the expected value - average value- of an operator with respect to a state vector $|\psi\rangle$ as:

$$\langle\hat{A}\rangle \equiv \langle\psi|\hat{A}|\psi\rangle$$

Show that if $|\psi\rangle$ is expanded in terms of the eigenbasis of $\hat{A}$, then the expected value is the sum of the eigenvalues of the operator a_n, i.e.

$$\langle\hat{A}\rangle = \sum_n c_n a_n$$

for some constants c_n.

PROBLEM (7)

Recall that we defined the identity operator as the outer product of the basis :

$$I = \sum_i |i\rangle\langle i|$$

Using this definition, show that this operator sends the ket $|\psi\rangle$ to itself (does not change the ket), then show this for the Bra vector $\langle\psi|$.

PROBLEM(8)

Discuss why if $[\hat{A},\hat{B}]|\psi\rangle \neq 0$ one cannot find a mutual eigenbasis to expand $|\psi\rangle$ with ?