Contents

Preface

This manual provides solutions to the exercises in the parts of *Actuarial Mathematics for Life Contingent Risks* that are on the MLC syllabus, as well as to the exercises in the Supplementary Notes. The Supplementary Notes can be downloaded from the the SOA website at

`http://www.soa.org/files/pdf/edu-2012-spring-mlc-studynotes.pdf`

Here are some considerations for students taking the MLC exam:

1. Most of the exercises in the textbook require use of spreadsheets, or involve derivations of formulas. Thus they are not exam-type questions. However, they will give you a lot of practice with the concepts, and teach you skills needed in actuarial work. The following exercises do not require spreadsheets and are not formula derivations, and thus are exam-like, although some of these questions require more calculations than a typical exam question:

   2.1, 2.2, 2.3, 2.4(d)(i),(ii),(iii), 2.6, 2.10
   3.1, 3.2 3.3, 3.4, 3.5, 3.6, 3.7, 3.8
   4.1, 4.6(c), 4.7, 4.10, 4.11, 4.14, 4.17(c)
   5.1, 5.3, 5.4, 5.5
   6.1, 6.8, 6.12
   7.1
   8.10(a),(d), 8.20 (a),(b),(c), 8.22
   9.1, 9.2
   10.1
   11.1, 11.3

   In addition, some exercises can be done without a spreadsheet if you are given a table of values for the Standard Ultimate or Select Survival Model.

   Notice that few exercises on the later material are doable without a spreadsheet. Thus the above list will not give you a balanced set of practice questions for the exam.

2. This manual uses the same terminology and notation as *Actuarial Mathematics for Life Contingent Risks* . This terminology and notation is mostly British, and may not be used on Exam MLC. See

   `http://www.soa.org/files/pdf/edu-2012-spring-mlc-terminology.pdf`

   for a comparison between terminology and notation used in the textbooks and used on the exam.

Since most exercises are meant to be done on a spreadsheet, the solutions given in the textbook involve no intermediate rounding other than the computer's internal rounding. Often the textbook gives an intermediate rounded answer with inadequate precision to obtain a later answer. I generally maintain a lot of precision in the solutions given in this manual. Despite this precision, you will often find that the values given are slightly off when calculated from earlier values. In those cases, you will get the exact answer given here only if you reproduce the intermediate calculations in a spreadsheet. Nevertheless, you should have no trouble duplicating my work by following the solutions.

Errata

Undoubtedly there are errors in this manual. Please send error reports to `mail@studymanuals.com` or directly to me at `errata@aceyourexams.net`. Errata will be posted at `errata.aceyourexams.net`.

In some cases, the answers provided here differ from those in the textbook. The errata list will include corrections to my solutions, or confirmation that the textbook's solution is incorrect, when such information is known.

Solutions to Chapter 3 Exercises

3.1 (a) Figures 3.1 and‘3.2, with linear and logarithmic scales, are taken from the ASM MLC study manual. The exercise asks for Figure 3.1. But a clearer way to see the pattern is to use a logarithmic scale, as in Figure 3.2. The force of mortality is high at birth and then drops until about age 10, then rises approximately exponentially, although it may have a hump in the 20’s.

(b) Figure 3.3 for $S_x(t)$ is taken from the ASM MLC study guide. l_x is a constant multiple of $S_0(t)$, so the curve looks the same as the $S_0(t)$ curve. The curve drops slowly at early ages, then humps down starting at age 60 and drops rapidly, slowing down a bit at very high ages.

(c) Figure 3.4 is a set of graphs for the probability density function, taken from the ASM MLC study guide. Since d_x is a difference of l_x’s, just like $f(x)$ is a negative derivative of $S_0(x)$, the curve of d_x’s looks the same as the $f_0(t)$ curve. The curve first rises slowly, then more rapidly after age 60, peaking around age 83, then dropping.

3.2 (a) Interpolate in the life table to obtain $l_{52.4}$ and $l_{52.6}$.

$$l_{52.4} = 0.6(89{,}948) + 0.4(89{,}089) = \boxed{\mathbf{89{,}604.4}}$$

$$l_{52.6} = 0.4(89{,}948) + 0.6(89{,}089) = \boxed{\mathbf{89{,}432.6}}$$

$${}_{0.2}q_{52.4} = \frac{l_{52.4} - l_{52.6}}{l_{52.4}} = \frac{89{,}604.4 - 89{,}432.6}{89{,}604.4} = \boxed{\mathbf{0.00191732}}$$

(b) $p_{52} = 89{,}089/89{,}948 = 0.990450$. Then ${}_{0.2}q_{52.4} = 1 - p_{52}^{0.2} = \boxed{\mathbf{0.0019173}}$.

(c) Interpolate in the life table to obtain $l_{58.1}$.

$$l_{58.1} = 0.9(83{,}940) + 0.1(82{,}719) = 83{,}817.9$$

$${}_{5.7}p_{52.4} = \frac{l_{58.1}}{l_{52.4}} = \frac{83{,}817.9}{89{,}604.4} = \boxed{\mathbf{0.9354217}}$$

(d) ${}_{5.7}p_{52.4} = {}_{0.6}p_{52.4}\ {}_{5}p_{53}\ {}_{0.1}p_{58}$. The first factor is $p_{52}^{0.6}$. The second factor can be calculated directly off the life table as l_{58}/l_{53}. For the third factor:

$$p_{58} = \frac{82{,}719}{83{,}940} = 0.985454$$

and we extract the tenth root of that. So

$${}_{5.7}p_{52.4} = (0.990450^{0.6})\left(\frac{83{,}940}{89{,}089}\right)(0.985454^{0.1}) = \boxed{\mathbf{0.9354230}}$$

(e)

$${}_{3.2|2.5}q_{52.4} = \frac{l_{55.6} - l_{58.1}}{l_{52.4}}$$

We’ve already calculated $l_{52.4}$ and $l_{58.1}$. We just need $l_{55.6}$.

$$l_{55.6} = 0.4(87{,}208) + 0.6(86{,}181) = 86{,}591.8$$

$${}_{3.2|2.5}q_{52.4} = \frac{86{,}591.8 - 83{,}817.9}{89{,}604.4} = \boxed{\mathbf{0.0309572}}$$

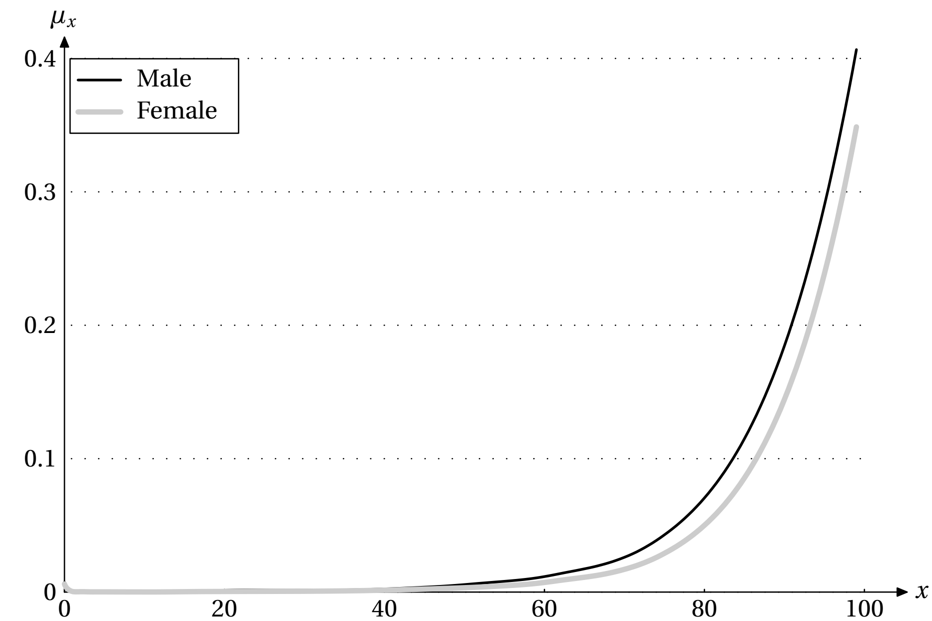

Figure 3.1: Force of mortality for males and females

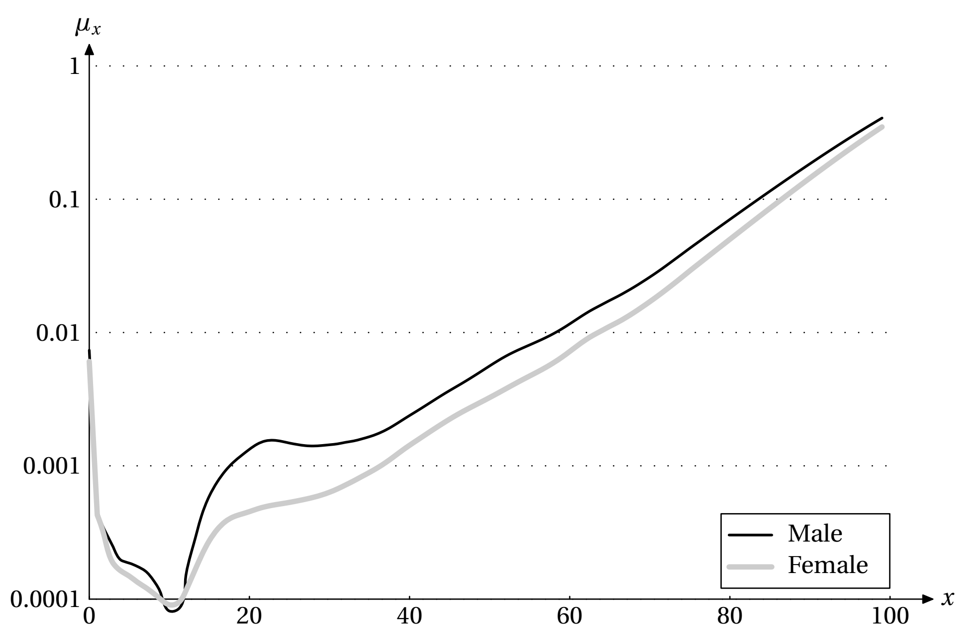

Figure 3.2: Force of mortality for males and females with logarithmic scale

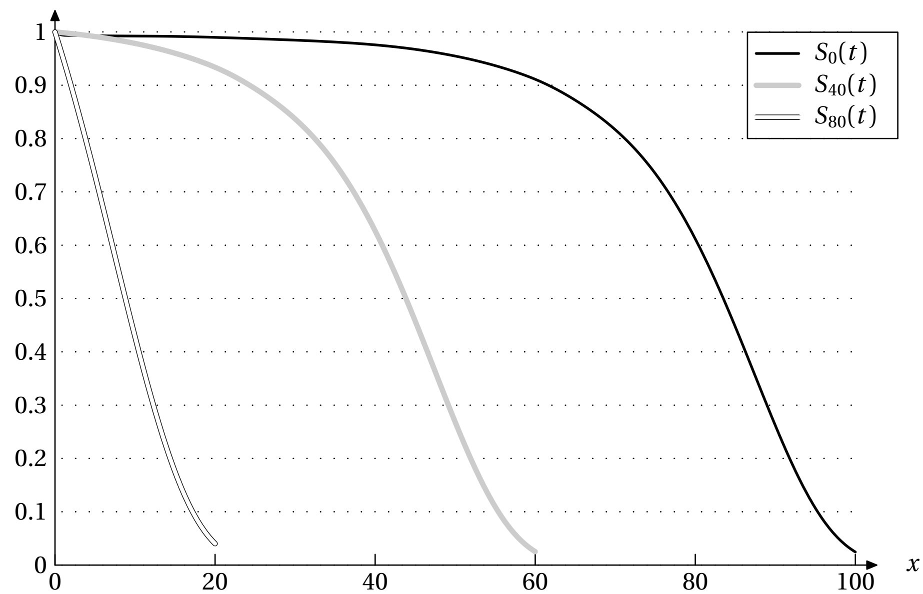

Figure 3.3: Survival function for 3 ages

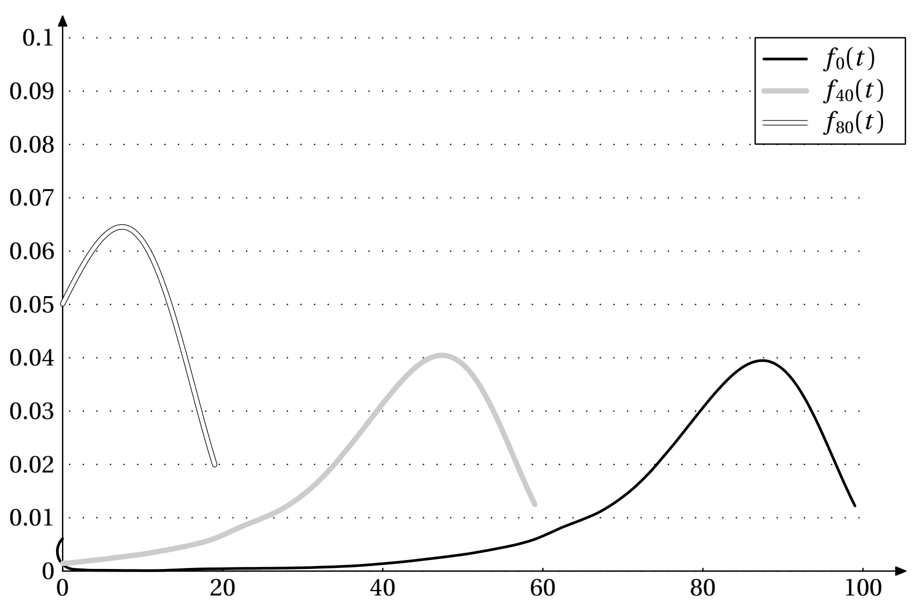

Figure 3.4: Probability density function for three ages

(f) We'll use ${}_{3.2|2.5}q_{52.4} = {}_{3.2}p_{52.4}\ {}_{2.5}q_{55.6}$.

$$p_{55} = \frac{86{,}181}{87{,}208} = 0.988224$$

$${}_{3.2}p_{52.4} = (0.990450^{0.6})\left(\frac{87{,}208}{89{,}089}\right)(0.988224^{0.6}) = 0.9663733$$

$${}_{2.5}p_{55.6} = (0.988224^{0.4})\left(\frac{83{,}940}{86{,}191}\right)(0.985454^{0.1}) = 0.9679728$$

$${}_{3.2|2.5}q_{52.4} = (0.9663733)(1 - 0.9679728) = \boxed{\mathbf{0.0309502}}$$

3.3 (a) $10{,}542/15{,}930 = \boxed{\mathbf{0.661770}}$

(b)

$$\frac{l_{85} - l_{87}}{l_{[75]+1}} = \frac{10{,}542 - 9{,}064}{15{,}668} = \boxed{\mathbf{0.094332}}$$

(c)

$${}_{4|2}q_{[77]+1} = \frac{l_{82} - l_{84}}{l_{[77]+1}} = \frac{12{,}576 - 11{,}250}{14{,}744} = \boxed{\mathbf{0.089935}}$$

3.4 (a)

$${}_{2}p_{[72]} = (1 - q_{[72]})(1 - q_{[72]+1}) = (1 - 0.005236)(1 - 0.007456) = \boxed{\mathbf{0.987347}}$$

(b)

$$\begin{aligned}{}_{3}q_{[73]+2} &= 1 - (1 - q_{[73]+2})(1 - q_{[73]+3})(1 - q_{[73]+4})\\ &= 1 - (1 - 0.011370)(1 - 0.014988)(1 - 0.019316) = \boxed{\mathbf{0.044998}}\end{aligned}$$

(c)

$${}_{1|}q_{[65]+4} = (1 - 0.007994)(0.010599) = \boxed{\mathbf{0.010514}}$$

(d)

$$\begin{aligned}{}_{7}p_{[70]} &= (1 - 0.004285)(1 - 0.005967)(1 - 0.008066)(1 - 0.010629)(1 - 0.013698)(1 - 0.018774)(1 - 0.021053)\\ &= \boxed{\mathbf{0.920271}}\end{aligned}$$

3.5 (a)

$$\begin{aligned}{}_{7}p_{[70]} &= (1 - 0.010373)(1 - 0.014330)(1 - 0.019192)(1 - 0.025023)(1 - 0.031859)(1 - 0.043686)(1 - 0.048270)\\ &= \boxed{\mathbf{0.821929}}\end{aligned}$$

(b)

$$\begin{aligned}{}_{1|2}q_{[70]+2} &= p_{[70]+2}\big(1 - (1 - q_{[70]+3})(1 - q_{[70]+4})\big)\\ &= (1 - 0.019192)\big(1 - (1 - 0.025023)(1 - 0.031859)\big) = \boxed{\mathbf{0.055008}}\end{aligned}$$

(c) Let's first calculate ${}_{0.8}q_{[70]+0.2}$.

$$ {}_{0.8}q_{[70]+0.2} = \frac{0.8q_{[70]}}{1 - 0.2q_{[70]}} = \frac{0.8(0.010373)}{1 - 0.2(0.010373)} = 0.0083157 $$

Then

$$ {}_{3.8}q_{[70]+0.2} = 1 - (1 - 0.0083157)(1 - 0.014330)(1 - 0.019192)(1 - 0.025023) = \boxed{\mathbf{0.065276}} $$

3.6 ${}_{2|3}q_{[50]+1} = {}_{2}p_{[50]+1}\, {}_{3}q_{53}$, so if we can calculate ${}_{2}p_{[50]+1}$ we'll be done. From $q_{[50]}$ and ${}_{2}p_{[50]}$, we have

$$ p_{[50]+1} = \frac{{}_{2}p_{[50]}}{p_{[50]}} = \frac{0.96411}{1 - 0.01601} = 0.979797 $$

From ${}_{2}p_{[50]}$ and ${}_{2|}q_{[50]}$ we have

$$ q_{[50]+2} = \frac{{}_{2|}q_{[50]}}{{}_{2}p_{[50]}} = \frac{0.02410}{0.96411} = 0.024997 $$

So ${}_{2}p_{[50]+1} = (0.979797)(1 - 0.024997) = 0.955304$, and ${}_{3}p_{53} = 1 - 0.09272/0.955304 = \boxed{\mathbf{0.902942}}$.

3.7 The following table computes d_x, $d^A_{[30]+x-30}$ and $q^A_{[30]+x-30}$ based on the U.S. Life Table for ages 30–34. In the $d^A_{[30]+x-30}$ column, the d_x's are multiplied by 6, 5, 4, 3, and 2. Then the q^A's are calculated by dividing these d^A's by the l_x's.

x	l_x	d_x	$d^A_{[30]+x-30}$	$q^A_{[30]+x-30}$
30	98,424	62	372	0.003780
31	98,362	66	330	0.003355
32	98,296	71	284	0.002889
33	98,225	77	231	0.002352
34	98,148	84	168	0.001712

Then

$$ {}_{5}p_{[30]} = (1 - 0.003780)(1 - 0.003355)(1 - 0.002889)(1 - 0.002352)(1 - 0.001712) = 0.985991 $$

Multiplying the force of mortality by 1.5 results in raising the probability of survival to the 1.5 power, so ${}_{5}p_{35} = (97{,}500/98{,}064)^{1.5} = 0.991385$. So

$$ {}_{10}p_{[30]} = (0.985991)(0.991385) = \boxed{\mathbf{0.977497}} $$

3.8 (a) The l^*_{x+3} column is the same as the original l_{x+4} column, since in effect the age in the special table is 1 higher than in the standard table; l^*_{25} is set equal to l_{26} and $p^*_x = p_{x+1}$. And the select mortality probabilities interlock:

$$ p^*_{[x]+2} = {}_{2}p_{x+2} = \frac{l_{x+4}}{l_{x+2}} $$

and since the l^*_{x+3} column is the same as the l_{x+4} column, the $l^*_{[x]+2}$ column is the same as l_{x+2}.

$$ p^*_{[x]+1} = {}_{3}p_{x-1} = \frac{l_{x+2}}{l_{x-1}} $$

and since the $l^*_{[x]+2}$ column is the same as the l_{x+2} column, the $l^*_{[x]+1}$ column is the same as l_{x-1}. Finally, the $l^*_{[x]}$ column is the same as l_{x-5}. So copying the needed values, we get

x	$l^*_{[x]}$	$l^*_{[x]+1}$	$l^*_{[x]+2}$	l^*_{x+3}	$x+3$
20	99,180	98,942	98,700	98,529	23
21	99,135	98,866	98,615	98,444	24
22	99,079	98,785	98,529	98,363	25

(b) Note that $l^*_{62} = l_{63} = 86{,}455$.

$$
\begin{aligned}
{}_{2|38}q^*_{[21]+1} &= \frac{l^*_{24} - l^*_{62}}{l^*_{[21]+1}} = \frac{98{,}444 - 86{,}455}{98{,}866} = \boxed{\mathbf{0.121265}}\\
{}_{40}p_{[22]} &= \frac{l^*_{62}}{l^*_{[22]}} = \frac{86{,}455}{99{,}079} = \boxed{\mathbf{0.872587}}\\
{}_{40}p_{[21]+1} &= \frac{l^*_{62}}{l^*_{[21]+1}} = \frac{86{,}455}{98{,}866} = \boxed{\mathbf{0.874465}}\\
{}_{40}p_{[20]+2} &= \frac{l^*_{62}}{l^*_{[20]+2}} = \frac{86{,}455}{98{,}700} = \boxed{\mathbf{0.875937}}\\
{}_{40}p_{22} &= \frac{l^*_{62}}{l^*_{22}} = \frac{l_{63}}{l_{23}} = \frac{86{,}455}{98{,}615} = \boxed{\mathbf{0.876692}}
\end{aligned}
$$

3.9 (a) Evaluate conditional probability from the definition.

$$
\begin{aligned}
\Pr(K_x = k) &= {}_kp_x(1 - p_{x+k}) = {}_kp_x\left(1 - e^{-\mu^*_{x+k}}\right)\\
\Pr(K_x = k \,\&\, R_x < s) &= {}_kp_x(1 - {}_sp_{x+k}) = {}_kp_x\left(1 - e^{-s\mu^*_{x+k}}\right)\\
\Pr(R_x \le s \mid K_x = k) &= \frac{{}_kp_x\left(1 - e^{-s\mu^*_{x+k}}\right)}{{}_kp_x\left(1 - e^{-\mu^*_{x+k}}\right)}\\
&= \frac{1 - e^{-s\mu^*_{x+k}}}{1 - e^{-\mu^*_{x+k}}}
\end{aligned}
$$

The asterisks are not needed for this part, since you are given that the μ's are constant between integral ages. They are needed for the next part of the exercise.

(b) By definition of μ^*_{x+k}, $q_x = \Pr(K_x = k \mid K_x \ge k) = 1 - exp(-\mu^*_{x+k})$. For $K_x = k$, $R_x = T_x + k$. So we have

$$
\Pr(T_x \le k + s \mid T_x \ge k) = \Pr(R_x \le s \mid K_x = k)\Pr(K_x = k \mid K_x \ge k) = 1 - \exp(-\mu^*_{x+k}s)
$$

where $K_x > k$ and $T_x > k$ are equivalent since k is an integer. Then $\Pr(T_x > k + s \mid T_x \ge k) = \exp(-\mu^*_{x+k}s)$. However, $\Pr(T_x > k + s \mid T_x \ge k) = S_k(s)$. Logging $S_k(s)$ and differentiating, we see that $\mu_{x+s} = \mu^*_{x+k}$, a constant.

3.10 We have to verify the integral of $\mu_{[x]+s}$.

$$\begin{aligned}
\int_0^t \mu_{[x]+s}\,\mathrm{d}s &= \int_0^t (0.9^{2-s})(A + Bc^{x+s})\mathrm{d}s \\
&= 0.9^2 A \int_0^t 0.9^{-s}\mathrm{d}s + 0.9^2 Bc^x \int_0^t \left(\frac{0.9}{c}\right)^{-s} \mathrm{d}s \\
&= 0.9^2 A\left(\frac{1 - 0.9^{-t}}{\ln 0.9}\right) + 0.9^2 Bc^x\left(\frac{1 - (0.9/c)^{-t}}{\ln(0.9/c)}\right) \\
&= 0.9^{2-t}\left(A\left(\frac{0.9^t - 1}{\ln 0.9}\right) + Bc^x\left(\frac{0.9^t - c^t}{\ln(0.9/c)}\right)\right)
\end{aligned}$$

Since ${}_tp_x = \exp\left(-\int_0^t \mu_{[x]+s}\,\mathrm{d}s\right)$, we negate this expression and exponentiate it. The result is equation (3.15).