# Modern Physics PHYS 351 — “Summery of wave nature of matter ”

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*In quantum physics, the distinction between a wave and a particle is not present.*

## 1 Wavepackets

Just like how light has a dual *ondulaire* and *corpuscular* nature, we can apply this duality on matter as well. Such that every lump of matter having a momentum $p$ has a De Broglie wavelength

$$\lambda_{db} = \frac{h}{p} \tag{1}$$

By applying this wave theory of matter to electrons in atoms, de Broglie was able to explain the appearance of integers in certain Bohr orbits as a natural consequence of electron wave interference in the double-slit experiment with electrons, see figure 1. Although the wavelength of matter waves can be experimentally determined, it is important to understand that they are not just like other waves because their frequency and phase velocity cannot be directly measured. In particular, the phase velocity of an individual matter wave is greater than the velocity of light and varies with wavelength or wavenumber.

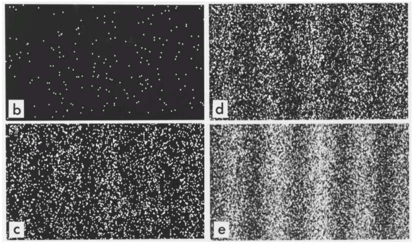

Figure 1: Build-up of an interference pattern. The number of detected electrons is 100 (b), 3000 (c), 20000 (d), 70000 (e).Tonomura et al. (1989)

$$v_p = \nu\lambda = \left(\frac{E}{h}\right)\left(\frac{h}{p}\right) = c\sqrt{1+(mc/\hbar k)^2} \tag{2}$$

In order to represent a particle properly, a superposition of matter waves with different wavelengths, amplitudes, and phases must be chosen to interfere constructively over a limited region of space. The resulting wave packet or group can then be shown to travel with the same speed as the classical particle.

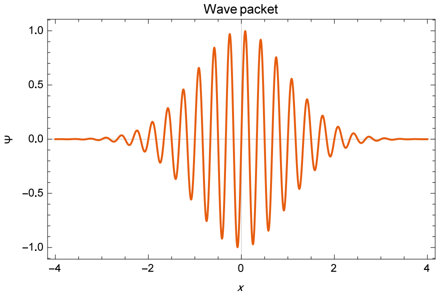

Figure 2: How localised wave packet is constructed from superposition of many waves of wavenumber $k$, a Gaussian wavepacket $e^{-x^2/2}\sin(8\pi x)$, is an example

## 2 The Uncertainty Principle

In any experimental setting, instrumentation could result an uncertainty in measuring a physical quantity $\phi$, which is calculated from:

$$\Delta\phi = \sigma_\phi = \sqrt{\langle\phi^2\rangle - \langle\phi\rangle^2} \tag{3}$$

However, this uncertainty is due to experimental setup and our imperfect measuring conditions. In principle, classical systems are fully deterministic and the laws of classical physics has NO uncertainty. Measurement of ‘ quantum’ systems cannot be gentile enough not to affect the state of the system. This creates a genuine *uncertainty* that cannot be avoided regardless of instrumentation used, or the experiment conducted. An example of this is the *Heisenberg Microscope thought experiment*, see figure **??**. Another way to explain the uncertainty principle is that a wave packet localised in a region $\Delta x$ contains a range of wavenumbers $\Delta k$ , such that $\Delta x \geq 1/2k$. And since $p = \hbar k$ we have the uncertainty relation :

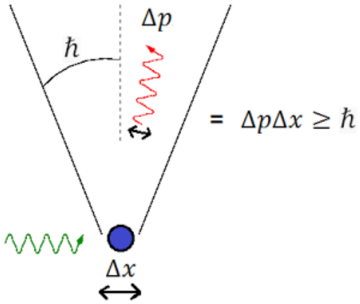

Figure 3: Heisenberg Microscope

$$\Delta x \Delta p \geq \frac{\hbar}{2} \tag{4}$$

Implying we cannot measure the position and momentum of a quantum particle with infinite accuracy at the same time. Another uncertainty relation can be established between energy and time

$$\Delta t \Delta E \geq \frac{\hbar}{2} \tag{5}$$

## 3 Wavefunctions

In quantum mechanics matter waves are represented by a **wavefunction** $\Psi(x, y, z)$ The probability of finding a particle represented by $\Psi$ in a volume $V$ at a given time $t$ is given by the square of the norm of the wave function integrated over this small volume

$$W(V) = \int_V \Psi^* \Psi dxdydz = \int_V |\Psi|^2 dxdydz \tag{6}$$

We call the quantity $|\Psi|^2$ the probability density. The waveâĂŞparticle duality of electrons may be seen by considering the passage of electrons through two narrow slits and their arrival at a viewing screen. We find that although the electrons are detected as particles at a localized spot on the screen, the probability of arrival at that spot is determined by finding the intensity of two interfering matter waves.

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