



الزمن: ساعة ونصف.
الدرجة:

الإسم:
الرقم الجامعي:
أستاذ المقرر:

ملاحظات: 1. عدد الورقات 3 و ورقة مسودة 2. ممنوع استخدام الآلة الحاسبة.

السؤال الأول (4 درجات): احسب $\frac{dy}{dx}$ فيما يلي:

(درجتان)

$$y = \ln(\sinh(2x)) \quad (1)$$

$$\frac{dy}{dx} = \frac{2 \cosh(2x)}{\sinh(2x)} = 2 \coth(2x)$$

(درجتان)

$$y = \cosh^{-1}(\sqrt{x}) \quad (2)$$

① + ①

$$\frac{dy}{dx} = \frac{1}{\sqrt{(\sqrt{x})^2 - 1}} \cdot \frac{1}{2\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x(x-1)}}$$

السؤال الثاني (21 درجة): احسب التكاملات التالية:

(درجتان)

$$\int \frac{e^x}{\sqrt{e^{2x} - 16}} dx \quad (1)$$

①

نضع $u = e^x$ فإن $du = e^x dx$

$$\int \frac{e^x}{\sqrt{e^{2x} - 16}} dx = \int \frac{du}{\sqrt{u^2 - 4^2}} = \cosh^{-1}\left(\frac{u}{4}\right) + c$$

①

$$\int \frac{e^x}{\sqrt{e^{2x} - 16}} dx = \cosh^{-1}\left(\frac{e^x}{4}\right) + c$$

$$\int 2x \tan^{-1} x \, dx \quad (2)$$

(درجتان)

دستخیزم طریقه انتکامل باینجزبه:

$$u(x) = x^2 \Rightarrow v'(x) = \frac{1}{1+x^2}$$

$$u'(x) = 2x \Rightarrow v(x) = \tan^{-1} x$$

$$\int 2x \tan^{-1} x \, dx = x^2 \tan^{-1} x - \int \frac{x^2}{1+x^2} dx$$

$$= x^2 \tan^{-1} x - \int \left[1 - \frac{1}{1+x^2} \right] dx$$

$$= x^2 \tan^{-1} x + \tan^{-1} x - x + C, \quad C \in \mathbb{R}$$

①

①

$$\int \cos x \sin^3 x \, dx \quad (3)$$

(درجتان)

ذبح $u = \sin x$ فان $du = \cos x \, dx$

$$\int \cos x \sin^3 x \, dx = \int u^3 \, du = \frac{u^4}{4} + C$$

$$= \frac{\sin^4 x}{4} + C, \quad C \in \mathbb{R}$$

(درجتان)

$$\int \tan^3 x \sec^6 x \, dx \quad (4)$$

①

①

ذبح $u = \tan x$ فان $du = \sec^2 x \, dx$

$$\int \tan^3 x \sec^6 x \, dx = \int \tan^3 x (\sec^2 x)^2 \sec^2 x \, dx = \int \tan^3 x (1 + \tan^2 x)^2 \sec^2 x \, dx$$

$$\int \tan^3 x \sec^6 x \, dx = \int u^3 (1 + u^2)^2 \, du = \int u^3 (1 + u^2 + 2u^2 + u^4) \, du = \int [u^3 + u^5 + 2u^5 + u^7] \, du$$

$$= \frac{u^4}{4} + \frac{u^6}{6} + \frac{2u^6}{6} + \frac{u^8}{8} + C = \frac{\tan^4 x}{4} + \frac{\tan^6 x}{8} + \frac{\tan^6 x}{3} + C$$

(درجتان)

①

①

مع العلم أن: $\sin a \cos b = \frac{1}{2} [\sin(a-b) + \sin(a+b)]$

$$\sin 2x \cos 4x = \frac{1}{2} [\sin(-2x) + \sin 6x]$$

$$= \frac{1}{2} [\sin 6x - \sin 2x]$$

$$\int \sin 2x \cos 4x \, dx = \frac{1}{2} \int [\sin 6x - \sin 2x] \, dx$$

$$\int \sin(2x) \cos(4x) \, dx = \frac{1}{2} \left[-\frac{\cos 6x}{6} + \frac{\cos 2x}{2} \right] + C$$

(3 درجات)

①

①

$$\int \frac{dx}{x^2 \sqrt{4-x^2}} \quad (6)$$

ذبح $x = 2 \sin \theta$ فان $dx = 2 \cos \theta \, d\theta$

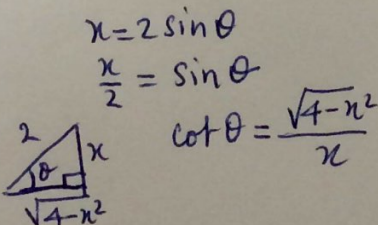
$$\sqrt{4-x^2} = \sqrt{4-4\sin^2 \theta} = 2 \cos \theta$$

$$\int \frac{dx}{x^2 \sqrt{4-x^2}} = \int \frac{2 \cos \theta \, d\theta}{4 \sin^2 \theta \cdot 2 \cos \theta} = \frac{1}{4} \int \csc^2 \theta \, d\theta$$

$$= -\frac{1}{4} \cot \theta + C$$

$$= -\frac{1}{4} \frac{\sqrt{4-x^2}}{x} + C$$

①



(3 درجات)

$$\int \frac{dx}{\sqrt{10x - x^2 - 21}} \quad (7)$$

$$10x - x^2 - 21 = -[x^2 - 10x + 21]$$

$$= -[(x-5)^2 - 25 + 21]$$

$$10x - x^2 - 21 = 4 - (x-5)^2$$

$$\int \frac{dx}{\sqrt{10x - x^2 - 21}} = \int \frac{dx}{\sqrt{4 - (x-5)^2}} = \sin^{-1}\left(\frac{x-5}{2}\right) + \text{const}$$

(3 درجات)

$$\int \frac{x-2}{x^3+x} dx \quad (8)$$

(هالة كسرية) $f(x) = \frac{x-2}{x^3+x} = \frac{(x-2)}{x(x^2+1)} ; x \neq 0$

$$f(x) = \frac{A}{x} + \frac{Bx+C}{x^2+1} = \frac{A(x^2+1) + (Bx+C)x}{x^3+x}$$

$$x-2 = A(x^2+1) + x(Bx+C)$$

$$x-2 = (A+B)x^2 + cx + A$$

$$\begin{cases} A = -2 \\ B = 2 \\ C = 1 \end{cases}$$

$$\int \frac{x-2}{x^3+x} dx = \int \left[-\frac{2}{x} + \frac{2x+1}{x^2+1} \right] dx = -2 \ln|x| + \ln(x^2+1) + \tan^{-1}x$$

(درجتان)

$$\int \sqrt{1+\sqrt{x}} dx \quad (9)$$

$$u = \sqrt{1+\sqrt{x}} \quad \text{عوض}$$

$$u^2 = 1+\sqrt{x}$$

$$(u^2-1)^2 = x$$

$$dx = 2(u^2-1)2u du \quad \text{فإن}$$

$$dx = 4u(u^2-1)du$$

$$\int \sqrt{1+\sqrt{x}} dx = 4 \int u^2(u^2-1) du$$

$$\int \sqrt{1+\sqrt{x}} dx = 4 \int (u^4 - u^2) du$$

$$\int \sqrt{1+\sqrt{x}} dx = 4 \left[\frac{u^5}{5} - \frac{u^3}{3} \right] + c = \frac{4}{5} (1+\sqrt{x})^{5/2} - \frac{4}{3} (1+\sqrt{x})^{3/2} + c$$