Home work (Math 106) Sec 1054

Ex 1 Evaluate the following integrals:

$$I = \int \frac{dx}{x\sqrt{\ln x}}$$

$$J = \int \frac{e^{3/x}}{x^2}\,dx$$

$$K = \int x\,5^{-x^2}\,dx$$

$$L = \int \frac{dx}{x\sqrt{9x^2-1}}$$

$$M = \int \frac{x^2}{4+x^6}\,dx$$

$$N = \int \frac{dx}{\sqrt{1+e^{2x}}}$$

$$P = \int \frac{dx}{x\sqrt{16-x^4}}$$

Ex2 Find the limit if there exists:

1. $$\lim_{x\to\pi/2} \frac{1-\sin x}{\cos x}$$

2. $$\lim_{x\to\infty} \left(1+\frac{5}{x}\right)^{2x}$$

Answer HW1 (Math 106)

Ex1

- $I = \int \frac{dx}{x\sqrt{\ln x}} = 2\int \frac{1}{2\sqrt{u}}\,du = 2\sqrt{u} + cst$
  $= 2\sqrt{\ln x} + cst$

  $u = \ln x$
  $du = \frac{dx}{x}$

- $J = \int \frac{e^{3/x}}{x^2}\,dx = -\frac{1}{3}\int e^u\,du = -\frac{1}{3}e^u + cst$
  $= -\frac{1}{3}e^{3/x} + cst$

  $u = 3/x$
  $du = -\frac{3}{x^2}dx \Leftrightarrow \frac{dx}{x^2} = -\frac{1}{3}du$

- $K = \int x\,5^{-x^2}dx = -\frac{1}{2}\int 5^u\,du = -\frac{1}{2\ln 5}5^u + cst$
  $= -\frac{1}{2\ln 5}5^{-x^2} + cst$

  $u = -x^2$
  $du = -2x\,dx \Leftrightarrow x\,dx = -\frac{1}{2}du$

- $L = \int \frac{dx}{x\sqrt{9x^2-1}} = \int \frac{dx}{x\sqrt{(3x)^2-1}} = \int \frac{du/3}{u/3\sqrt{u^2-1}}$
  $= \int \frac{du}{u\sqrt{u^2-1}} = \sec^{-1}(u) + cst$
  $= \sec^{-1}(3x) + cst.$

  $u = 3x \quad \Leftrightarrow \quad x = \frac{u}{3}$
  $du = 3\,dx \quad dx = \frac{du}{3}$

- $M = \int \frac{x^2}{4+x^6}dx = \int \frac{x^2dx}{2^2+(x^3)^2}$

  $u = x^3$
  $du = 3x^2dx \Leftrightarrow x^2dx = \frac{1}{3}du$

  $M = \frac{1}{3}\int \frac{du}{2^2+u^2} = \frac{1}{3}\cdot\frac{1}{2}\cdot\tan^{-1}\left(\frac{u}{2}\right) + cst$
  $= \frac{1}{6}\tan^{-1}\left(\frac{x^3}{2}\right) + cst.$

- $N = \int \frac{dx}{\sqrt{1+e^{2x}}} = \int \frac{dx}{\sqrt{1+(e^x)^2}} = \int \frac{du}{u\sqrt{1+u^2}}$
  $= -\operatorname{csch}^{-1}(u) + cst$
  $= -\operatorname{csch}^{-1}(e^x) + cst$

  $u = e^x$
  $du = e^x dx = u\,dx \Leftrightarrow dx = \frac{du}{u}$

$$P = \int \frac{dx}{x\sqrt{16-x^4}} = \int \frac{dx}{x\sqrt{4^2-(x^2)^2}}$$

$u = x^2$

$du = 2x\,dx = 2x^2 \frac{dx}{x} = 2u\frac{dx}{x}$

$$\frac{dx}{x} = \frac{du}{2u}$$

So $$P = \frac{1}{2}\int \frac{du}{u\sqrt{4^2-u^2}} = -\frac{1}{8}\,\mathrm{sech}^{-1}\left(\frac{u}{4}\right) + cst$$

$$= -\frac{1}{8}\,\mathrm{sech}^{-1}\left(\frac{x^2}{4}\right) + cst.$$

Ex2 :

① $$\lim_{x\to\frac{\pi}{2}} \frac{1-\sin x}{\cos x} = \frac{1-\sin(\pi/2)}{\cos(\pi/2)} = \frac{0}{0} \quad \text{I.F}$$

by Hospital rule $$= \lim_{x\to\pi/2} \frac{\frac{d}{dx}(1-\sin x)}{\frac{d}{dx}(\cos x)}$$

$$= \lim_{x\to\pi/2} \frac{-\cos x}{-\sin x} = \frac{0}{1} = 0$$

② $$\lim_{x\to\infty}\left(1+\frac{5}{x}\right)^{2x} = \lim_{x\to\infty}\left(1+\frac{5}{\infty}\right)^{2\infty} = 1^{\infty} \quad \text{I.F}$$

So $$\lim_{x\to+\infty} \ln\left[\left(1+\frac{5}{x}\right)^{2x}\right] = \lim_{x\to+\infty} 2x\ln(1+5/x)$$

$$= 2\lim_{x\to+\infty} \frac{\ln(1+5/x)}{1/x}$$

$t = 1/x$ $$= 2\lim_{t\to 0^+} \frac{\ln(1+5t)}{t} = \frac{0}{0}$$

Hospital rule $$= 2\lim_{t\to 0^+} \frac{5/1+5t}{1} = 10$$

We deduce that $$\lim_{x\to+\infty}\left(1+\frac{5}{x}\right)^{2x} = e^{10}.$$