

King Saud University  
Faculty of Sciences  
Department of Mathematics

Second Examination

Math 106

Semester I

1438-1439

Time: 1H30

Exercise 1 : (3+2+3)

a) Find  $\lim_{x \rightarrow 0} (\cos x)^{\csc^2 x}$ .

b) Compute  $\int x \sec^2 x dx$ .

c) Evaluate  $\int \frac{dx}{x\sqrt{9-x^4}}$ .

Exercise 2 : (3+3+2)

a) Compute  $\int \sec^6 x \tan^4 x dx$ .

b) Find  $\int \frac{x^2}{(4-x^2)^{\frac{3}{2}}} dx$ .

c) Find  $\int \frac{dx}{\sqrt{x^2+4x+3}}$ .

Exercise 3 : (3+3+3)

a) Compute  $\int \frac{x+1}{(x-2)(x^2-3x+2)} dx$ .

b) Does the integral  $\int_0^{+\infty} e^{-\sqrt{x}} dx$  converge? Find its value if possible.

c) Find the area of the region bounded by  $y = x^2$ ,  $y = x + 2$ ,  $x = 0$ ,  $x = 3$ .

## Grading scheme

① a)  $\ln y = \frac{\ln \cos x}{\sin^2 x} \quad (1)$

$$\lim_{x \rightarrow 0} \ln y = \lim_{x \rightarrow 0} \frac{-\sin x}{2 \cos^2 x \cdot \sin x} = -\frac{1}{2} \quad (1,5)$$

so  $\lim_{x \rightarrow 0} y = e^{-\frac{1}{2}} \quad (0.5)$

b)  $\int x \sec^2 x dx = x \tan x - \int \tan x dx \quad (1)$   
 $= x \tan x + \ln |\cos x| + C \quad (1)$

c)  $\int \frac{dx}{x \sqrt{9-x^4}} = \frac{1}{2} \int \frac{2x dx}{x^2 \sqrt{9-(x^2)^2}} \quad u = x^2 \quad (1)$   
 $= -\frac{1}{6} \operatorname{sech}^{-1} \left( \frac{x^2}{3} \right) + C \quad (2)$

$$(2) \quad a) \int \sec^6 x \tan^4 x \, dx = \int (1 + \tan^2 x)^2 \tan^4 x \sec^2 x \, dx$$

$$u = \tan x \quad = \int (1 + u^2)^2 u^4 \, du \quad (1.5)$$

$$= \int (u^4 + 2u^2 + 1) u^4 \, du$$

$$(1.5) \quad = \frac{\tan^9 x}{9} + \frac{2}{7} \tan^7 x + \frac{1}{5} \tan^5 x + C$$

$$b) \quad x = 2 \sin \theta \quad dx = 2 \cos \theta \, d\theta \quad (1)$$

$$\int \frac{x^2}{(4-x^2)^{3/2}} \, dx = \int \frac{4 \sin^2 \theta \cdot 2 \cos \theta \, d\theta}{8 \cos^3 \theta}$$

$$= \int \tan^2 \theta \, d\theta = \int (\sec^2 \theta - 1) \, d\theta \quad (1)$$

$$= \tan \theta - \theta + C$$

$$= \frac{x}{\sqrt{4-x^2}} - \sin^{-1} \left( \frac{x}{2} \right) + C \quad (1)$$

$$c) \int \frac{dx}{\sqrt{x^2 + 4x + 3}} = \int \frac{dx}{\sqrt{(x+2)^2 - 1}} \quad (1)$$

$$= \cosh^{-1}(x+2) + C \quad (1)$$

( $x+2 = \sec \theta$  is also ok)

3)

(3) a)

$$\frac{x+1}{(x-2)(x^2-3x+2)} = \frac{x+1}{(x-1)(x-2)^2}$$

$$(1.5) \quad = \frac{2}{x-1} - \frac{2}{x-2} + \frac{3}{(x-2)^2}$$

$$\int \frac{x+1}{(x-2)(x^2-3x+2)} dx = \underset{(1.5)}{2 \ln|x-1|} - 2 \ln|x-2| - \frac{3}{x-2} + C$$

b) Set  $u = \sqrt{x}$  (1)

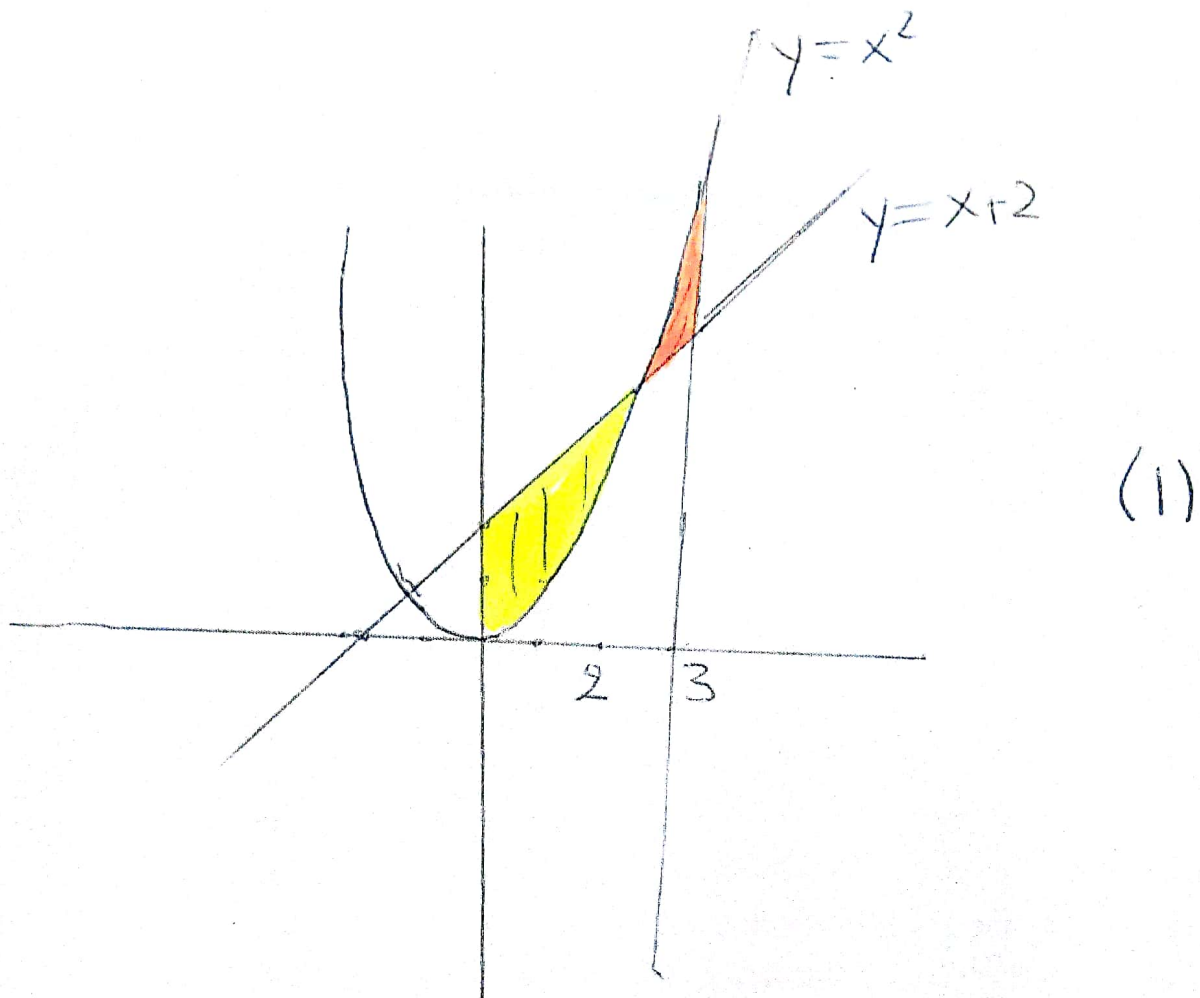
$$\int_0^c e^{-\sqrt{x}} dx = \int_0^{\sqrt{c}} 2u e^{-u} du$$

$$= 2 \left[ -u e^{-u} - e^{-u} \right]_0^{\sqrt{c}} \quad (1)$$

$$= 2 \left( -\sqrt{c} e^{-\sqrt{c}} - e^{-\sqrt{c}} + 1 \right) \xrightarrow{c \rightarrow \infty} 2$$

So  $\int_0^{+\infty} e^{-\sqrt{x}} dx$  converges and  $\int_0^{+\infty} e^{-\sqrt{x}} dx = 2$  (1)

4)



intersection points

$$x^2 = x + 2 \iff (x - 2)(x + 1) = 0$$

$$x = 2 \text{ or } x = -1 \quad (0.5)$$

$$A' = \int_0^2 (x + 2 - x^2) dx + \int_2^3 (x^2 - x - 2) dx$$

$$= \left[ \frac{x^2}{2} + 2x - \frac{x^3}{3} \right]_0^2 + \left[ \frac{x^3}{3} - \frac{x^2}{2} - 2x \right]_2^3$$

$$= \frac{10}{3} + \frac{11}{6} = \frac{31}{6} \quad (1.5)$$