كلية العلوم قسم الرياضيات.	الإمتحان الفصلي الثاني ٢٠٩ ريض الفصل الأول ١٤٤٠/١٤٣٩ هـ الزمن : ساعة ونصف.	جامعة الملك سعود King Saud University

السؤال الأول (٩) : أ) أوجد متسلسلة القوى في x للدالة : $f(x)=\dfrac{x^2}{8+x^3}$ ثم استنتج متسلسلة القوى في x للدالة : $g(x)=\ln\left(8+x^3\right)$ وماهي فترة تقاربها ؟

ب) أوجد متسلسلة ماكلورين للدالة : $f(x)=(1+x)^{\alpha}$ حيث α عدد حقيقي.

السؤال الثاني (١١) : أ) باستخدام المتسلسلة $\sin u=\sum_{n=0}^{\infty}(-1)^n\dfrac{u^{2n+1}}{(2n+1)!}$ لكل $u\in\mathbb{R}$ احسب القيمة التقريبية للتكامل : $\int_{1/2}^{1}\dfrac{\sin x}{x}dx$ وذلك بمكاملة الحدود الخمس الأولى في المتسلسلة .

ب) أوجد متسلسلة فورييه للدالة $f(x)=x^2$ على الفترة $[-\pi,\pi]$ ثم استنتج أنّ: $\sum_{n=1}^{\infty}\dfrac{1}{n^2}=\dfrac{\pi^2}{6}$ و $\sum_{n=1}^{\infty}\dfrac{(-1)^{n+1}}{n^2}=\dfrac{\pi^2}{12}$.

السؤال الثالث(٥): أوجد تكامل فورييه للدالة : $f(x)=\begin{cases}-2, & -1\le x\le 0\\ 1, & 0<x\le 1\\ 0, & |x|>1\end{cases}$ ، ثم استنتج أنّ:

$$\int_0^{+\infty}\frac{(3-4\cos\alpha)\sin\alpha}{\alpha}d\alpha=\frac{\pi}{2}.$$

(أ)

$$f(x) = \frac{x^2}{8+x^3}$$

$$f(x) = x^2 \frac{1}{8+x^3} = \frac{x^2}{8} \frac{1}{\left[1+\left(\frac{x}{2}\right)^3\right]}$$

نعلم $\frac{1}{1-u} = \sum_{n=0}^{\infty} u^n$ لكل $-1 < u < 1$

0,5

0,5

نضع $u = -\left(\frac{x}{2}\right)^3$

$$\frac{1}{1+\left(\frac{x}{2}\right)^3} = \frac{1}{1-\left[-\left(\frac{x}{2}\right)^3\right]} = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{3n}} x^{3n}$$

0,5

$$-1 < -\left(\frac{x}{2}\right)^3 < 1$$

$$-2 < x < 2$$

0,5

$$f(x) = \frac{x^2}{8+x^3} = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{3n+3}} x^{3n+2}$$

لكل $-2 < x < 2$

$$\ln(8+x^3) - \ln 8 = \int_0^x \frac{3t^2}{8+t^3} dt$$

0,5

باستعمال تكامل المتسلسلة

لكل $-2 < x < 2$

1

$$\int_0^x \frac{t^2}{8+t^3} dt = \int_0^x \left(\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{3n+3}} t^{3n+2}\right) dt$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{3n+3}} \left(\int_0^x t^{3n+2} dt\right)$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{3n+3}} \frac{\left[t^{3n+3}\right]_0^x}{3n+3}$$

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1 $(a+b)^n=\sum_{k=0}^{n}\frac{n!}{k!(n-k)!}a^k b^{n-k}$

$$(1+x)^n=\sum_{k=0}^{n}\frac{n!}{k!(n-k)!}x^k$$

$$(1+x)^n=\sum_{k=0}^{n}\frac{n(n-1)\cdots(n-k+1)}{k!}x^k$$

$$\boxed{(1+x)^\alpha=\sum_{n=0}^{\infty}\frac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!}x^n}$$

$$(1+x)^\alpha=\sum_{n=0}^{\infty}\frac{\alpha(\alpha-1)(\alpha-2)\cdots(\alpha-n+1)}{n!}x^n$$

[illegible] α جواب

1

$$f(x)=(1+x)^\alpha=\sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}x^n$$

1

$$=f(0)+f'(0)x+\frac{f''(0)}{2!}x^2+\cdots$$

$$f(0)=1$$

2

$$f'(x)=\alpha(1+x)^{\alpha-1};\quad f'(0)=\alpha$$

$$f''(x)=\alpha(\alpha-1)(1+x)^{\alpha-2};\quad f''(0)=\alpha(\alpha-1)$$

$$f^{(k)}(x)=\alpha(\alpha-1)(\alpha-2)\cdots(\alpha-k+1)(1+x)^{\alpha-k}$$

$$f^{(k)}(0)=\alpha(\alpha-1)(\alpha-2)\cdots(\alpha-k+1)$$

1,5

$$\frac{1}{3}\left(g(x)-\ln 8\right)=\sum_{n=0}^{\infty}\frac{(-1)^n}{2^{3n+3}(3n+3)}x^{3n+3}$$

$$\boxed{g(x)=\ln(8+x^3)=\ln 8+\sum_{n=0}^{\infty}\frac{(-1)^n}{(n+1)2^{3n+3}}x^{3n+3}}$$

$-2<x<2$ [illegible]

(ب) $f(x)=(1+x)^\alpha$

ماکلورن f را [illegible]

$$\frac{\pi^4}{90} = \sum_{n=0}^{\infty} \frac{1}{n^4}$$

نجد

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} x^2\,dx$$

1

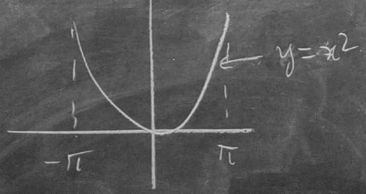
$$a_0 = \frac{2}{\pi}\left[\frac{x^3}{3}\right]_0^{\pi} = \frac{2\pi^2}{3}$$

لأجل $n \geq 1$

0,5

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \cos(nx)\,dx$$

(ب) $f(x) = x^2$ على $[-\pi, \pi]$



بما أن f متصلة على $[-\pi, \pi]$ و زوجية فهي تحصل بمفكوك فورييه لما $-\pi < x < \pi$

$$x^2 = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx)$$

1

لما $x \in \mathbb{R}$ $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$ 1

0,5

لأجل $x \neq 0$

1 $\frac{\sin x}{x} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots$

1 $\int_{1/2}^{1} \frac{\sin x}{x}\,dx = \int_{1/2}^{1} \left[1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots\right] dx$

$$\approx \left[x - \frac{x^3}{3.3!} + \frac{x^5}{5.5!}\right]_{1/2}^{1}$$

0,5

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} x^4\,dx$$

$$f = \frac{a_0}{2} + \sum_{1}^{\infty} a_n \cos \qquad u = a_1\vec{i} + b_1\vec{j}$$

$$\|f\|^2 = \sum_{n=0}^{\infty} a_n^2 \qquad \|u\|^2 = a_1^2 + b_1^2$$

Plancherel - Fourier

$-\pi \le x \le \pi$ در

$$\boxed{x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}\cos(nx)}$$

0.5

$x = 0$ [illegible]

$$0 = \frac{\pi^2}{3} + 4\left(\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}\right)$$

0.5

$$-\frac{\pi^2}{3} = 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$$

$$\int_0^{\pi} x\sin(nx)\,dx = -\frac{1}{n}\Big[x\cos(nx)\Big]_0^{\pi} + \frac{1}{n}\underbrace{\int_0^{\pi}\cos(nx)\,dx}_{0}$$

$$= -\frac{1}{n}\Big[\pi(-1)^n\Big]$$

$$= \frac{\pi(-1)^{n+1}}{n}$$

$$J_n = \frac{2(-1)^n\pi}{n^2}$$

1

$$\boxed{a_n = \frac{4(-1)^n}{n^2}}$$

$$a_n = \frac{2}{\pi}\underbrace{\int_0^{\pi} x^2\cos(nx)\,dx}_{J_n}$$

0.5

$$u = x^2 \Rightarrow u'(x) = 2x$$

$$v' = \cos(nx) \quad V(x) = \frac{\sin(nx)}{n}$$

$$J_n = \int_0^{\pi} x^2\cos(nx)\,dx = \frac{1}{n}\underbrace{\Big[x^2\sin(nx)\Big]_0^{\pi}}_{=0} - \frac{2}{n}\int_0^{\pi} x\sin(n$$

$$J_n = -\frac{2}{n}\int_0^{\pi} x\sin(nx)\,dx.$$

0.5

$$u_1(x) = x \Rightarrow u_1' = 1$$

$$v_1'(x) = \sin(nx) \quad v_1(x) = -\frac{\cos(nx)}{n}.$$

$$\frac{\pi^4}{90} = \sum_{n=0}^{\infty} \frac{1}{n^4}$$

لأي $x \in D$

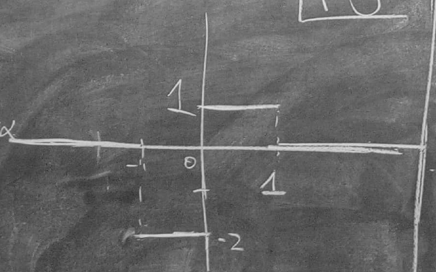
$$f(x) = \frac{1}{\pi}\int_0^{+\infty} A(\alpha)\cos(\alpha x) + B(\alpha)\sin(\alpha x)\, d\alpha$$

(1)

$$A(\alpha) = \int_{-\infty}^{+\infty} f(x)\cos(\alpha x)\, dx$$

$$B(\alpha) = \int_{-\infty}^{+\infty} f(x)\sin(\alpha x)\, dx$$

س ٣



f متصلة على $D = \mathbb{R}\setminus\{-1, 0, 1\}$

f تحقق شروط تكامل فورييه

بوضع $x = \pi$;

$$\pi^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}\underbrace{\cos(n\pi)}_{(-1)^n}$$

$$\pi^2 - \frac{\pi^2}{3} = 4\sum_{n=1}^{\infty}\frac{1}{n^2}$$

(15)

$$\frac{2}{3}\pi^2 = 4\left(\sum_{n=1}^{\infty}\frac{1}{n^2}\right)$$

$$\frac{\pi^2}{6} = \sum_{n=1}^{\infty}\frac{1}{n^2}$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} x^4 \, dx$$

$$f = \frac{a_0}{2} + \sum_{1}^{\infty} a_n \cdots$$

$$\|f\|^2 = \sum_{n=0} a_n^2 \quad \text{Plancherel - Fourier}$$

$$u = a_1 \vec{i} + b_1 \vec{j}$$

$$\|u\|^2 = a_1^2 + b_1^2$$

$$B(\alpha) = -2\int_{-1}^{0} \sin(\alpha x)\,dx + \int_{0}^{1} \sin(\alpha x)\,dx$$

$$= +2\left[\frac{\cos(\alpha x)}{\alpha}\right]_{-1}^{0} - \left[\frac{\cos(\alpha x)}{\alpha}\right]_{0}^{1}$$

$$= \frac{2}{\alpha}\left[1 - \cos\alpha\right] - \frac{1}{\alpha}\left[\cos\alpha - 1\right]$$

$$\boxed{B(\alpha) = \frac{3}{\alpha}(1-\cos\alpha)}$$

1,5

$$A(\alpha) = \int_{-1}^{0} -2\cos(\alpha x)\,dx + \int_{0}^{1} 1\cos(\alpha x)\,dx$$

$$= -2\left[\frac{\sin(\alpha x)}{\alpha}\right]_{-1}^{0} + \left[\frac{\sin(\alpha x)}{\alpha}\right]_{0}^{1}$$

$$= -2\frac{\sin\alpha}{\alpha} + \frac{\sin\alpha}{\alpha}$$

$$\boxed{A(\alpha) = -\frac{\sin\alpha}{\alpha}}$$

1,5

$$f(x) = \frac{1}{\pi}\int_{0}^{+\infty}\left[\left(-\frac{\sin\alpha}{\alpha}\right)\cos(\alpha x) + \frac{3}{\alpha}(1-\cos\alpha)\sin\alpha x\right]d\alpha$$

1

عند ما $x = 1$;

$$\frac{1+0}{2} = \frac{1}{\pi}\int_{0}^{+\infty}\left[-\frac{\cos\alpha\sin\alpha}{\alpha} + \frac{3}{\alpha}(\sin\alpha - \cos\alpha\sin\alpha)\right]d\alpha$$

$$\frac{\pi}{2} = \int_{0}^{+\infty}\frac{(3-4\cos\alpha)\sin\alpha}{\alpha}\,d\alpha$$