Solution M 107 ***(SEMESTER I, 1439 -1440) FIRST MID-TERM*** **Total Marks :50**

Question: 1(a) Use Gauss - Jordan method to solve the system of linear equations

$$3x + y - z = 1$$
$$x - y + z = -3 \qquad [10]$$
$$2x + y + z = 0$$

(b) Find the equation of circle $x^2 + y^2 + ax + by + c = 0$ **whose graph passes through the points** $(-1,3)$, $(2,2)$ **and** $(3,1)$.

[10]

Solution (a) Augmented matrix

$$[A \mid b] \equiv \left[\begin{array}{ccc|c} 3 & 1 & -1 & 1 \\ 1 & -1 & 1 & 3 \\ 2 & 1 & 1 & 0 \end{array}\right] \equiv \left[\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 0 & 4 & -4 & 10 \\ 0 & 3 & -1 & 6 \end{array}\right]$$

$$\equiv \left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & -1 & \frac{5}{2} \\ 0 & 3 & -1 & 6 \end{array}\right] \equiv \left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 & 7/4 \\ 0 & 0 & 1 & -3/4 \end{array}\right] \qquad 10$$

$x = -\frac{1}{2}$, $y = \frac{7}{4}$, $z = -\frac{3}{4}$

(b)

$$-a + 3b + c = -10$$
$$2a + 2b + c = -8$$
$$3a + b + c = -10$$

Augmented matrix is

$$\left[\begin{array}{ccc|c} -1 & 3 & 1 & -10 \\ 2 & 2 & 1 & -8 \\ 3 & 1 & 1 & -10 \end{array}\right] \equiv \left[\begin{array}{ccc|c} -1 & 3 & 1 & -10 \\ 0 & 8 & 3 & -28 \\ 0 & 10 & 4 & -40 \end{array}\right]$$

$$\equiv \left[\begin{array}{ccc|c} 1 & -3 & -1 & -10 \\ 0 & 1 & \frac{3}{8} & -\frac{28}{8} \\ 0 & 10 & 4 & -40 \end{array}\right] \equiv \left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{8} & -\frac{1}{2} \\ 0 & 1 & \frac{3}{8} & -\frac{28}{8} \\ 0 & 0 & \frac{1}{4} & -5 \end{array}\right] \qquad 8$$

$$\equiv \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -20 \end{array}\right]$$

$a = 2$, $b = 4$, $c = -20$ 2

Equation of circle is $x^2 + y^2 + 2x + 4y - 20 = 0$

Question:2 . (a) Find the matrix A so that

$$(AB)^{-1} = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}, \text{where } B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$ **[6]**

(b) Find $\det(A^{-1})$ **for the matrix**

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & -1 & 2 \\ 2 & 1 & 0 \end{bmatrix}$$ **[6]**

Solution: (a) $(AB)^{-1} = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$

Taking inverse of both sides.

$AB = \frac{1}{6}\begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$ (2)

$B^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$ (2)

$A = AB\,B^{-1} = \frac{1}{6}\begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 3 & 0 \\ -3 & 2 \end{bmatrix}$ (2)

(b) $\det A^{-1} = \frac{1}{\det A}$ (2)

$\det A = \begin{vmatrix} 1 & 0 & 3 \\ 0 & -1 & 2 \\ 2 & 1 & 0 \end{vmatrix} = 1(0-2) - 0 + 3(0+2)$

$= -2 + 6 = 4.$ (3)

$\det A^{-1} = \frac{1}{4}$ (1)

Question: 3 . (a)) Use row reduction to show that

$$\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z)$$

[10]

(b) Solve the system of linear equations by using Crammer's Rule

$$x + y = -1$$
$$2x - z = 3$$
$$y + 2z = -1$$

[8]

Solution. (a) Taking Transpose

$$\begin{vmatrix} 1 & x & x^3 \\ 1 & y & y^3 \\ 1 & z & z^3 \end{vmatrix} = \begin{vmatrix} 1 & x & x^3 \\ 0 & y-x & y^3-x^3 \\ 0 & z-x & z^3-x^3 \end{vmatrix}$$

$$= (y-x)(z-x) \begin{vmatrix} 1 & x & x^3 \\ 0 & 1 & x^2+xy+y^2 \\ 0 & 1 & x^2+xz+z^2 \end{vmatrix}$$

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$$= (y-x)(z-x) \begin{vmatrix} 1 & x & x^3 \\ 0 & 1 & x^2+xy+y^2 \\ 0 & 0 & x(y-z)+y^2-z^2 \end{vmatrix}$$

$$= (y-x)(z-x) \begin{vmatrix} 1 & x & x \\ 0 & 1 & x^2+xy+y^2 \\ 0 & 0 & (y-z)(x+y+z) \end{vmatrix}$$

$$= (y-x)(z-x)(y-z)(x+y+z)$$

(b)
$$\begin{bmatrix} 1 & 1 & 0 \\ 2 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}$$

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$\det A = \begin{vmatrix} 1 & 1 & 0 \\ 2 & 0 & -1 \\ 0 & 1 & 2 \end{vmatrix} = -3$, $\det A_1 = \begin{vmatrix} -1 & 1 & 0 \\ 3 & 0 & -1 \\ 1 & 1 & 2 \end{vmatrix} = -6$, $\det A_2 = \begin{vmatrix} 1 & -1 & 0 \\ 2 & 3 & -1 \\ 0 & 1 & 2 \end{vmatrix} = 9$

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$\det A_3 = \begin{vmatrix} 1 & 1 & -1 \\ 2 & 0 & 3 \\ 0 & 1 & 1 \end{vmatrix} = -3$

$x = \frac{-6}{-3} = 2$

$y = \frac{9}{-3} = -3$

$z = \frac{-3}{-3} = +1$