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King Saud University

First Midterm Exam Math151
(Discrete Mathematics)
Spring Semester 2018-2019
Name:..
ID:..

Q1: Complete the following truth table (2marks)

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

Q2: Using logic laws, show that $[\neg p \wedge (p \vee q)] \rightarrow q$ is a tautology. (2 marks)

$[\neg p \wedge (p \vee q)] \rightarrow q \equiv \neg[\neg p \wedge (p \vee q)] \vee q$

$\equiv p \vee \neg (p \vee q) \vee q$

$\equiv (p \vee q) \vee \neg (p \vee q) \equiv A \vee \neg A \equiv T$

Q3: Determine the truth value of each of these statements and justify your answer. (2 marks)

(i) $\forall x \in \mathbb{R}, \quad x^2 - 5x + 6 \geq 0.$

False, $x^2 - 5x + 6 = (x-2)(x-3)$; take $x \in (2,3)$

(ii) $\exists x \in \mathbb{R}, \quad x^4 < x^2.$

True, $x^4 < x^2 \Leftrightarrow x^2(x^2 - 1) < 0$ take $x \in (0,1)$

Q4: Use a proof by contraposition to show that if (xy) is an even number where $x, y \in \mathbb{Z}$ then x is even or y is even. (2 marks)

We assume that x and y are odd Then

$x = 2k+1$ and $y = 2L+1$ with $k, L \in \mathbb{Z}$

So $xy = (2k+1)(2L+1) = 4kL + 2k + 2L + 1$

$= 2(\underbrace{2kL + k + L}_{M}) + 1$

$= 2 \quad M \quad + 1$

Then (xy) is odd.

Q5: Let $x, y, z \in \mathbb{R}$ such $x + y + z = 21$, use a proof by contradiction to show that $x \geq 8$ or $y \geq 7$ or $z \geq 6$. (2 marks)

Assume that $x < 8$ and $y < 7$ and $z < 6$

Then $x + y + z < 8 + 7 + 6 = 21$

So $x + y + z \neq 21$ ↯ with $x + y + z = 21$

Q6: Let $\{a_n\}_{n \geq 0}$ be a sequence defined as: $\begin{cases} a_0 = 2, \quad a_1 = 4 \\ a_n = 4a_{n-1} - 3a_{n-2}, \text{ for } n \geq 2 \end{cases}$.

Show that $a_n = 1 + 3^n$ for all integers $n \geq 0$. (3 marks)

Put $P(n)$: $a_n = 1 + 3^n$

Basis step: $n = 0$: $2 = a_0 \stackrel{?}{=} 1 + 3^0 = 2$ | $n = 1$: $4 = a_1 \stackrel{?}{=} 1 + 3^1 = 4$

So $P(0)$, $P(1)$ are true

Inductive step: let $k \geq 2$, we suppose that $P(2), \ldots, P(k)$ are all true, Now we prove that $P(k+1)$ remains true. $a_{k+1} \stackrel{?}{=} 1 + 3^{k+1}$

as $a_{k+1} = 4a_k - 3a_{k-1}$

$= 4(1 + 3^k) - 3(1 + 3^{k-1})$ because $P(k)$ and $P(k-1)$ are true.

$= 4 + 4 \cdot 3^k - 3 - 3^k = 1 + 3 \cdot 3^k = 1 + 3^{k+1}$

we deduce that $a_n = 1 + 3^n$ for $n \geq 0$.

Q7: Let R be a relation defined on $A = \{-2, -1, 0, 1, 2, 3, 4\}$ as: for $a, b \in A$, $a \, R \, b \Leftrightarrow a^2 = b$.

(i) Write R as a set of ordered pairs. (1.5 marks)

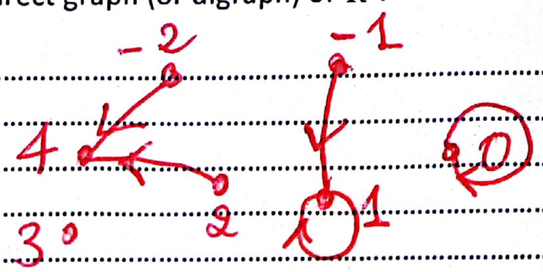
$R = \{(-2, 4); (-1, 1); (0, 0); (1, 1); (2, 4)\}$

(ii) Determine the domain and the image of R. (1 mark)

domain R: $D_R = \{-2, -1, 0, 1, 2\}$

Range R: $\text{Im } R = \{4, 1, 0\}$

(iii) Draw the direct graph (or digraph) of R. (1 mark)



Q8: Let S= $\{(1,1),(1,2),(3,1),(3,3)\}$ be a relation defined on $B=\{1,2,3\}$.

(i) Find M_S (the matrix of S). (1 mark)

$$M_S = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

(i) Find $S - S^{-1}$. (1 mark)

$S - S^{-1} = \{(1,1); (1,2); (3,1); (3,3)\} - \{(1,1); (2,1); (1,3), (3,$

$= \{(1,2); (3,1)\}$

(ii) Find $S \circ S^{-1}$. (1.5 marks)

$S^{-1} = \{(1,1); (2,1); (1,3); (3,3)\}$

$S \circ S^{-1} = \{(1,1), (1,2); (2,1); (2,2); (1,3); (3,1); (3,3)\}$