King Saud University

Faculty of Sciences

Department of Mathematics

Final Examination	Math 106	Semester I	1438-1439	Time: 3H

Exercise 1 : (2+3)

a) If $F(x) = \int_{2x}^{x^2} t \ln t dt$, $x > 0$, find $F'(x)$.

b) Approximate $\int_0^5 \frac{dx}{\sqrt{1+x^4}}$ using the trapezoidal rule with $n = 5$.

Exercise 2 : (3+3+2)

a) Evaluate $\int x 3^{2x^2} (3^{2x^2} + 1)^{-4} dx$.

b) Find $\int \frac{(\log_2 x)^2 + \sqrt{x}}{x} dx$.

c) Compute $\int \frac{dx}{\sqrt{e^{4x} - 36}}$.

Exercise 3 : (3+3+3)

a) Find $\int \ln(x^2+1) dx$.

b) Evaluate $\int \frac{dx}{x^3\sqrt{x^2-1}}$.

c) Compute $f'(x)$ if $f(x) = x^2(x^2+1)^{(x^3+1)}$.

Exercise 4 : (3+3+3)

a) Find $\int \frac{3x-1}{x^2+4x+8} dx$.

b) Find the area of the region bounded by the curves $x = y^2$, $x + y = 6$
$y = -4$, $y = 2$.

c) Set up integrals for the volume obtained by revolving the region bounded by
$y = 2x^2$, $y = 8x$ about the lines
(i) $x = 5$,
(ii) $y = -1$.

Exercise 5 : (3+3+3)

a) Find the area of the surface obtained by revolving the curve $x = 2y^3$,
$0 \leq y \leq 1$, about the $y-$ axis .

b) Find the area of the region inside the polar curve $r = 1$ and outside
$r = 1 - \cos\theta$.

c) Compute the arc length of the curve $r = 1 + \cos\theta$.

Exercise 1 : (5 points)

a) $F'(x) = 4x^3\ln(x) - 4x\ln(2x)$, (1) + (1)

b) $\int_0^5 \frac{dx}{\sqrt{1+x^4}} \approx \frac{1}{2}(3.284871) \approx 1.642435$

k	x_k	$f(x_k)$	m	$mf(x_k)$
0	0	1	1	1
1	1	$\frac{1}{\sqrt{2}}$	2	$\sqrt{2} \approx 1.414213$
2	2	$\frac{1}{\sqrt{17}}$	2	$\frac{2}{\sqrt{17}} \approx 0.485071$
3	3	$\frac{1}{\sqrt{82}}$	2	$\frac{2}{\sqrt{82}} \approx 0.220863$
4	4	$\frac{1}{\sqrt{257}}$	2	$\frac{2}{\sqrt{257}} \approx 0.124756$
5	5	$\frac{1}{\sqrt{626}}$	1	$\frac{1}{\sqrt{626}} \approx 0.039968$
				3.284871

(1) correct formula
(1.5) correct numbers
(0.5) final answer

Exercise 2 : (8 pts)

a)

$$\int x3^{2x^2}(3^{2x^2}+1)^{-4}dx \overset{u=3^{2x^2}+1)}{=} \frac{1}{4\ln 3}\int u^{-4}du \quad (2)$$
$$= -\frac{1}{12\ln 3}u^{-3}+c$$
$$= -\frac{1}{12\ln 3}(3^{2x^2}+1)^{-3}+c. \quad (1)$$

b)

$$\int \frac{(\log_2 x)^2+\sqrt{x}}{x}dx = \int \frac{\ln^2(x)}{x\ln^2 2}dx + \int x^{-\frac{1}{2}}dx \quad (1)$$
$$= \frac{\ln^3(x)}{3\ln^2 2} + 2\sqrt{x} + c. \quad (1)+(1)$$

c) $\int \frac{dx}{\sqrt{e^{4x}-36}} \overset{6u=e^{2x}}{=} \frac{1}{12}\int \frac{du}{u\sqrt{u^2-1}} = \frac{1}{12}\sec^{-1}(\frac{e^{2x}}{6}) + c.$ (1) + (1)

Exercise 3 : (9 pts)

a) By parts

$$\int \ln(x^2+1)dx \overset{u=\ln(x^2+1), v'=1}{=} x\ln(x^2+1) - \int \frac{2x^2}{x^2+1}dx \quad (1.5)$$
$$= x\ln(x^2+1) - 2x + 2\tan^{-1}(x) + c. \quad (1.5)$$

b)

$$\int \frac{dx}{x^3\sqrt{x^2-1}} \overset{x=\sec(\theta)}{=} \int \frac{\sec(\theta)\tan(\theta)}{\sec^3(\theta)\tan(\theta)}d\theta = \int \cos^2(\theta)d\theta \quad (1)$$
$$= \frac{\theta}{2} + \frac{1}{2}\sin(\theta)\cos(\theta) + c \quad (1)$$
$$= \frac{1}{2}\sec^{-1}(x) + \frac{\sqrt{x^2-1}}{2x^2} + c. \quad (1)$$

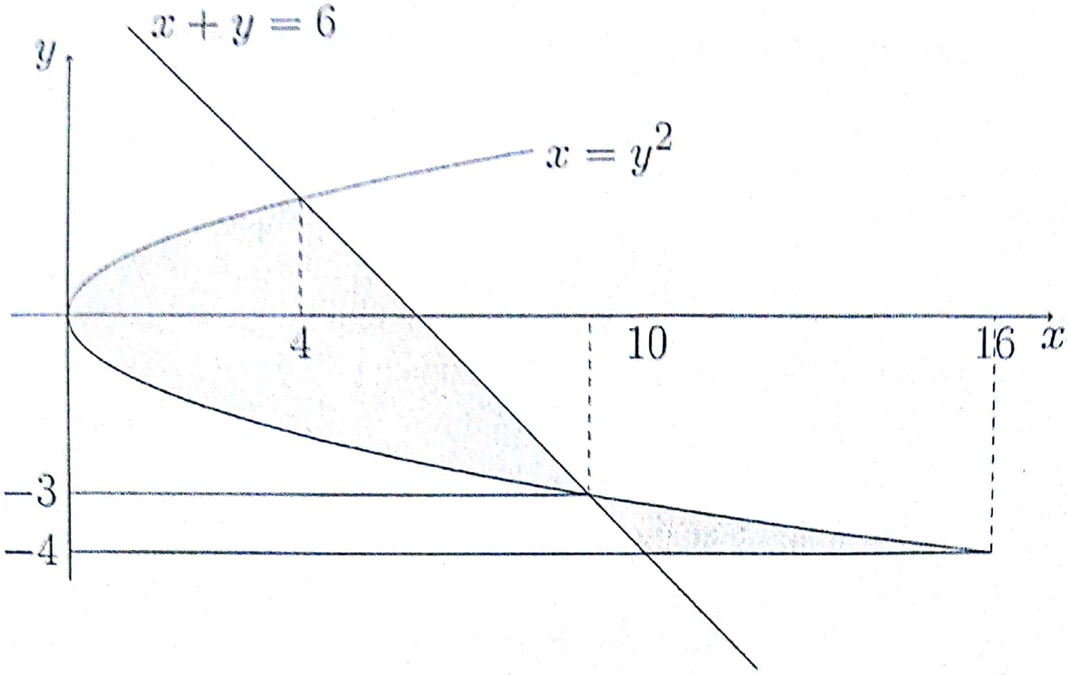
c) $$\frac{f'(x)}{f(x)} = \frac{2}{x} + 3x^2\ln(x^2+1) + \frac{2x^4+2x}{x^2+1} \quad (2)$$
$$f'(x) = \left(2x + 3x^4\ln(x^2+1) + \frac{2x^3(x^3+1)}{x^2+1}\right)(x^2+1)^{(x^3+1)}. \quad (1)$$

Exercise 4 : (9 pts)

a)

$$\int \frac{3x-1}{x^2+4x+8}dx = \frac{1}{2}\int \frac{3(2x+4)-14}{x^2+4x+8}dx \quad (1)$$
$$= \frac{3}{2}\ln|x^2+4x+8| - 7\int \frac{1}{(x+2)^2+4}dx \quad (1)$$
$$= \frac{3}{2}\ln|x^2+4x+8| - \frac{7}{2}\tan^{-1}(\frac{x+2}{2}) + c. \quad (1)$$

b) The area of the region bounded by the curves $x = y^2$, $x+y=6$, $y=-4$, $y=2$ is
$A = \int_{-3}^{2}\left(6-y-y^2\right)dy + \int_{-4}^{-3}(y^2-6+y)dy = 23 + \frac{2}{3}.$ (2)



(1)

c) $2x^2 = 8x \iff x(x-4) = 0$ (0.5)

(i) $(x = 5)$, $V = 2\pi \int_0^4 (5-x)(8x-2x^2)dx,$ (1)

(ii) $(y = -1)$, $V = \pi \int_0^4 \left((1+8x)^2 - (1+2x^2)^2\right) dx.$ (1.5)

Exercise 5 : (9 pts)

a)

$$\begin{aligned} SA &= 2\pi \int_0^1 2y^3\sqrt{1+36y^4}dy \quad (1) \\ &\overset{u=1+36y^4}{=} \frac{\pi}{36}\int_1^{37} \sqrt{u}du \quad (1) \\ &= \frac{\pi}{54}(\sqrt{37}-1). \quad (1) \end{aligned}$$

b)

$$\begin{aligned} A &= \frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1-(1-\cos(\theta))^2 d\theta \quad (1) \\ &= \frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(2\cos(\theta) - \frac{1}{2} - \frac{1}{2}\cos(2\theta)\right) d\theta = 2 - \frac{\pi}{2}. \quad (2) \end{aligned}$$

c)

$$\begin{aligned} L &= \int_{-\pi}^{\pi} \sqrt{(1+\cos\theta)^2 + \sin^2(\theta)d\theta} \quad (1) \\ &= \int_{-\pi}^{\pi} 2\cos(\frac{\theta}{2})d\theta = 4\sin(\frac{\theta}{2})]_{-\pi}^{\pi} = 8. \quad (2) \end{aligned}$$