



ممنوع استخدام الآلة الحاسبة

ملاحظات :

السؤال الأول (4 درجات): احسب $\frac{dy}{dx}$ فيما يلي :

(درجتان)

$$y = \sinh(\ln(x)) + \operatorname{sech}^{-1} x \quad (1)$$

(درجتان)

$$y = x^3 \tanh^2 \sqrt{x} \quad (2)$$

السؤال الثاني (21 درجة): احسب التكاملات التالية :

(درجتان)

$$\int \frac{\cosh(x)}{e^{2x}} dx \quad (1)$$

(درجتان)

$$\int \frac{dx}{e^{-x} \sqrt{e^{2x} - 1}} \quad (2)$$

(درجتان)

$$\int_1^e x^6 \ln x dx \quad (3)$$

(درجتان)

$$\int \sin^3 x \cos^5 x dx \quad (4)$$

(درجتان)

$$\int \frac{dx}{x^2 + 2x - 3} \quad (5)$$

(3 درجات)

$$\int \tan^5(x) \sec^4(x) dx \quad (6)$$

(3 درجات)

$$\int \sqrt{4 - x^2} dx \quad (7)$$

(3 درجات)

$$\int \frac{x+4}{x^3+4x} dx \quad (8)$$

(درجتان)

$$\int \frac{dx}{\sqrt{1+\sqrt{x}}} \quad (9)$$

السؤال الأول (4 درجات)

$$y = \sinh(\ln x) + \operatorname{sech}^{-1} x \quad (1)$$

$$\frac{dy}{dx} = \frac{1}{x} \cosh(\ln x) - \frac{1}{x\sqrt{1-x^2}} \quad (1)$$

$$y = x^3 \tanh^2(\sqrt{x}) \quad (2)$$

$$\frac{dy}{dx} = 3x^2 \tanh^2(\sqrt{x}) + \frac{2x^3}{2\sqrt{x}} \tanh(\sqrt{x}) \operatorname{sech}^2(\sqrt{x}) \quad (1)$$

السؤال الثاني: (12 درجة)

$$\int \frac{\cosh(x)}{e^{2x}} dx = \frac{1}{2} \int e^{-2x} (e^x + e^{-x}) dx \quad (1)$$

$$= \frac{1}{2} \int [e^{-x} + e^{-3x}] dx \quad (1)$$

$$= \frac{1}{2} [-e^{-x} - \frac{1}{3} e^{-3x}] + C \quad (1)$$

$u = e^x$ جف

(1/2)

$$\int \frac{du}{e^{-x} \sqrt{e^{2x}-1}} = \int \frac{e^x}{\sqrt{(e^x)^2-1}} dx \quad (1/2) \quad (2)$$

$$= \int \frac{du}{\sqrt{u^2-1}} = \cosh^{-1}(u) + C$$

$$= \cosh^{-1}(e^x) + C \quad (1)$$

$$I = \int_1^e x^6 \ln x dx = \left[\frac{x^7}{7} \ln x \right]_1^e - \int_1^e \frac{x^6}{7} dx \quad (3)$$

(1.5)

$$u(x) = \ln x \Rightarrow u'(x) = 1/x$$

$$v'(x) = x^6 \Rightarrow v(x) = \frac{x^7}{7}$$

(0.5)

$$I = \frac{e^7}{7} - \frac{1}{49} [x^7]_1^e = \frac{e^7}{7} - \frac{1}{49} [e^7 - 1]$$

$$\int \sin^3 x \cos^5 x \, dx = \int \sin^2 x \cos^5 x \sin x \, dx \quad (4)$$

(1/2) $u = \cos x$ & $du = -\sin x \, dx$

$$\begin{aligned} &= \int (1 - \cos^2 x) \cos^5 x \sin x \, dx \\ &= -\int (1 - u^2) u^5 \, du \\ &= -\int [u^7 - u^5] \, du \\ &= -\left[\frac{u^8}{8} - \frac{u^6}{6} \right] + C = \frac{\cos^6 x}{6} - \frac{\cos^8 x}{8} + C \end{aligned}$$

$$\int \frac{dx}{x^2 + 2x - 3} = \int \frac{dx}{x^2 + 2x + 1 - 4} \quad (5)$$

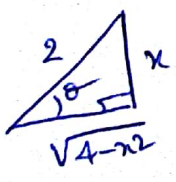
$$\begin{aligned} &= \int \frac{dx}{(x+1)^2 - 2^2} \\ &= -\int \frac{dx}{2^2 - (x+1)^2} = -\frac{1}{2} \tanh^{-1}\left(\frac{x+1}{2}\right) + C \end{aligned}$$

$$\int \tan^5 x \sec^4(x) \, dx = \int \tan^5 x \sec^2 x \sec^2 x \, dx \quad (6)$$

$u = \tan x$
 $du = \sec^2 x \, dx$ (0.5)

$$\begin{aligned} &= \int \tan^5 x (1 + \tan^2 x) \sec^2 x \, dx \\ &= \int u^5 (1 + u^2) \, du \\ &= \int [u^5 + u^7] \, du \\ &= \frac{u^6}{6} + \frac{u^8}{8} + C \\ &= \frac{\tan^6 x}{6} + \frac{\tan^8 x}{8} + C \end{aligned}$$

$$\int \sqrt{4-x^2} \, dx = \int 2^2 \cos^2 \theta \, d\theta = 4 \int \frac{1 + \cos 2\theta}{2} \, d\theta \quad (7)$$



$$\begin{aligned} x &= 2 \sin \theta \\ dx &= 2 \cos \theta \, d\theta \\ \sqrt{4-x^2} &= 2 \cos \theta \\ \sin(2\theta) &= 2 \cos \theta \sin \theta \\ &= \frac{2x\sqrt{4-x^2}}{4} = \frac{x\sqrt{4-x^2}}{2} \end{aligned}$$

$$\begin{aligned} &= 2 \left[\theta + \frac{\sin 2\theta}{2} \right] + C \\ &= 2 \left[\sin^{-1}\left(\frac{x}{2}\right) + \frac{x\sqrt{4-x^2}}{4} \right] + C \end{aligned}$$

$$\int \frac{x+4}{x^3+4x} dx = \int \frac{x+4}{x(x^2+4)} dx \quad (8)$$

$$= \int \left[\frac{A}{x} + \frac{Bx+C}{x^2+4} \right] dx \quad (0,5)$$

$$= A \ln|x| + \frac{B}{2} \int \frac{2x}{x^2+4} dx + C \int \frac{dx}{x^2+4}$$

$$= A \ln|x| + \frac{B}{2} \ln(x^2+4) + \frac{C}{2} \tan^{-1}\left(\frac{x}{2}\right) + \text{const}$$

∴ C و B, A استوابت \rightarrow

$$f(x) = \frac{x+4}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$$

$$A = \lim_{x \rightarrow 0} x f(x) = \lim_{x \rightarrow 0} \frac{x+4}{x^2+4} = 1$$

$$\lim_{x \rightarrow \infty} x f(x) = A + B = 0 \Rightarrow B = -1$$

$$f(1) = \frac{5}{5} = A + \frac{B+C}{4} = 1 + \frac{B+C}{4}$$

$$C = 1$$

$$\int \frac{dx}{\sqrt{1+\sqrt{x}}} = \int \frac{4u(u^2-1)du}{4u}$$

$$= 4 \int (u^2-1) du$$

$$= 4 \left[\frac{u^3}{3} - u \right] + C$$

$$= \frac{4}{3} (1+\sqrt{x})^{3/2} - 4\sqrt{1+\sqrt{x}} + \text{const} \quad (0,5)$$

$$u = \sqrt{1+\sqrt{x}}$$

$$u^2 = 1+\sqrt{x}$$

$$u^2-1 = \sqrt{x}$$

$$(u^2-1)^2 = x$$

$$2(u^2-1) 2u du = dx$$

(1,5)

(9)