ملاحظة : ممنوع استخدام الالة الحاسبة.

السؤال الأول (8 درجات):

(1) استخدم مجموع ريمان لحساب التكامل المحدد $\int_1^3 (1-2x)\,dx$. (3 درجات)

(2) أوجد قيمة c التي تحقق نظرية القيمة المتوسطة للدالة $f(x)=4x-x^2$ على الفترة $[0,3]$ (3 درجات)

(3) جد $F'(x)$ إذا كانت $F(x)=\int_{\ln x}^{\sqrt{x}} \frac{\sin t}{\sqrt{t^2-1}}dt$. (درجتان)

السؤال الثاني (5 درجات): احسب $\frac{dy}{dx}$ فيما يلي :

(1) $y=x^2\sin^{-1}\left(e^x\right)$ (درجتان)

(2) $y=\ln\left(\frac{e^{3x}\tan(x^2)}{\sqrt[5]{x}}\right)$ (3 درجات)

السؤال الثالث (12 درجة): احسب التكاملات التالية :

(1) $\int \frac{x^3-1}{\sqrt[3]{x}}\,dx$ (درجتان)

(2) $\int x^3\left(2-x^4\right)^7 dx$ (درجتان)

(3) $\int 2x\sec^2\left(x^2\right)dx$ (درجتان)

(4) $\int \frac{x-1}{x+1}\,dx$ (درجتان)

(5) $\int \frac{e^{6\ln x}}{x^6}\,dx$ (درجتان)

(6) $\int x^2\, 4^{x^3-1}\,dx$ (درجتان)

ثانياً (9 درجات)

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \left(\frac{b-a}{n}\right) \sum_{k=1}^{n} f(x_k) \quad 1$$

$$x_k = a + k\left(\frac{b-a}{n}\right)$$

$a = 1$ 1 $f(x) = 1 - 2x$

$b = 3$

$$\Delta x = \frac{b-a}{n} = \frac{2}{n}, \quad x_k = 1 + \frac{2k}{n}$$

$$\sum_{k=1}^{n} f(x_k) = \sum_{k=1}^{n} f\left(1 + \frac{2k}{n}\right) = \sum_{k=1}^{n} \left[1 - 2\left(1 + \frac{2k}{n}\right)\right]$$

1 $$= \sum_{k=1}^{n} \left(-1 - \frac{4}{n}k\right) = \left(\sum_{k=1}^{n} (-1)\right) - \frac{4}{n}\left(\sum_{k=1}^{n} k\right)$$

$$= -(n+1) - \frac{4}{n}\left(\frac{n(n+1)}{2}\right)$$

$$= -3(n+1)$$

$$\int_1^3 (1-2x)\,dx = \lim_{n\to\infty} \frac{2}{n}\left(-3(n+1)\right)$$

1 $$= -6\left(\lim_{n\to\infty} \frac{n+1}{n}\right) = -6$$

Check

$$\int_1^3 (1-2x)\,dx = \left[x - x^2\right]_1^3$$

$$= (3-9) - (1-1)$$

$$= -6 - 0$$

$$= -6$$

2 $f(x) = 4x - x^2$ متصلة على $[0,3]$ وباستخدام [illegible] x

$C \in (0,3)$ توجد

1 $$\int_0^3 (4x - x^2)\,dx = (3-0) f(c)$$

1 $$\left[2x^2 - \frac{x^3}{3}\right]_0^3 = 3(4c - c^2)$$

$$(18 - 9) = 3(4c - c^2)$$

$$3 = 4c - c^2$$

1 $$c^2 - 4c + 3 = 0$$

$$(c-1)(c-3) = 0$$

$\boxed{c = 1}$ أو $c = 3$ ↯

$$F(x) = \int_{\ln x}^{\sqrt{x}} \frac{\ln t}{\sqrt{t^2+1}}\,dt \quad 3$$

$$\frac{d}{dx}\left(\int_{g(x)}^{h(x)} f(t)\,dt\right) = f(h(x))\,h'(x) - f(g(x))\,g'(x)$$

1 $g(x) = \ln x, \quad h(x) = \sqrt{x}, \quad f(t) = \frac{\ln t}{\sqrt{t^2+1}}$

1 $$F'(x) = f(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}} - f(\ln x) \cdot \frac{1}{x}$$

$$= \frac{\ln(\sqrt{x})}{\sqrt{x+1}} \cdot \frac{1}{2\sqrt{x}} - \frac{\ln(\ln x)}{\sqrt{(\ln x)^2+1}} \cdot \frac{1}{x}$$

$$= \frac{\ln(\sqrt{x})}{2\sqrt{x^2+x}} - \frac{\ln(\ln x)}{\sqrt{[illegible]}}$$

$$\int \frac{x-1}{x+1}\,dx = \int \frac{(x+1)-2}{x+1}\,dx \quad 4$$

$$= \int \left[1 - \frac{2}{x+1}\right] dx$$

$$2 \qquad = x - 2\ln|x+1| + c$$

$$\int \frac{e^{6\ln x}}{x^6}\,dx = \int \frac{x^6}{x^6}\,dx \quad 5$$

$$2 \qquad = \int 1\,dx$$

$$= x + c$$

$$\int x^2\, 4^{x^3-1}\, dx = \quad 6$$

$u = x^3 - 1$

$du = 3x^2\,dx$

$$= \frac{1}{3}\int 4^u\,du$$

$$2 \qquad = \frac{1}{3\ln 4}\, 4^u + c = \frac{1}{3\ln 4}\, 4^{x^3-1} + c$$

$$du = -4x^3\,dx$$

$$x^3\,dx = -\frac{1}{4}\,du$$

$$\int x^3(2-x^4)^7\,dx = -\frac{1}{32}(2-x^4)^8 + c \quad 2$$

$$\int 2x \sec^2(x^2)\,dx \quad 3$$

$u = x^2$

$du = 2x\,dx$

$$= \int \sec^2 u\,du$$

$$= \tan u + c$$

$$2 \qquad = \tan(x^2) + c,\ c \in \mathbb{R}$$

س٣ (12 درجة)

$$\int \frac{x^3-1}{\sqrt[3]{x}}\,dx = \int (x^3-1)\,x^{-1/3}\,dx \quad 1$$

$$= \int \left[x^{3-1/3} - x^{-1/3}\right] dx$$

$$2 \qquad = \int (x^{8/3} - x^{-1/3})\,dx$$

$$= \frac{3}{11}x^{11/3} - \frac{3}{2}x^{2/3} + c$$

$$\int x^3(2-x^4)^7\,dx = -\frac{1}{4}\int u^7\,du \quad 2$$

$u = 2 - x^4$

$$= -\frac{1}{32}u^8 + c$$

س٤ (5 درجات)

$$y = x^2 \sin^{-1}(e^x) \quad 1$$

$$\frac{dy}{dx} = 2x\sin^{-1}(e^x) + x^2\,\frac{e^x}{\sqrt{1-e^{2x}}}$$

1 1

$$y = \ln\left[\frac{e^{3x}\tan(x^2)}{\sqrt[5]{x}}\right] \quad 2$$

$$1{,}5 \qquad y = \ln(e^{3x}) + \ln(\tan(x^2)) - \ln(x^{1/5})$$

$$y = 3x + \ln(\tan(x^2)) - \frac{1}{5}\ln x$$

$$1{,}5 \qquad \frac{dy}{dx} = 3 + \frac{2x\sec^2(x^2)}{\tan(x^2)} - \frac{1}{5}\,\frac{1}{x}$$