Answer sheet (Dr Borhen)

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Second Midterm Exam Math151
(Discrete Mathematics)
Spring Semester 2018-2019
Name:...
ID:...

Q1: Let R be a relation defined on the set $\mathbb{N}=\{1,2,3,\ldots\}$ such that for $a,b\in\mathbb{N}$,

$$a\,R\,b \Leftrightarrow \left(\sqrt{a}-\sqrt{b}\right)\in\mathbb{Z}=\{0,\pm1,\pm2,\ldots\}$$

(a) Show that R is an equivalence relation on $\mathbb{N}$. (3 marks)

• R is reflexive because $\sqrt{a}-\sqrt{a}=0\in\mathbb{Z}$ so aRa. 1

• R is symmetric because if aRb then $(\sqrt{a}-\sqrt{b})\in\mathbb{Z}$
so $(\sqrt{b}-\sqrt{a})\in\mathbb{Z}$. Hence bRa 1

• R is transitive because if aRb and bRc then
$(\sqrt{a}-\sqrt{b})=k\in\mathbb{Z}$ and $(\sqrt{b}-\sqrt{c})=h\in\mathbb{Z}$ by addition
we get $\sqrt{a}-\cancel{\sqrt{b}}+\cancel{\sqrt{b}}-\sqrt{c}=k+h$ so $\sqrt{a}-\sqrt{c}\in\mathbb{Z}$ 1
so aRc.
As R reflexive, symmetric and transitive then R is an equivalence relation on $\mathbb{N}$.

(b) Is $9\in[4]$? (1 mark)

$9R4$ because $\sqrt{9}-\sqrt{4}=3-2=1\in\mathbb{Z}$
So $9\in[4]$ 1

Q2: Let T be the equivalence relation defined on the set $A=\{1,2,3,4,5,6,7,8\}$, where $\Im(T)=\{\{1\},\{2,3\},\{4,5,6\},\{7,8\}\}$. Represent T in ordered pairs. (3 marks)

$T=\{(1,1);(2,2);(3,3);(4,4);(5,5);(6,6);(7,7);(8,8);$
$(2,3);(3,2);(4,5);(5,4);(4,6);(6,4);(5,6);(6,5);$
$(7,8);(8,7)\}$ 3

Q3: Let S be a relation defined on the set $A = \{1,2,3,4,5,6,7,8,9,10\}$ as: $a\,S\,b \Leftrightarrow a|b$.

(a) Show that S is a partial ordering relation on A. (3 marks)

• S is reflexive because if $a \in A$ we know that a/a so $a\,S\,a$ 1

• S is antisymmetric because if $a\,S\,b$ and $b\,S\,a$ then $a|b \Leftrightarrow \exists k \in \mathbb{N} \,/\, b = ak$, also $b/a \Leftrightarrow \exists k' \in \mathbb{N} \,/\, a = k'b$

By substitution, $b = k'bk \Leftrightarrow b(1-k'k) = 0$

As $b \neq 0$ then $kk' = 1$. We deduce that $k = k' = 1$ and $a = b$ 1

• S is transitive because if $a\,S\,b$ and $b\,S\,c$ then a/b & b/c

$b = ak$ and $c = bl$. By substitution, $c = akl = aM$ 1

So $a/c \Leftrightarrow a\,S\,c$.

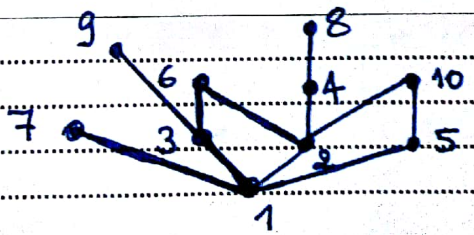
As S reflexive, antisymmetric and transitive then $(A, |)$ is a poset.

(b) Is S a totally ordering relation on A ? (1 mark)

No, S does not satisfy comparison property 1

take $a = 5$ and $b = 3$ we have $a \not S b$ and $b \not S a$.

(c) Draw the Hasse diagram for (A,S). (2 marks)



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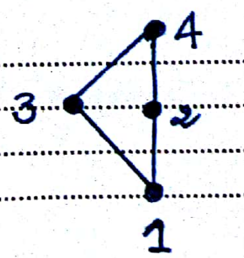
Q4: Draw the Hasse diagram representing the partial ordering relation

$R = \{(1,1),(2,2),(3,3),(4,4),(1,2),(1,3),(1,4),(2,4),(3,4)\}$ on the set $A = \{1,2,3,4\}$.

(3 marks)



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Q5: (a) Determine the number of edges for the **complement** of $K_{10,14}$. (2 marks)

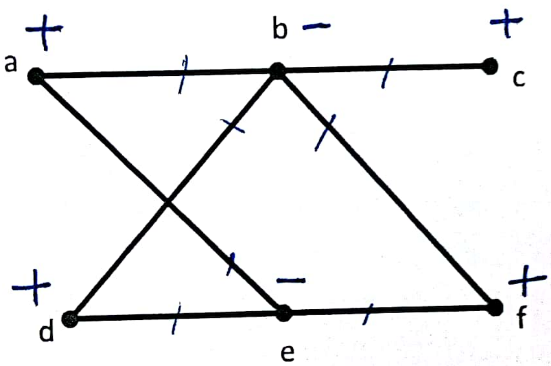
We know that $K_{10,14} \cup \overline{K_{10,14}} = K_{24}$ 0,5

Then $|E(K_{10,14})| + |E(\overline{K_{10,14}})| = |E(K_{24})| = \frac{24 \times 23}{2}$ 0,5

$140 + |E(\overline{K_{10,14}})| = 276$ 1

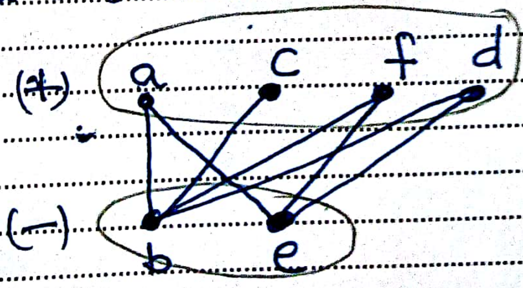
So $|E(\overline{K_{10,14}})| = 276 - 140 = 136$.

(b) Determine whether the graph below is bipartite or not. If so, provide a bipartite representation. (2 marks)



G is bipartite because it has not odd cycles 1

$G \simeq$:



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