M - 107 (SEMESTER I, 1439 -1440) SECOND MID-TERM EXAM

Question: 1. (a) Find the equation of plane containing lines

[6+6+6]

$$L_1: \ x = 1+2t, \ y = t, \ z = -1+2t, \ t \in R$$

$$L_2: \ x = -1+2s, \ y = 2+2s, \ z = 1+3s, \ s \in R$$

(b) Find the equation of line passing through points P(2, 1, -3) and Q(5, -1, 4). At what point this line intersects xz – plane.

(c) Identify the surface $36x^2 + 9y^2 - 16z^2 = 144$.
Find its traces on the coordinate planes and sketch the surface.

Solution (a) L_1: passing through point $(1, 0, -1)$ and vector parallel to line is $a = \langle 2, 1, 2\rangle$

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L_2, Passing through point $(-1, 2, 1)$ and vector parallel to line is $b = \langle 2, 2, 3\rangle$

Vector normal to plane containing L_1 and L_2 is

2

$$N = a \times b = \begin{vmatrix} i & j & k \\ 2 & 1 & 2 \\ 2 & 2 & 3 \end{vmatrix} = -i - 2j + 2k.$$

Equation the plane containing point $(-1, 2, 1)$ with normal vector N is

2

$$-1(x+1) - 2(y-2) + 2(z-1) = 0$$

(b)

$$-x - 2y + 2z + 1 = 0$$

Vector parallel to line $\vec{PQ} = \langle 3, -2, 7\rangle$, let point be $P(2, 1, -3)$

4 Eq. of line is $x = 2+3t, \ y = 1-2t, \ z = -3+7t, \ t \in R$.

Line intersect xz-plane at $y = 0 \Rightarrow t = \frac{1}{2} \Rightarrow x = \frac{7}{2}, z = \frac{1}{2}$

2 Required point is $(7/2, 0, 1/2)$

(c) (i) $\frac{x^2}{4} + \frac{y^2}{16} - \frac{z^2}{9} = 1$.

1

Surface is Hyperboloid of one Sheet

(ii) Traces

3

Plane	Equation	Name
xy	$x^2/4 + y^2/16 = 1$	Ellipse
xz	$y^2/16 - z^2/9 = 1$	Hyperbola
yz	$x^2/4 - z^2/9 = 1$	Hyperbola

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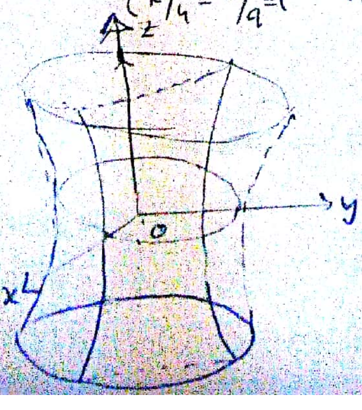

Question: 2 . (a) Given vectors

[6+6+6] $a = <1,1,1>$ *and* $b = <3,0,5>$. Find

(i) $Comp_b^a$

(ii) $Proj_b^a$

(b) Find the distance of point B(- 1,5,2) from the line through points A(0, 1, 2) and C(2,4,0).

(c) Find parametric equation of the tangent line to the curve

$x = t^3 - 1,\ y = -t^2 + 3,\ z = t - 2$ **at point A(0,2,-1)**

Solution. (a) $a \cdot b = 3 + 0 + 5 = 8$

$\|b\| = \sqrt{9 + 0 + 25} = \sqrt{34}$

3 (i) $Comp_b^a = \frac{a \cdot b}{\|b\|} = \frac{8}{\sqrt{34}}$

3 (ii) $Proj_b^a = \frac{a \cdot b}{\|b\|} \cdot \frac{b}{\|b\|} = \frac{8 <3,0,5>}{34} = \frac{4}{17} <3,0,5>$

(b) $d = \frac{\|\vec{AB} \times \vec{AC}\|}{\|AC\|}$, $\vec{AB} = <-1, 4, 0>$ $\vec{AC} = <2, 3, -2>$

$\vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ -1 & 4 & 0 \\ 2 & 3 & -2 \end{vmatrix} = <-8, -2, -11>$ 3

$\|\vec{AB} \times \vec{AC}\| = \sqrt{64 + 4 + 121} = \sqrt{189}$

$\|\vec{AC}\| = \sqrt{4 + 9 + 4} = \sqrt{17}$

$d = \frac{\sqrt{189}}{\sqrt{17}} = \sqrt{\frac{189}{17}}$ unit 3

(c) $r(t) = (t^3 - 1)\,i + (-t^2 + 3)\,j + (t - 2)\,k$

1 $r'(t) = 3t^2\,i - 2t\,j + k$, At $A(0, 2, -1)$

$\Rightarrow t = 1$ 1

2 $r'(1) = 3i - 2j + k$,

Equation of tangent line at point $A(0, 2, -1)$ is

2 $x = 3t, \quad y = 2 - 2t, \quad z = -1 + t, \quad t \in R.$

Question: 3. (a) Let the velocity be $v(t) = \cos t i + (t^2 - 1) j + e^t k$.
[6+8] Find the position vector r(t) such that $r(0) = i + j + k$.

(b) The position vector of a moving particle at time t is
$r(t) = \langle 2\sin t,\ 2\cos t,\ 4t \rangle$. Find the tangential and the normal components of acceleration and the curvature at $t = \frac{\pi}{2}$.

Solution. (a) $r(t) = \int (\cos t\, i + (t^2 - 1) j + e^t k)\, dt$

3 $= (\sin t + c_1) i + (\frac{t^3}{3} - t + c_2) j + (e^t + c_3) k.$

$r(0) = i + j + k$

$i + j + k = c_1 i + c_2 j + (1 + c_3) k \Rightarrow c_1 = 1, c_2 = 1, c_3 = 0$

3 $r(t) = (\sin t + 1) i + (\frac{t^3}{3} - t + 1) j + e^t k$

(b)

$r(t) = \langle 2\sin t, 2\cos t, 4t \rangle$

$r'(t) = \langle 2\cos t, -2\sin t, 4 \rangle$

2 $r''(t) = \langle -2\sin t, -2\cos t, 0 \rangle$

$r'(t) \cdot r''(t) = -4\sin t\cos t + 4\sin t\cos t = 0$

2 $r'(t) \times r''(t) = \begin{vmatrix} i & j & k \\ 2\cos t & -2\sin t & 4 \\ -2\sin t & -2\cos t & 0 \end{vmatrix} = -8\cos t\, i - 8\sin t\, j - 4k$

At $t = \frac{\pi}{2}$ $r'(\frac{\pi}{2}) \times r''(\frac{\pi}{2}) = -8j - 4k$

1 $\|r'(\frac{\pi}{2}) \times r''(\frac{\pi}{2})\| = \sqrt{64 + 16} = \sqrt{80}.$

$r'(\frac{\pi}{2}) = \langle 0, -2, 4 \rangle$

$\|r'(\frac{\pi}{2})\| = \sqrt{4 + 16} = \sqrt{20}.$

1. $a_T = \frac{r'(\frac{\pi}{2}) \cdot r''(\pi/2)}{\|r'(\pi/2)\|} = 0$

3 2 $a_N = \frac{\|r'(\frac{\pi}{2}) \times r''(\frac{\pi}{2})\|}{\|r'(\frac{\pi}{2})\|} = \frac{\sqrt{80}}{\sqrt{20}} = \sqrt{\frac{80}{20}} = \sqrt{4} = 2$

$K = \frac{\|r'(\frac{\pi}{2}) \times r''(\frac{\pi}{2})\|}{\|r'(\frac{\pi}{2})\|^3} = \frac{\sqrt{80}}{\sqrt{8000}} = \frac{1}{\sqrt{100}} = \frac{1}{10}$