الاختبار الفصلي الثّاني - المدّة: ساعة ونصف

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نعتبر على الكرة $\mathbb{S}^2$ المستقيمين الكرويّين $\ell_1 : x+y=0$ ، $\ell_2 : x+z=0$ ، والنّقط $\xi_1\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2},0\right)$ ، $\xi_2\left(\frac{1}{2},\frac{1}{2},\frac{\sqrt{2}}{2}\right)$ ، $\xi_3\left(\frac{\sqrt{3}}{3}+\frac{\sqrt{6}}{6},\frac{\sqrt{3}}{3}-\frac{\sqrt{6}}{6},0\right)$.

١- اعط معادلة المستقيم الكروي المار بالنّقطتين ξ_1 و ξ_2.

٢- اعط صيغة المسار $p : [0,\frac{\pi}{4}] \to \mathbb{S}^2$ الّذي يحقّق، لكلّ $0 \le t_1 < t_2 \le \frac{\pi}{4}$:

$p(0)=\xi_1$ ، $p\left(\frac{\pi}{4}\right)=\xi_2$ و $d(p(t_1),p(t_2)) = |t_1-t_2|$ ،

حيث d هي المسافة الكرويّة على $\mathbb{S}^2$.

٣- احسب مساحة المثلّث الكروي $\xi_1\xi_2\xi_3$.

٤- تأكّد من تحقق قاعدة الجيوب الكرويّة في المثلّث $\xi_1\xi_2\xi_3$.

٥- نعتبر على $\mathbb{S}^2$ النّقط $\xi_1'(1,0,0)$ ، $\xi_2'\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2},0\right)$ ، $\xi_3'\left(\frac{\sqrt{6}}{3},0,\frac{\sqrt{3}}{3}\right)$. بيّن أنّ المثلّثين $\xi_1\xi_2\xi_3$ و $\xi_1'\xi_2'\xi_3'$ متطابقان واعط صيغة التّقايس الّذي يحوّل $\xi_1'\xi_2'\xi_3'$ إلى $\xi_1\xi_2\xi_3$.

٦- اعط صيغة الانعكاس الكروي Ω_{ℓ_1} بالنّسبة للمستقيم ℓ_1.

٧- عيّن طبيعة تركيب الانعكاسين $\Omega_{\ell_2}\circ\Omega_{\ell_1}$ بالنّسبة للمستقيمين ℓ_1 ، ℓ_2 وحدّد عناصره.

٨- بيّن أنّ التّحويل $T : \mathbb{S}^2 \to \mathbb{S}^2$ المعرّف بـ

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 2 & -3 & -6 \\ 6 & -2 & 3 \\ 3 & 6 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

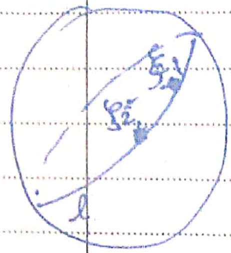
تقايس كروي، عيّن نوعه، وحدّد عناصره.

د/ ابرهان

٢٨/٣/١٤٣٩ لا تكتب في هذا الهامش

تصحيح الاختبار الفصلي الثاني ٣٣٩ ريض

① $\xi_1 = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right)$, $\xi_2 = \left(\frac{1}{2}, \frac{1}{2}, \frac{\sqrt{2}}{2}\right)$



$$l = \{X \in S^2 / \langle X | \xi \rangle = 0\}$$

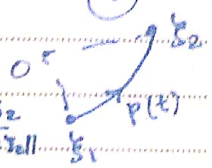
$$\xi = \xi_1 \times \xi_2 = \frac{1}{2}\begin{pmatrix} \sqrt{2} \\ \sqrt{2} \\ 0 \end{pmatrix} \times \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ \sqrt{2} \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix}$$

$$l = \left\{X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in S^2 / 2x - 2y = 0\right\}$$

$$l = \{X = (x, y, z) \in S^2 / x = y\}$$

② Geodesic جيوديسك

$$p(t) = \cos t \, \xi_1 + \sin t \, \vec{u}$$



$\vec{u} = \frac{\xi \times \xi_1}{\|\xi \times \xi_1\|}$; $\xi = \frac{\xi_1 \times \xi_2}{\|\xi_1 \times \xi_2\|}$

$t = 0$; $p(0) = \xi_1$

$t = \frac{\pi}{4}$; $p\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}(\xi_1 + \vec{u}) = \xi_2 \Leftrightarrow \vec{u} = \sqrt{2}\,\xi_2 - \xi_1$

$$= \sqrt{2}\begin{pmatrix} 1/2 \\ 1/2 \\ \sqrt{2}/2 \end{pmatrix} - \begin{pmatrix} \sqrt{2}/2 \\ \sqrt{2}/2 \\ 0 \end{pmatrix}$$

$\vec{u} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$; $p(t) = \cos t \, \xi_1 + \sin t \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

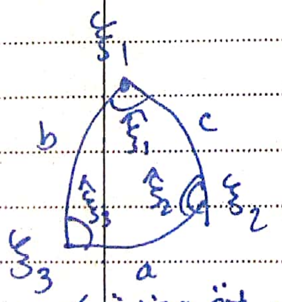
$$p(t) = \begin{pmatrix} \frac{\sqrt{2}}{2}\cos t \\ \frac{\sqrt{2}}{2}\cos t \\ \sin t \end{pmatrix} \in S^2, \quad 0 \le t \le \frac{\pi}{4}$$

$$d(p(t_1), p(t_2)) = \cos^{-1}(\langle p(t_1) | p(t_2) \rangle)$$

$$= \cos^{-1}\left(\frac{1}{2}\cos t_1 \cos t_2 + \frac{1}{2}\cos t_1 \cos t_2 + \sin t_1 \sin t_2\right)$$

$$= \cos^{-1}(\cos|t_1 - t_2|) = |t_1 - t_2|$$

③ نضع



طول الضلع بين ξ_2 و ξ_3 $a = \overset{\frown}{\xi_2 \xi_3}$

بين ξ_1 و ξ_3 // $b = \overset{\frown}{\xi_1 \xi_3}$

بين ξ_1 و ξ_2 // // $c = \overset{\frown}{\xi_1 \xi_2}$

مساحة المثلث كروي $\xi_1 \xi_2 \xi_3$ لطلابنا خفه الم

$A(\Delta) = \hat{\xi}_1 + \hat{\xi}_2 + \hat{\xi}_3 - \pi$

نعلم أن

$$\cos \hat{\xi}_1 = \frac{\cos a - \cos b \cos c}{\sin b \sin c}$$

$$\cos \hat{\xi}_2 = \frac{\cos b - \cos a \cos c}{\sin a \sin c}$$

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$$\cos \hat{\xi}_3 = \frac{\cos c - \cos a \cdot \cos b}{\sin a \sin b}$$

$$a = \widehat{\xi_2 \xi_3} = \cos^{-1}(\langle \xi_2 | \xi_3 \rangle)$$

$$= \cos^{-1} \left\langle \begin{pmatrix} 1/2 \\ 1/2 \\ \sqrt{2}/2 \end{pmatrix} \middle| \begin{pmatrix} \frac{\sqrt{3}}{3} + \frac{\sqrt{6}}{6} \\ \sqrt{3}/3 - \sqrt{6}/6 \\ 0 \end{pmatrix} \right\rangle$$

$\cos a = \frac{\sqrt{3}}{3}$ وبالتالي $a = \cos^{-1}\left(\frac{\sqrt{3}}{6} + \cancel{\frac{\sqrt{6}}{12}} + \frac{\sqrt{3}}{6} - \cancel{\frac{\sqrt{6}}{12}}\right) = \cos^{-1}\left(\frac{\sqrt{3}}{3}\right)$

$$\sin a = \sqrt{1 - \cos^2 a} = \sqrt{1 - \frac{1}{3}} = \sqrt{\frac{2}{3}} = \frac{\sqrt{6}}{3}$$

$$b = \widehat{\xi_1 \xi_3} = \cos^{-1} \langle \xi_1 | \xi_3 \rangle = \cos^{-1} \left\langle \begin{pmatrix} \frac{\sqrt{2}}{2} \\ \sqrt{2}/2 \\ 0 \end{pmatrix} \middle| \begin{pmatrix} \frac{\sqrt{3}}{3} + \frac{\sqrt{6}}{6} \\ \frac{\sqrt{3}}{3} - \frac{\sqrt{6}}{6} \\ 0 \end{pmatrix} \right\rangle$$

$\cos b = \frac{\sqrt{6}}{3}$ وبالتالي $b = \cos^{-1}\left(\frac{\sqrt{6}}{6} + \cancel{\frac{\sqrt{12}}{12}} + \frac{\sqrt{6}}{6} - \cancel{\frac{\sqrt{12}}{12}}\right) = \cos^{-1}\left(\frac{\sqrt{6}}{3}\right)$

$$\sin b = \sqrt{1 - \cos^2 b} = \sqrt{1 - \frac{6}{9}} = \frac{1}{\sqrt{3}}$$

$$c = \widehat{\xi_1 \xi_2} = \cos^{-1} \left\langle \begin{pmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \\ 0 \end{pmatrix} \middle| \begin{pmatrix} 1/2 \\ 1/2 \\ \sqrt{2}/2 \end{pmatrix} \right\rangle$$

$\cos c = \frac{\sqrt{2}}{2}$ $\quad c = \cos^{-1}\left(\frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{4} + 0\right) = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

$\sin c = \frac{\sqrt{2}}{2}$

$$\cos(\hat{\xi}_1) = \frac{\sqrt{3}/3 - \frac{\sqrt{6}}{3} \cdot \frac{\sqrt{2}}{2}}{1/\sqrt{3} \quad \sqrt{2}/2} = \frac{2\sqrt{3} - \sqrt{12}}{\sqrt{6}} = \frac{2\sqrt{3} - 2\sqrt{3}}{\sqrt{6}} = 0$$ وبالتالي

إذن $\boxed{\hat{\xi}_1 = \frac{\pi}{2}}$

$$\cos \hat{\xi}_2 = \frac{\frac{\sqrt{6}}{3} - \frac{\sqrt{3}}{3} \frac{\sqrt{2}}{2}}{\frac{\sqrt{6}}{3} \frac{\sqrt{2}}{2}} = \frac{2\sqrt{6} - \sqrt{6}}{\sqrt{12}} = \frac{\sqrt{6}}{\sqrt{2 \times 6}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\boxed{\hat{\xi}_2 = \frac{\pi}{4}}$$

$$\cos \hat{\xi}_3 = \frac{\sqrt{2}/2 - \sqrt{3}/3 \cdot \sqrt{6}/3}{\frac{\sqrt{6}}{3} \cdot \frac{1}{\sqrt{3}}} = \frac{\sqrt{2}/2 - \frac{\sqrt{2}}{3}}{\frac{\sqrt{2}}{3}} = \frac{3\sqrt{2}/2 - \sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2\sqrt{2}}$$

$$\boxed{\hat{\xi}_3 = \frac{\pi}{3}}$$

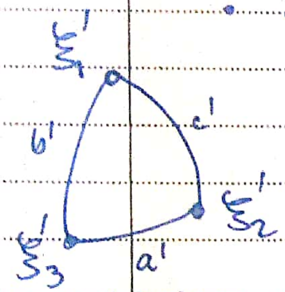
فإن مساحة المثلث $\xi_1 \xi_2 \xi_3$ هو $\frac{\pi}{12} = \left[\frac{\pi}{2} + \frac{\pi}{4} + \frac{\pi}{3} - \pi\right]$

④

$$\frac{\sin a}{\sin \hat{\xi}_1} \overset{?}{=} \frac{\sin b}{\sin \hat{\xi}_2} \overset{?}{=} \frac{\sin c}{\sin \hat{\xi}_3}$$

- $\dfrac{\sin a}{\sin \hat{\xi}_1} = \dfrac{\sqrt{6}/3}{1} = \dfrac{\sqrt{6}}{3}$
- $\dfrac{\sin b}{\sin \hat{\xi}_2} = \dfrac{1/\sqrt{3}}{\frac{\sqrt{2}}{2}} = \dfrac{2}{\sqrt{6}} = \dfrac{2\sqrt{6}}{6} = \dfrac{\sqrt{6}}{3}$ ✓
- $\dfrac{\sin c}{\sin \hat{\xi}_3} = \dfrac{\sqrt{2}/2}{\sqrt{3}/2} = \sqrt{\dfrac{2}{3}} = \dfrac{\sqrt{6}}{3}$ ✓

⑤



مثلث كروي

نضع طول الضلع $a' = \overset{\frown}{\xi'_2 \xi'_3}$

- $a' = \cos^{-1} \langle \xi'_2 \mid \xi'_3 \rangle$

$$a' = \cos^{-1} \left\langle \begin{pmatrix} \sqrt{2}/2 \\ \sqrt{2}/2 \\ 0 \end{pmatrix} \middle| \begin{pmatrix} \frac{\sqrt{6}}{3} \\ 0 \\ \frac{\sqrt{3}}{3} \end{pmatrix} \right\rangle$$

$a' = \cos^{-1}\left(\frac{\sqrt{12}}{6} + 0 + 0\right) = \cos^{-1}\left(\frac{\sqrt{3}}{3}\right)$ و بالتالي $a' = a$

$l(\overset{\frown}{\xi'_2 \xi'_3}) = l(\overset{\frown}{\xi_2 \xi_3})$

- طول الضلع الذي يربط ξ'_1 بـ ξ'_3 : $b' = \overset{\frown}{\xi'_1 \xi'_3} = \cos^{-1} \langle \xi'_1 \mid \xi'_3 \rangle$

$b' = \cos^{-1} \left\langle \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \middle| \begin{pmatrix} \sqrt{6}/3 \\ 0 \\ \frac{\sqrt{3}}{3} \end{pmatrix} \right\rangle = \cos^{-1}\left(\frac{\sqrt{6}}{3}\right)$ و بالتالي $b' = b$

- $c' = \overset{\frown}{\xi'_1 \xi'_2}$
$= \cos^{-1} \langle \xi'_1 \mid \xi'_2 \rangle$

$c' = \cos^{-1} \left\langle \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \middle| \begin{pmatrix} \frac{\sqrt{2}}{2} \\ \sqrt{2}/2 \\ 0 \end{pmatrix} \right\rangle = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$ و بالتالي $c' = c$

بما أن جميع أطوال الأضلاع متساوية

$\overset{\frown}{\xi'_1 \xi'_2} = \overset{\frown}{\xi_1 \xi_2}$

$\overset{\frown}{\xi'_2 \xi'_3} = \overset{\frown}{\xi_2 \xi_3}$

$\overset{\frown}{\xi'_1 \xi'_3} = \overset{\frown}{\xi_1 \xi_3}$

فإن المثلثين $\xi_1 \xi_2 \xi_3$ و $\xi'_1 \xi'_2 \xi'_3$ متطابقين

نضع φ التقايس الذي يحول $\xi_1 \xi_2 \xi_3$ إلى $\xi'_1 \xi'_2 \xi'_3$:

$$M_1 = \begin{array}{c} \begin{array}{ccc} \xi_1 & \xi_2 & \xi_3 \end{array} \\ \begin{pmatrix} \frac{\sqrt{2}}{2} & 1/2 & \frac{\sqrt{3}}{3} + \frac{\sqrt{6}}{6} \\ \sqrt{2}/2 & 1/2 & \frac{\sqrt{3}}{3} - \frac{\sqrt{6}}{6} \\ 0 & \frac{\sqrt{2}}{2} & 0 \end{pmatrix} \end{array} \begin{array}{c} e_1 \\ e_2 \\ e_3 \end{array} \quad ; \quad M_2 = \begin{array}{c} \begin{array}{ccc} \xi'_1 & \xi'_2 & \xi'_3 \end{array} \\ \begin{pmatrix} 1 & \frac{\sqrt{2}}{2} & \sqrt{6}/3 \\ 0 & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & \sqrt{3}/3 \end{pmatrix} \end{array} \begin{array}{c} e_1 \\ e_2 \\ e_3 \end{array}$$

فإن مصفوفة التقايس φ هي $A_\varphi = M_2 M_1^{-1}$

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$$|M_1| = \begin{vmatrix} \frac{\sqrt2}{2} & 1/2 & \frac{\sqrt3}{3}+\frac{\sqrt6}{6} \\ \frac{\sqrt2}{2} & 1/2 & \frac{\sqrt3}{3}-\frac{\sqrt6}{6} \\ 0 & \frac{\sqrt2}{2} & 0 \end{vmatrix} = -\frac{\sqrt2}{2}\begin{vmatrix} \frac{\sqrt2}{2} & \frac{\sqrt3}{3}+\frac{\sqrt6}{6} \\ \frac{\sqrt2}{2} & \frac{\sqrt3}{3}-\frac{\sqrt6}{6} \end{vmatrix}$$

$$= -\frac{\sqrt2}{2}\left[\frac{\sqrt2}{2}\left(\frac{\sqrt3}{3}-\frac{\sqrt6}{6}\right) - \frac{\sqrt2}{2}\left(\frac{\sqrt3}{3}+\frac{\sqrt6}{6}\right)\right]$$

$$= -\frac{\sqrt2}{2}\left[\cancel{\frac{\sqrt6}{6}} - \frac{\sqrt{12}}{12} - \cancel{\frac{\sqrt6}{6}} - \frac{\sqrt{12}}{12}\right] = +\frac{\sqrt2}{2}\cdot\frac{\sqrt{12}}{6} = \frac{1}{\sqrt6} = \frac{\sqrt6}{6}$$

$$M_1^{-1} = \sqrt6 \begin{pmatrix} -\frac{\sqrt2}{2}\left(\frac{\sqrt3}{3}-\frac{\sqrt6}{6}\right) & 0 & \frac12 \\ \frac{\sqrt2}{2}\left(\frac{\sqrt3}{3}+\frac{\sqrt6}{6}\right) & 0 & -1/2 \\ -\frac{\sqrt6}{6} & \frac{\sqrt{12}}{6} & 0 \end{pmatrix}^T = \sqrt6 \begin{pmatrix} \frac{\sqrt6}{12}(\sqrt2-2) & 0 & \frac12 \\ \frac{\sqrt6}{12}(2+\sqrt2) & 0 & -\frac12 \\ -\frac{\sqrt6}{6} & \frac{\sqrt{12}}{6} & 0 \end{pmatrix}^T$$

$$= \sqrt6 \begin{pmatrix} \frac{\sqrt6}{12}(\sqrt2-2) & \frac{\sqrt6}{12}(2+\sqrt2) & -\sqrt6/6 \\ 0 & 0 & \sqrt{12}/6 \\ 1/2 & -1/2 & 0 \end{pmatrix}$$

$$M_1^{-1} = \begin{pmatrix} \frac{\sqrt2-2}{2} & \frac{2+\sqrt2}{2} & -1 \\ 0 & 0 & \sqrt2 \\ \frac{\sqrt6}{2} & -\frac{\sqrt6}{2} & 0 \end{pmatrix}$$

$$A_\varphi = \begin{pmatrix} 1 & \sqrt2/2 & \sqrt6/3 \\ 0 & \sqrt2/2 & 0 \\ 0 & 0 & \frac{\sqrt3}{3} \end{pmatrix}\begin{pmatrix} \frac{\sqrt2-2}{2} & \frac{2+\sqrt2}{2} & -1 \\ 0 & 0 & \sqrt2 \\ \frac{\sqrt6}{2} & \frac{-\sqrt6}{2} & 0 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt2}{2} & \frac{\sqrt2}{2} & 0 \\ 0 & 0 & 1 \\ \frac{\sqrt2}{2} & -\frac{\sqrt2}{2} & 0 \end{pmatrix}$$

مصفوفة التشابه التي يحول $\vec{e}_1'\vec{e}_2'\vec{e}_3'$ إلى $\vec{e}_1\vec{e}_2\vec{e}_3$

$$A_{\varphi^{-1}} = (A_\varphi)^{-1} = \begin{pmatrix} \frac{\sqrt2}{2} & \frac{\sqrt2}{2} & 0 \\ 0 & 0 & 1 \\ \frac{\sqrt2}{2} & -\frac{\sqrt2}{2} & 0 \end{pmatrix}^{-1} \text{ طلب}$$

$$A_{\varphi^{-1}} = \begin{pmatrix} \frac{\sqrt2}{2} & 0 & \sqrt2/2 \\ \sqrt2/2 & 0 & -\sqrt2/2 \\ 0 & 1 & 0 \end{pmatrix} = (M_2 M_1^{-1})^{-1} = M_1 M_2^{-1}$$

⑥ $l_1 : \quad x + y = 0$

$\xi = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\\0\end{pmatrix}$ متجه الوحدة $\xi \perp l_1$

$$\Omega_{l_1}(X) = X' \Leftrightarrow X' = X - 2\langle X \mid \xi\rangle \xi$$

$X = \begin{pmatrix}x\\y\\z\end{pmatrix} \in S^2$; $\quad \langle X \mid \xi \rangle = \frac{x+y}{\sqrt{2}}$

وبالتالي

$$\begin{pmatrix}x'\\y'\\z'\end{pmatrix} = \begin{pmatrix}x\\y\\z\end{pmatrix} - (x+y)\begin{pmatrix}1\\1\\0\end{pmatrix}$$

$$\Omega_{l_1} : \begin{pmatrix}x'\\y'\\z'\end{pmatrix} = \begin{pmatrix}-y\\-x\\z\end{pmatrix} = \begin{pmatrix}0 & -1 & 0\\-1 & 0 & 0\\0 & 0 & 1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}$$

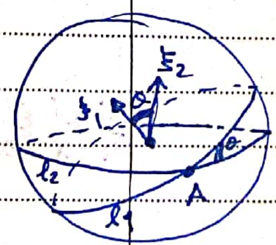
⑦ $\Omega_{l_2} \circ \Omega_{l_1}$ هو دوران لأن $l_1 \cap l_2 = \{\pm A\}$

نقاط التقاطع $\begin{cases} x+y=0 \\ x+z=0 \end{cases}$ $\quad x = -y = -z$

$(x, -x, -x) \in S^2$; $\quad 3x^2 = 1 \Leftrightarrow x = \pm \frac{1}{\sqrt{3}}$

$A = \left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$ مركزه

$\pi(A) = A' = \left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} ; \frac{1}{\sqrt{3}}\right)$



نضع $\theta = \widehat{(l_1, l_2)}$

$\xi_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\\0\end{pmatrix} \perp l_1$

$\xi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\1\end{pmatrix} \perp l_2$. $\quad \cos\theta = \langle \xi_1 \mid \xi_2 \rangle = \frac{1}{2}$

وبالتالي $\theta = \pi/3$ فإن زاوية الدوران

هو $2\theta = \frac{2\pi}{3}$ ومركزه $A = \left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$

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$$A_T = \frac{1}{7}\begin{pmatrix}2&-3&-6\\6&-2&3\\3&6&-2\end{pmatrix}$$

$$A_T^T \cdot A_T = \frac{1}{49}\begin{pmatrix}2&6&3\\-3&-2&6\\-6&3&-2\end{pmatrix}\begin{pmatrix}2&-3&-6\\6&-2&3\\3&6&-2\end{pmatrix} = \frac{1}{49}\begin{pmatrix}49&0&0\\0&49&0\\0&0&49\end{pmatrix}$$

$$= \begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} = I$$

وبالتالي التحويل T هو تقايس على S^2

$$\left(d_{S^2}(T(X),T(Y)) = d_{S^2}(X,Y)\right)$$

$$|A_T| = \frac{1}{7^3}\begin{vmatrix}2&-3&-6\\6&-2&3\\3&6&-2\end{vmatrix}$$

$A_T^T \neq A_T$

$$= \frac{1}{7^3}\left[\underbrace{(8-27-216)}_{-235}-\underbrace{(36+36+36)}_{108}\right] = \frac{-343}{343} = -1$$

وبالتالي T هو تحصيل انعكاس ودوران.

$$A_T \simeq \begin{pmatrix}\cos\theta&-\sin\theta&0\\\sin\theta&\cos\theta&0\\0&0&-1\end{pmatrix}$$

$$2\cos\theta - 1 = tr(A_T) = -\frac{2}{7}$$

$$2\cos\theta = \frac{5}{7} \Rightarrow \cos\theta = \frac{5}{14}$$

$\theta = \cos^{-1}\left(\frac{5}{14}\right)$ هي زاوية الدوران

$(Ker(A_T+I))$

$A_T\Omega = -\Omega \Leftrightarrow \Omega\left(\frac{-1}{\sqrt{19}}, \frac{-3}{\sqrt{19}}, \frac{3}{\sqrt{19}}\right)$ هو مركز الدوران

والمستقيم الانعكاس معادلته $l: 9x-9y+5z=0$

$$\begin{pmatrix}1\\1\\0\end{pmatrix} \times \frac{1}{7}\begin{pmatrix}-1\\4\\9\end{pmatrix} = \frac{1}{7}\begin{pmatrix}9\\-9\\5\end{pmatrix} \quad ; \quad T\begin{pmatrix}1\\1\\0\end{pmatrix} = \frac{1}{7}\begin{pmatrix}-1\\4\\9\end{pmatrix}$$