جامعة الملك سعود King Saud University	كلية العلوم – قسم الرياضيات	الاختبار الشهري الثاني للمقرر 111 ريض للفصل الاول 1441-1440 هـ الزمن: ساعة ونصف

ملاحظه : ممنوع استخدام الالة الحاسبة

السؤال الأول (4 درجات): احسب $\frac{dy}{dx}$ فيما يلي :

(1) $y = \tanh^{-1}(\ln(x)) + \operatorname{sech}\left(\sqrt{x}\right)$. (درجتان)

(2) $y = \sqrt{4x + \cosh^2(5x)}$. (درجتان)

السؤال الثاني (21 درجة): احسب التكاملات التالية :

(1) $\int \frac{e^{\cosh(x)}}{\operatorname{csc} h(x)} dx$ (درجتان)

(2) $\int \frac{dx}{x\sqrt{x^8+1}}$ (درجتان)

(3) $\int_1^e x^8 \ln x \, dx$ (درجتان)

(4) $\int e^x \sin x dx$ (درجتان)

(5) $\int \sin(7x)\cos(3x)\, dx$ مع العلم أن : $\sin a \cos b = \frac{1}{2}[\sin(a-b)+\sin(a+b)]$ (درجتان)

(6) $\int \frac{dx}{(4+x^2)^{3/2}}$ (3 درجات)

(7) $\int \frac{x}{x^2-4x+8} dx$ (3 درجات)

(8) $\int \frac{x-1}{x^2+3x+2} dx$ (3 درجات)

(9) $\int \frac{dx}{\sqrt{x}+\sqrt[4]{x}}$ (درجتان)

حل اختبار الشهري الثاني

111 رعين الفصل الأول.

- السؤال الأول:

① $y = \tanh^{-1}(\ln(x)) + \operatorname{sech}(\sqrt{x})$

$$\frac{dy}{dx} = \frac{\frac{1}{x}}{1-(\ln x)^2} - \frac{1}{2\sqrt{x}} \operatorname{sech}\sqrt{x}\tanh\sqrt{x}$$ (1) (1)

② $y = \sqrt{4x + \cosh^2(5x)}$

$$\frac{dy}{dx} = \frac{4 + 10\sinh(5x)\cosh(5x)}{2\sqrt{4x+\cosh^2(5x)}}$$ (2)

- السؤال الثاني:

① $$\int \frac{e^{\cosh(x)}}{\operatorname{csch}(x)}\,dx = \int e^{\cosh(x)} \cdot \sinh(x)\,dx$$ (1)

$$= e^{\cosh(x)} + c$$ (1)

② $$\int \frac{dx}{x\sqrt{x^8+1}} = \frac{1}{4}\int \frac{4x^3}{x^4\sqrt{(x^4)^2+1}}\,dx = -\frac{1}{4}\operatorname{csch}^{-1}x^4 + c$$ (1) (1)

③ $$\int_1^e x^8 \ln x\,dx = \left[\frac{x^9}{9}\ln x - \frac{x^9}{81}\right]_1^e = \left[\frac{e^9}{9}\ln e - \frac{e^9}{81}\right] - \left[\frac{1}{9}\ln 1 - \frac{1}{81}\right]$$ (1/2) (1/2)

$$= \frac{e^9}{9} - \frac{e^9}{81} + \frac{1}{81} = \frac{9e^9 - e^9 + 1}{81}$$ (1)

$$= \frac{8e^9+1}{81}$$

④ $I = \int e^x \sin x\,dx =$

$u = e^x \Rightarrow du = e^x dx$ (1/2)

$dv = \sin x\,dx \Rightarrow v = -\cos x$ (1/2)

$$I = -e^x\cos x + \underbrace{\int e^x \cos x\,dx}_{I_1}$$ (1/2)

$u = e^x \Rightarrow du = e^x dx$

$dv = \cos x\,dx \Rightarrow v = \sin x$

$\Rightarrow I = -e^x\cos x + e^x\sin x - \int e^x \sin x\,dx$ (1/2)

$\int e^x \sin x\,dx = \frac{e^x}{2}[\sin x - \cos x]$.

(5) $\int \sin(7x)\cos(3x)\,dx = \frac{1}{2}\int (\sin 4x + \sin 10x)\,dx$ (1)

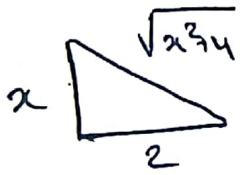
$= -\frac{1}{8}\cos 4x - \frac{1}{20}\cos 10x + c$. (1)

(6) $\int \frac{dx}{(4+x^2)^{3/2}} =$

$x = 2\tan\theta \Rightarrow dx = 2\sec^2\theta\,d\theta$ (1/2)

$I = 2\int \frac{\sec^2\theta}{(4+4\tan^2\theta)^{3/2}}\,d\theta = \frac{2}{8}\int \frac{\sec^2\theta}{(1+\tan^2\theta)^{3/2}}\,d\theta$ (1/2) (1/2)

$= \frac{1}{4}\int \frac{\sec^2\theta}{\sec^3\theta}\,d\theta$



$= \frac{1}{4}\int \cos\theta\,d\theta$ (1/2) (1/2)

$= \frac{1}{4}\sin\theta + c = \frac{1}{4}\frac{x}{\sqrt{x^2+4}} + c$ (1/2)

(7) $\int \frac{x}{x^2-4x+8}\,dx = \int \frac{x}{(x-2)^2+(2)^2}\,dx = \int \frac{x-2+2}{(x-2)^2+4}\,dx$ (1/2) (1/2)

$= \int \frac{x-2}{(x-2)^2+4}\,dx + 2\int \frac{dx}{(x-2)^2+4} = \frac{1}{2}\ln|(x-2)^2+4|$ (1/2) (1/2) (1/2)

$+ \tan^{-1}\frac{(x-2)}{2} + c$ (1/2)

(8) $\int \frac{x-1}{x^2+3x+2}\,dx$ (1/2)

$\frac{x-1}{x^2+3x+2} = \frac{x-1}{(x+2)(x+1)} = \frac{A}{x+2} + \frac{B}{x+1} = \frac{A(x+1)+B(x+2)}{(x+2)(x+1)}$ (1/2) (1/2)

$A(x+1)+B(x+2) = x-1 \Rightarrow B=-2,\ A=3$ (1/4)

$\int \frac{x-1}{x^2+3x+2}\,dx = 3\int \frac{dx}{x+2} - 2\int \frac{dx}{x+1} = 3\ln|x+2| - 2\ln|x+1| + c$ (1/2) (1/2)

(9) $\int \frac{dx}{\sqrt{x}+\sqrt[4]{x}}$,

$u = x^{1/4} \Rightarrow u^4 = x \Rightarrow 4u^3 du = dx.$ (1/2)

$$I = \int \frac{4u^3}{u^2+u}\, du = 4\int \frac{u^2}{u+1}\, du$$

$$= 4\int \frac{u^2+1-1}{u+1}\, du \quad (1/2)$$

$$= 4\left[\int (u-1)\, du + \int \frac{1}{u+1}\, du\right] \quad (1/2)$$

$$= 4\left[\frac{u^2}{2} - u\right] + 4 \ln|u+1| + C \quad (1/2)$$

$$= 4\left[\frac{x^{1/2}}{2} - x^{1/4}\right] + 4\ln|x^{1/4}+1| + C$$