Amplitude and Frequency Modulation

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EE 421

- Instructor: Dr. Anas Al-Hussayen, ahussayen@ksu.edu.sa
- Office Hours: By appointment, Room 2C27 Engineering Building
- Grading:
 - 40% Lab reports
 - 20% Quizzes
 - 40% Final exam (20% written, 20% practical)



Reporting Activities

- Summary: A succinctly written single paragraph given a general description of objective, methodology and main conclusion of the experiment.
- Introduction: Provide some background material about the nature of the experiment and the significance of the questions to be answered by the experiment design.
- Theory: Key mathematical formulas underpinning the reported results, e.g., SNR, channel capacity, etc. Your theory section should be a recapitulation, not a regurgitation of textbook material. Formulas described fully in the textbook can be referenced by their numbers in the textbook, but key formulas most relevant to the measurements/results described in the report should be included in a concise form.
- Equipment: A list of the modules used in the experiment should be provided.



Reporting Activities

- Procedure: This should reflect the Procedure given in the Lab Manual. The report should include documentation from your lab notebook summarizing your observations (e.g. oscilloscope screen shots) associated with each step.
- Results: A concise summary of the measurements in table and/or figure form should be included. The figures and/or tables should give the reader an idea about the mean and variance of specific measurements.
- Discussion: The report should give the reader an idea about any anomalies in the observations or measurements. Give a brief statement about possible reasons and significance. If significant, speculate on possible mitigations even if there is no time to test them in the lab.
- Conclusions: Summarize the main results, margins or statistics of error, significant discrepancies (if any) and possible mitigations.



Reporting Activities

- Grading:
 - 20% Summary, introduction, theory, equipment and procedure
 - 60% Results, analysis, and discussion
 - 20% Conclusions



List of Experiments

- Amplitude and Frequency Modulation
- Pulse Code Modulation
- Shift Keying
- Phase-Locked-Loop
- Demonstration of Wireless Communications



Quiz #1

- 1) In AM, the message modulates the carrier's frequency.
- 2) When $|K_a m(t)| < 1$, the AM signal is <u>over-modulated</u>.
- 3) The massage can <u>not</u> be recovered from <u>under-modulated</u> AM signals.
- 4) AM provides better noise discrimination than FM.
- 5) AM requires more transmission BW than FM.



Why Modulation?

- Upconvert Baseband signal to a passband signal
- Frequency Division Multiplexing (FDM)
- Smaller Antennas
 - For audio (20 KHz) $\rightarrow \lambda$ = 15 Km \rightarrow antenna size = 1.5 Km
- Carrier signal

$$c(t) = A(t)cos(w_c t + \theta(t))$$

Amplitude Modulation

$$\Theta_{AM}(t) = (A + m(t))cos(w_c t)$$

$$\Phi_{AM}(f) = \frac{1}{2} [M(f + f_c) + M(f - f_c)] + \frac{A}{2} [\delta(f + f_c) + \delta(f - f_c)]$$

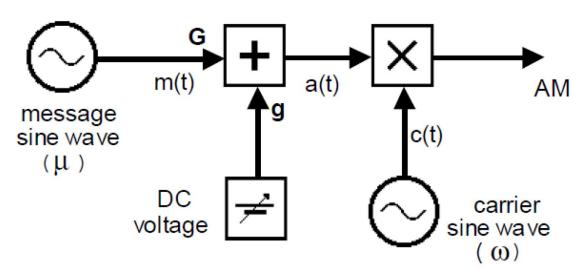


Fig. 1: Conventional AM generator.

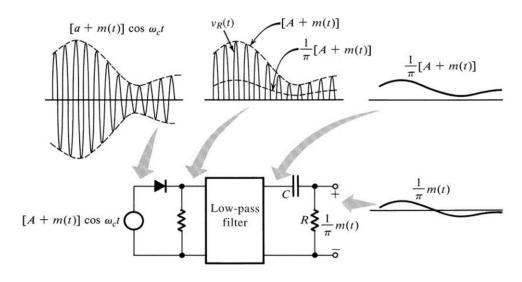


Fig. 2: Envelope detection.



Modulation Index

$$A + m(t) \ge 0$$

$$\Rightarrow A \ge -m_{min}$$

$$\mu = \frac{m_p}{A} \Rightarrow 0 \le \mu \le 1$$

For nonzero offset

$$\mu = \frac{m_{max} - m_{min}}{2A + m_{max} + m_{min}}$$

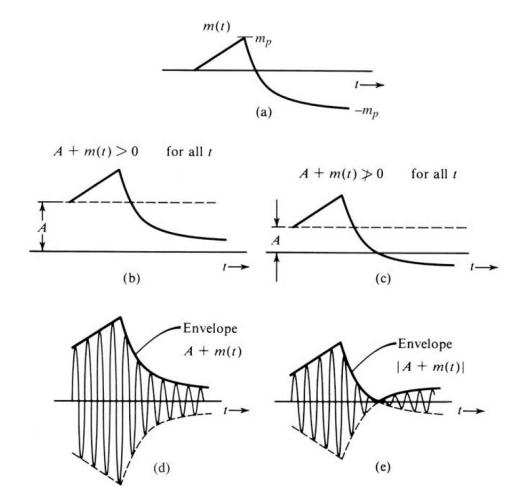


Fig. 3: Illustration of the role of A in AM.

Double Side Band-Suppressed Carrier (DSB-SC)

$$\Theta_{DSB-SC}(t) = m(t)cos(w_c t)$$

$$\Phi_{DSB-SC}(f) = \frac{1}{2} [M(f + f_c) + M(f - f_c)]$$



Bandwidth of AM Signals

For a single-tone message:

$$B_{AM} = 2 f_m$$

• For an arbitrary message with bandwidth W $B_{AM} = 2W$

Frequency Modulation

Carrier signal

$$c(t) = A(t)cos(w_c t + \theta(t))$$

Angular Frequency (Instantaneous Angle)

$$\phi(t) = w_c t + \theta_0 \qquad = \int w(t) dt$$

Instantaneous Frequency

$$w(t) = \frac{d\phi(t)}{dt}$$

$$f(t) = \frac{w(t)}{2\pi}$$

Frequency Deviation

 Frequency Deviation Δf: the maximum difference between instantaneous frequency and carrier frequency.

$$\Delta f = max \left| \frac{1}{2\pi} \frac{d\theta(t)}{dt} \right|$$

• Range of instantaneous frequency: $2\Delta f$ around f_c

$$[f_c - \Delta f, f_c + \Delta f]$$

Frequency Modulation

• In FM, the instantaneous frequency is varied linearly with the message signal, i.e.

$$f(t) = f_c + k_f m(t)$$

• The instantaneous phase:

$$\phi(t) = 2\pi \int_0^t f(t)dt = 2\pi f_c t + 2\pi k_f \int_0^t m(t)dt$$

The FM wave is then:

$$\Theta_{FM}(t) = A_c \cos(2\pi f_c t + 2\pi k_f \int m(t)dt + \theta_0)$$

Frequency Modulation

The frequency deviation becomes

$$\Delta f = max \left| \frac{1}{2\pi} \frac{d\theta(t)}{dt} \right| = k_f max |m(t)|$$

Modulation index

$$\beta \triangleq \frac{\Delta f}{f_m}$$

Single-Tone FM

$$\Theta_{FM}(t) = A_c \cos \left(2\pi f_c t + \beta \sin(2\pi f_m t)\right)$$

$$\Theta_{FM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \ 2\pi \cos \left(2\pi (f_c + nf_m)t\right)$$

$$\Phi_{FM}(f) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \left[\delta \left(f - \left(f_c + n f_m \right) \right) + \delta \left(f + \left(f_c + n f_m \right) \right) \right]$$



Single-Tone FM

A property of Bessel functions:

$$\lim_{n\to\infty} J_n(\beta) = 0$$

 BW of single-tone FM: separation between two frequencies beyond which all

$$|J_n(\beta) < 0.01|$$

The corresponding BW

$$B = 2 n_{max} f_m$$

Carson's Rule

For a single-tone message:

$$B_{FM} \approx 2(f_m + \Delta f) = 2(\beta + 1)f_m$$

• For an arbitrary message with bandwidth *W*

$$B_{FM} \approx 2(W + \Delta f) = 2(\beta + 1)W$$