**Chapter 2: Individual risk model.**

We consider a portfolio of n insurance policies with value , where is the claim of policy i. Suppose that the ’s are independent and identically distributed (iid). The aim of chapter 2 is to compute or approximate the distribution of the portfolio value .

1. Review.

* A function is a cumulative distribution function (cdf for short) if it satisfies the conditions:
* and .
* is increasing,
* is right-continuous.

In this case we associate a random variable (rv) to such that . For a real function g we have .

* For discrete random variables, the cdf is a step function and then the mass function is defined by , where is the left limit of the function at . For a real function g we have
* For continuous random variables, the cdf is absolutely continuous and then the density function is defined by , where is the derivative of the function at . For a real function g we have .

1. Mixed distribution.

* We say that a random variable has a mixed distribution if there exist independent random variables such that is discrete, is continuous, has a Bernoulli distribution with parameter q and . We shall say that Z is the q-mixture of X and Y.
* In order to compute the cdf, mgf and expectation of , we prove the following formula:

1. for a real function .

Indeed we have

.

In particular we get

1. ,
2. ,

and

1. .

**Example1**. Suppose Z is the q-mixture of X and Y with and . Compute the mean, the variance and the mgf of Z.

The mean . The second moment . The mgf .

**Example2**. Let Z has the following cdf:

.

Compute the mean, the variance and the mgf of Z.

The mean

.

The second moment

.

The mgf

, for .

We remark that Z is the 1/2-mixture of X and Y with and .

**Example3**. Let Z has the following cdf:

with .

1. Compute the mean, the variance and the mgf of Z.
2. Z is the q-mixture of some X and Y. Find q and the distributions of X and Y.
3. Convolution.

Suppose X and Y are two independent random variables. Then

.

The product \* is called the convolution product.

* In particular for discrete random variables, we have

,

* For continuous random variables we have

.

**Example1**. Find the cdf of the sum when and are supposed independent in the following cases:

* and .
* and .
* and with given in (#).

1. Transformations.

* We define the following:
* Moment generating function (mgf): , used mainly to compute the moments as follows
* Probability generating function (pgf): , used mainly to compute the mass functions for discrete random variables as follows
* Cumulant generating function (cgf): , used mainly to compute the cumulants and in particular the third central moment as follows
* For two independent random variables X and Y, we have .

**Example1**. Find the cdf of the sum when and are supposed independent in the following cases:

* and .
* and .
* and .

1. Approximations.

5.1- Normal approximation.

The central limit theorem (CLT) says that for a sequence of iid random variables with mean and finite variance , we get for large values of that

.

**Example1**. Suppose that *n* = 1000 young men take out a life insurance policy for a period of one year. The probability of dying within this year is 0*.*01 for everyone and the payment for every death is 1. Calculate the probability that the total payment is at least 4, using the normal approximation.

5.2- Translated Gamma approximation.

Let a random variable with mean , finite variance and skewness . We look for three parameters and such that for , the two random variables and have the same first three moments, which means that: , and . In this case we approximate the cdf of by the cdf of : .

**Example1**. Suppose . Calculate , using the Translated Gamma approximation.

5.3- Normal Power (NP) approximation.

Let a random variable with mean , finite variance and skewness . For we have

and for we have

,

where is the standard normal cdf.

**Example1**. Suppose . Calculate , using the NP approximation.

**Example2**. A total claim amount has expected value 10,000, standard deviation 1,000 and

skewness 1. Calculate the minimum capital that covers loss with probability 95%, using the NP approximation.

1. Application to reinsurance.

An insurance company facing possible big losses or a budget shortage may decide to reinsure one part of the losses. Suppose the total claim amount has expected value , finite variance and skewness . For a prefixed retention , the problem is to compute the minimum capital for which the remaining losses are covered with known probability .

**Example1**. Suppose an insurer is looking for an optimal reinsurance for a portfolio consisting of 20,000 one-year life insurance policies that are grouped as follows:

Insured amount Number of policies

1 10,000

2 5,000

3 5,000

The probability of dying within one year is 0*.*01 for each insured, and the policies are independent. Calculate the minimum capital B that covers losses with probability 95% using the NP approximation:

1. Under no reinsurance.
2. under reinsurance with priority .