King Saud University
College of Sciences
Mathematics Department

Academic Year (G) 2017–2018
Academic Year (H) 1438–1439
Bachelor AFM: M. Eddahbi

Final exam ACTU-362 (40%) (two pages)

December 22, 2018 (three hours 8–11 AM)

Use ballpoint or ink–jet pens and keep three digits after dot

Problem 1. (8 marks)

1. Assume that the force of mortality of life aged x is of the form $\mu_x = F + e^{2x}$, $x \geq 0$ and ${}_{0.4}p_0 = 0.50$. Calculate the constant F.

2. Given $S_0(t) = \sqrt{1 - \frac{t}{100}}$, for $0 \leq t \leq 100$, calculate the probability that a life age 36 will die between ages 51 and 64.

3. Assume that the force of mortality of life aged x is given by :

$$\mu_x = \begin{cases} 0.05 & 50 \leq x < 60 \\ 0.04 & 60 \leq x < 70 \end{cases}$$

   Calculate ${}_{4|14}q_{50}$. (Hint decompose ${}_{18}p_{50} = {}_{10+8}p_{50}$)

4. You are given the following data from a life table: (i) $\ell_{63} = 500$~~0~~ (ii) $q_{63} = 0.050$ (iii) ${}_{1|}q_{63} = 0.070$ (iv) ${}_{2|}q_{63} = 0.042$, ~~[illegible]~~. Complete the following table:

x	ℓ_x	d_x
63	500	
64		
65		
66		—

Problem 2. (8 marks)
You are given the following select–and–ultimate table with a select period of 2 years:

x	$\ell_{[x]}$	$\ell_{[x]+1}$	ℓ_{x+2}	$x+2$
70	22507	22200	21722	72
71	21500	21188	20696	73
72	20443	20126	19624	74
73	19339	19019	18508	75
74	18192	17871	17355	76

1. Define ${}_{2}p_{[71]}$ and find its value

2. Compute the probability that a life age 71 dies between ages 75 and 76, **given that the life was selected at age 70.**

3. Assuming uniform distribution of deaths between integral ages, calculate ${}_{0.5}p_{[70]+0.7}$?.

4. Assuming constant force of mortality between integral ages, calculate ${}_{0.5}p_{[70]+0.7}$?.

Problem 3. (8 marks)

1. Z is the present–value random variable for a whole life insurance of b payable at the moment of death of (x). You are given:
   (i) $\delta = 0.04$, (ii) $\mu_{x+t} = 0.02$ for $t > 0$.
   (ii) The single benefit premium for this insurance is equal to $\text{Var}(Z)$. Calculate the benefit b.

2. You are given: (i) $(DA)^{1}_{40:\overline{10}|} = 5.8$ (ii) $p_{40} = 0.9$ (iii) $i = 0.05$. Find $(DA)^{1}_{41:\overline{9}|}$.

3. You are given: (i) $(IA)_{40} = 6.4$ (ii) $(IA)_{50} = 4.5$ (iii) $(IA)_{40:\overline{10}|}^{\;\;\;\;\;\;\;1} = 0.6$ (iv) $(IA)^{1}_{40:\overline{10}|} = 2.5$. Calculate A_{50} by decomposing the increasing insurance at (40) into an increasing 10 year term insurance 1 an increasing insurance at (50), and 10 units of whole life insurance at (50).

4. You are developing a table of net single premiums for whole life insurance with unit face amount and benefits payable at the end of the year of death. The assumed interest rate is $i = 0.04$. You are given $A_{68} = 0.3285$, and the following life table: $\ell_{65} = 9852$, $\ell_{66} = 9742$, $\ell_{67} = 9625$ and $\ell_{68} = 9500$. Calculate $A^{1}_{65:\overline{3}|}$ and then A_{65}.

Problem 4. (8 marks)

1. For a special fully discrete life insurance on (45), you are given:
   (i) Mortality follows the Illustrative Life Table with $i = 6\%$
   (ii) The death benefit is 1000 until age 65, and 500 thereafter.
   Calculate the actuarial present value of this life insurance.

2. A deferred endowment insurance provides the following benefits:
   (i) A payment of 1 at the moment of death if death occurs between time $t = 5$ and time $t = 12$.
   (ii) A payment of 2 if the insured survives 12 years. No benefit is paid if death occurs before time $t = 5$.
   (iii)
   $$\mu_{x+t} = \begin{cases} 0.05 & 0 \leq t \leq 8 \\ 0.10 & 8 < t \end{cases} \quad \text{and } \delta = 10\%.$$
   Calculate the expected present value of this endowment.

Problem 5. (8 marks)

1. Aicha aged 30 has a choice between two continuous life annuities:
   1. A whole life annuity priced assuming a constant force of mortality $\mu_1 = 0.05$ and a constant force of interest $\delta_1 = 0.03$.
   2. A two–year deferred whole life annuity priced assuming a constant force of mortality $\mu_2 = 0.04$ and a constant force of interest δ_2. Both annuities pay benefits at the same annual rate. The solution to the unique equation $e^{-2(0.04+x)} = 12.5x + 0.5$, is: $x = 0.029604$.
   Determine δ_2 such that the expected present values of the two annuities are equal.

2. (Ia) and $(I\ddot{a})$ represent standard increasing annuities. A person aged 20 buys a special five–year temporary life annuity–due, with payments of 1, 3, 5, 7 and 9. You are given: (i) $\ddot{a}_{20:\overline{4}|} = 3.41$; $a_{20:\overline{4}|} = 3.04$; $(I\ddot{a})_{20:\overline{4}|} = 8.05$; $(Ia)_{20:\overline{4}|} = 7.17$.
   Calculate the actuarial present value of this annuity.

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Useful formulas ACTU-362: December 22, 2018 (three hours 8–11 AM)

$$
\begin{aligned}
{}_tp_x &= S_x(t) = \frac{\ell_{x+t}}{\ell_x}, \quad {}_{t+u}p_x = {}_tp_x \times {}_up_{x+t}, \quad {}_tp_x = e^{-\int_0^t \mu_{x+r}dr} = e^{-\int_x^{x+t} \mu_r dr} \text{ and } S_x(t) = \frac{S_0(x+t)}{S_0(x)} \\
{}_{t|u}q_x &= {}_tp_x \times {}_uq_{x+t} = {}_tp_x - {}_{t+u}p_x = {}_{t+u}q_x - {}_tq_x
\end{aligned}
$$

$$
\begin{aligned}
{}_np_x &= \frac{\ell_{x+n}}{\ell_x} = P((x) \text{ remains alive at age } x+n), \text{ and } q_x = \frac{d_x}{\ell_x} = P((x) \text{ dies between age } x \text{ and } x+1) \\
{}_{m|n}q_x &= \frac{\ell_{x+m} - \ell_{x+m+n}}{\ell_x} = \frac{{}_nd_{x+m}}{\ell_x} = \frac{\sum_{k=0}^{n-1} d_{x+m+k}}{\ell_x} = P((x) \text{ dies between age } x+m \text{ and } x+m+n)
\end{aligned}
$$

and

$$
\text{UDD } ({}_rq_x = rq_x), \quad \text{CFM } ({}_rp_{x+u} = p_x^r), \; \forall\, x \in \mathbb{N},\ 0 < r < 1 \text{ and } 0 < u + r \leq 1
$$

Under **CFM** assumption $\mu_{x+r} = \dfrac{q_x}{1 - rq_x}$ for all integer x and $0 < r < 1$.

$$
A_x = vq_x + vp_x\, A_{x+1}, \quad \ddot{a}_x = \frac{1 - A_x}{d} \text{ and } {}_{n|}\ddot{a}_x = {}_nE_x \ddot{a}_{x+n}, \quad a_x = \ddot{a}_x - 1, \quad \ddot{a}_x = 1 + vp_x\ddot{a}_{x+1}
$$

$$
a_{x:\overline{n-1}|} = \ddot{a}_{x:\overline{n}|} - 1, \quad \text{and } \ddot{a}_{x:\overline{n}|} = \sum_{k=0}^{n-1} v^k \,{}_kp_x
$$

$$
A^1_{x:\overline{n}|} = \sum_{k=0}^{n-1} v^{k+1}\,{}_{k|}q_x = \sum_{k=0}^{n-1} v^{k+1}\,{}_kp_xq_{x+k} \text{ and } A^1_{x:\overline{n}|} = A_x - {}_{n|}A_x = A_x - {}_nE_xA_{x+n} \text{ and } \bar{a}_{x:\overline{n}|} = \bar{a}_x - {}_nE_x\bar{a}_{x+n}
$$

Recursions:

$$
\begin{aligned}
&\ddot{a}_{x:\overline{n}|} = 1 + vp_x\ddot{a}_{x+1:\overline{n-1}|}, \quad A_{x:\overline{n}|} = vq_x + vp_xA_{x+1:\overline{n-1}|}, \quad A^1_{x:\overline{n}|} = vq_x + vp_xA^{\;1}_{x+1:\overline{n-1}|} \\
&(IA)^1_{x:\overline{n}|} = vq_x + vp_x\left(A^{\;1}_{x+1:\overline{n-1}|} + (IA)^{\;1}_{x+1:\overline{n-1}|}\right), \quad (DA)^1_{x:\overline{n}|} = nvq_x + vp_x(DA)^{\;1}_{x+1:\overline{n-1}|} \\
&(IA)^1_{x:\overline{n}|} = \textstyle\sum_{k=0}^{n-1}(k+1)\,v^{k+1}\,{}_kp_xq_{x+k}; \quad (DA)^1_{x:\overline{n}|} = \textstyle\sum_{k=0}^{n-1}(n-k)\,v^{k+1}\,{}_kp_xq_{x+k} \\
&\bar{A}_{x:\overline{n}|} = \bar{A}^1_{x:\overline{n}|} + {}_nE_x, \quad \bar{A}_{x:\overline{n}|} = \bar{A}^1_{x:\overline{n}|} + {}_nE_x, \quad {}_nE_x = v^n\,{}_nP_x; \text{ and } (I\bar{A})_{x:\overline{n}|} = (I\bar{A})^1_{x:\overline{n}|} + n\;{}_nE_x
\end{aligned}
$$

Under **UDD** $\bar{A}_x = \dfrac{i}{\delta}A_x$ and $\bar{A}^1_{x:\overline{n}|} = \dfrac{i}{\delta}A^1_{x:\overline{n}|}$.

Under **CRM** $\bar{A}^1_{x:\overline{n}|} = \dfrac{\mu}{\delta + \mu}(1 - {}_nE_x)$ and ${}_nE_x = e^{-(\delta+\mu)n}$