King Saud University
College of Sciences
Mathematics Department

Academic Year (G) 2018–2019
Academic Year (H) 1439–1440
Bachelor AFM: M. Eddahbi

Final exam ACTU. 362 ; Spring 2019 (40%) (two pages)

May 2, 2019 (three hours 8–11 AM)

Problem 1. (8 marks)

1. An individual aged 40 is subject to the survival function

$$S_{40}(t) = \begin{cases} 1 - \dfrac{1}{200}t & \text{if} \quad t < 20 \\ \\ \dfrac{13}{10} - \dfrac{1}{50}t & \text{if} \quad 20 \le t \le 65 \end{cases}$$

Calculate the probability that an individual aged 50 survives at least 30 years.

2. According to the following mortality table:

x	ℓ_x	d_x	$_{x-60\|}q_{60}$
60	1000		
61	—	100	
62	—	—	0.07
63	780	—	—

Calculate q_{60}

3. Given the density $f_0(t) = \dfrac{20-t}{200}$ for $0 \le t \le 20$, calculate the force of mortality at age 10.

4. Assume that $_tp_x = 1 - \dfrac{t^2}{100}$ for $0 < t \le 10$. Find μ_{x+5}.

Problem 2. (8 marks)

1. Future lifetime of (20) is subject to force of mortality $\mu_x = \frac{1}{100-x}$ for $x < 100$. Calculate $\mathring{e}_{20:\overline{50}|}$

2. For a life whose survival function is $S_0(t) = 1 - \frac{t}{\omega}$ you are given that $e_{20:\overline{50}|} = 18$. Determine the limiting age ω.

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3. A person age 70 is subject to the following force of mortality:

$$\mu_{70+t} = \begin{cases} 0.01 & \text{if} \quad t \le 5 \\ \\ 0.02 & \text{if} \quad t > 5 \end{cases}$$

Calculate $\mathring{e}_{70}$ for this person.

4. Consider the following Mortality is select and ultimate table

x	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	q_{x+3}	$x+3$
40	0.002	0.005	0.008	0.012	43
41	0.003	0.006	0.009	0.015	44
42	0.004	0.007	0.010	0.018	45

Calculate $_{2|3}q_{[41]}$.

Problem 3. (8 marks)

1. If $\ell_x = 103 - x$ for $0 \le x \le 103$, and the force of interest is $\overset{\delta}{\cancel{i}} = 0.06$, calculate $\bar{A}_{45:\overline{20}|}$.
2. A 20–year deferred insurance on (35) pays a benefit of 5000 at the moment of death if death occurs no earlier than 20 years from now. You are given: (i) Mortality for (35) is uniformly distributed with $\omega = 100$. (ii) $\delta = 0.06$, (iii) Z is the random variable for the present value of the insurance. Calculate $E[Z]$.
3. For a special whole life insurance policy on (55) with benefits payable at the moment of death, you are given:
(i) $b_t = \dfrac{50}{50-t}$, for $t < 50$. (ii) $\mu_x = \dfrac{2}{105-x}$, for $x < 105$. (iii) $\delta = 0.04$. Calculate the net single premium for this insurance.
4. The net single premium for a 20–year term insurance on a person currently age 40 with benefit of 1 payable at the moment of death is 0.065. Now assume that (i) $\delta = 0.04$, (ii) ${}_{20}p_{40} = 0.88$, (iii) $p_{60} = 0.99$ and (iv) Deaths occurring between ages 60 and 61 are uniformly distributed.
Calculate the net single premium for a 21–year endowment insurance on a person currently age 40 with benefit of 1 payable at the moment of death.

Problem 4. (8 marks)

1. You are given the following: (i) $A_x = 0.25$, (ii) $A_{x+15} = 0.40$, (iii) $A_{x:\overline{15}|} = 0.50$, determine $A^{1}_{x:\overline{15}|}$.
2. For a 20–year term insurance on a life currently age 40: (i) $q_{40+k} = 0.01$, $k \ge 0$, (ii) if death occurs in the interval $(k, k+1]$, the benefit is $2000\,(1.03)^k$. (iii) The benefit is paid at the end of the year of death. Calculate the expected present value of the insurance when $i = 6\%$..
3. A continuously increasing whole life insurance on (40) pays a benefit of t at the moment of death if death occurs at time t. Calculate the net single premium for this insurance when $\delta = 0.02$ and the mortality follows $\ell_x = 120 - x$, $0 \le x \le 120$.
4. For a whole life insurance of 1000 on (80), with death benefits payable at the end of the year of death, you are given:
(i) Mortality follows a select and ultimate mortality table with a **one–year select period**.
(ii) $q_{[80]} = 0.5 q_{80}$, $i = 0.06$, (iv) $1000 A_{80} = 679.80$, (v) $1000 A_{81} = 689.52$. Calculate $1000 A_{[80]}$.

Problem 5. (8 marks)

1. A special temporary 3–year life annuity–due on (30) pays k at the beginning of year k, $k = 1, 2, 3$. Given: (i) $q_{30} = 0.01$, (ii) $q_{31} = 0.015$, (iii) $q_{32} = 0.02$ (iv) $i = 0.04$, compute the expected present value of this annuity.
2. You are given: $\bar{A}_{40} = 0.4$, $\bar{A}_{40:\overline{10}|} = 0.7$, ${}_{10}p_{40} = 0.9$ and the force of interest at time t is given by
$$\delta_t = \begin{cases} 0.05 & \text{if} \quad t \le 10 \\ 0.04 & \text{if} \quad t > 10 \end{cases}$$
Calculate $\bar{a}_{40}$
3. You are given the following:
(i) $A_{x:\overline{n}|} = 0.693$ (ii) ${}_nE_x = 0.566$ (iii) $\mu_x = 0.008$ (iv) $\mu_{x+n} = 0.025$, (v) $\delta = 0.04$.
Estimate $\ddot{a}^{(4)}_{x:\overline{n}|}$
   (a) using Woolhouse's formula with two terms.
   (b) using Woolhouse's formula with three terms.
4. Assume that the force of mortality is given by $\mu_x = \dfrac{1}{100-x}$, $0 \le x < 100$. Calculate $\bar{a}_{75:\overline{20}|}$ for $\delta = 0.06$.

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Useful formulas ACTU-362: May 2, 2019

$$
\begin{aligned}
{}_tp_x &= S_x(t) = \frac{\ell_{x+t}}{\ell_x}, \quad {}_{t+u}p_x = {}_tp_x \times {}_up_{x+t}, \quad {}_tp_x = e^{-\int_0^t \mu_{x+r}dr} = e^{-\int_x^{x+t} \mu_r dr} \text{ and } S_x(t) = \frac{S_0(x+t)}{S_0(x)} \\
{}_{t|u}q_x &= {}_tp_x \times {}_uq_{x+t} = {}_tp_x - {}_{t+u}p_x = {}_{t+u}q_x - {}_tq_x, \quad \mathring{e}_x = \int_0^\infty {}_tp_x dt, \quad \mathring{e}_{x:\overline{n}|} = \int_0^n {}_tp_x dt, \quad e_x = \sum_{k=1}^{\infty} {}_kp_x
\end{aligned}
$$

$$
\begin{aligned}
{}_np_x &= \frac{\ell_{x+n}}{\ell_x} = P((x) \text{ remains alive at age } x+n), \text{ and } q_x = \frac{d_x}{\ell_x} = P((x) \text{ dies between age } x \text{ and } x+1 \\
{}_{m|n}q_x &= \frac{\ell_{x+m} - \ell_{x+m+n}}{\ell_x} = \frac{{}_nd_{x+m}}{\ell_x} = \frac{\sum_{k=0}^{n-1} d_{x+m+k}}{\ell_x} = P((x) \text{ dies between age } x+m \text{ and } x+m+n)
\end{aligned}
$$

under **UDD** ${}_rq_x = rq_x$, $f_x(r) = q_x$ and $\mathring{e}_x = e_x + \frac{1}{2}$, $\mathring{e}_{x:\overline{n}|} = e_{x:\overline{n}|} + 0.5\ {}_nq_x$and $\mathring{e}_{x:\overline{n}|} = {}_np_x(n) + {}_nq_x\left(\frac{n}{2}\right)$.
under **CFM** $\mu_{x+r} = \frac{q_x}{1-rq_x}$ $\forall x \in \mathbb{N}$ and $0 < r < 1$. $({}_rp_{x+u} = p_x^r \quad \forall x \in \mathbb{N},\ 0 < r < 1 \text{ and } 0 < u + r \leq 1)$.

$$
A_x = vq_x + vp_x\, A_{x+1}, \quad \ddot{a}_x = \frac{1 - A_x}{d} \quad \text{and} \quad {}_{n|}\ddot{a}_x = {}_nE_x \ddot{a}_{x+n}, \quad a_x = \ddot{a}_x - 1, \quad \ddot{a}_x = 1 + vp_x \ddot{a}_{x+1}
$$

$$
\begin{aligned}
a_{x:\overline{n-1}|} &= \ddot{a}_{x:\overline{n}|} - 1, \quad \ddot{a}_{x:\overline{n}|} = \sum_{k=0}^{n-1} v^k\ {}_kp_x \quad \text{and} \quad (I\bar{a})_{x:\overline{n}|} = \int_0^n \lfloor t+1 \rfloor e^{-t\delta}\ {}_tp_x dt, \\
(I\ddot{a})_{x:\overline{n}|} &= \sum_{k=0}^{n-1} (k+1)v^k\ {}_kp_x, \quad (Ia)_{x:\overline{n}|} = \sum_{k=1}^{n} kv^k\ {}_kp_x,
\end{aligned}
$$

$$
A^1_{x:\overline{n}|} = \sum_{k=0}^{n-1} v^{k+1}\ {}_{k|}q_x = \sum_{k=0}^{n-1} v^{k+1}\ {}_kp_x q_{x+k} \text{ and } A^1_{x:\overline{n}|} = A_x - {}_{n|}A_x = A_x - {}_nE_x A_{x+n} \text{ and } \bar{a}_{x:\overline{n}|} = \bar{a}_x - {}_nE_x \bar{a}_{x+n}
$$

Recursions:

$\mathring{e}_x = \mathring{e}_{x:\overline{n}|} + {}_np_x \mathring{e}_{x+n}$, (Under UDD $\mathring{e}_{x:\overline{n}|} = e_{x:\overline{n}|} + 0.5\ {}_nq_x$)

$\ddot{a}_{x:\overline{n}|} = 1 + vp_x \ddot{a}_{x+1:\overline{n-1}|}$, $A_{x:\overline{n}|} = vq_x + vp_x A_{x+1:\overline{n-1}|}$, $A^1_{x:\overline{n}|} = vq_x + vp_x A^1_{x+1:\overline{n-1}|}$

$(IA)^1_{x:\overline{n}|} = vq_x + vp_x\left(A^1_{x+1:\overline{n-1}|} + (IA)^1_{x+1:\overline{n-1}|}\right)$, $(DA)^1_{x:\overline{n}|} = nvq_x + vp_x(DA)^1_{x+1:\overline{n-1}|}$

$(IA)^1_{x:\overline{n}|} = \sum_{k=0}^{n-1}(k+1)\,v^{k+1}\ {}_kp_x\ q_{x+k}$; $(DA)^1_{x:\overline{n}|} = \sum_{k=0}^{n-1}(n-k)\,v^{k+1}\ {}_kp_x q_{x+k}$,

$\bar{A}_{x:\overline{n}|} = \bar{A}^1_{x:\overline{n}|} + {}_nE_x$, $A_{x:\overline{n}|} = A^1_{x:\overline{n}|} + {}_nE_x$, ${}_nE_x = v^n\,{}_np_x$; and $(I\bar{A})_{x:\overline{n}|} = (I\bar{A})^1_{x:\overline{n}|} + n\ {}_nE_x$;

$$
\textbf{UDD } \bar{A}_x = \frac{i}{\delta} A_x \text{ and } \bar{A}^1_{x:\overline{n}|} = \frac{i}{\delta} A^1_{x:\overline{n}|}. \ \textbf{CRM } \bar{A}^1_{x:\overline{n}|} = \frac{\mu}{\delta + \mu}(1 - {}_nE_x) \text{ and } {}_nE_x = e^{-(\delta+\mu)n}
$$

Woolhouse's formula with two terms	Woolhouse's formula with three terms
$\ddot{a}^{(m)}_{x:\overline{n}|} = \ddot{a}_{x:\overline{n}|} - \frac{m-1}{2m}(1 - {}_nE_x)$	$\ddot{a}^{(m)}_{x:\overline{n}|} = \ddot{a}_{x:\overline{n}|} - \frac{m-1}{2m}(1 - {}_nE_x) - \frac{m^2-1}{12m^2}\left(\mu_x + \delta - {}_nE_x\left(\mu_{x+n} + \delta\right)\right)$