حل التمارين 3,4

Chs. 3,4

one Population

for μ

① $n=10$, from the sample that we have we find $\bar{X}=1.0465$ and $S=.03103$ → معلومات عن العينة

population is normal, σ^2 unknown → معلومات عن المجتمع

90% C.I for μ ?!

$\mu \in \bar{X} \pm t_{n-1,1-\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$, $(1-\alpha)100=90 \Rightarrow 1-\alpha=.9 \Rightarrow \alpha=.1 \Rightarrow t_{n-1,1-\frac{\alpha}{2}}=t_{9,.95}=1.8331$

$\therefore \mu \in (1.02851, 1.06449)$

النتائج باستخدام Minitab:

One Sample T: 1

Variable	N	Mean	StDev	90% CI
1	10	1.0465	.03103	(1.02851, 1.06449)

② $n=14$, from the sample that we have we find $\bar{X}=47.1429$ and $S=3.2548$ → معلومات عن العينة

population is normal, σ^2 unknown → معلومات عن المجتمع

95% C.I for μ ?!

$\mu \in \bar{X} \pm t_{n-1,1-\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$, $(1-\alpha)100=95 \Rightarrow 1-\alpha=.95 \Rightarrow \alpha=.05 \Rightarrow t_{n-1,1-\frac{\alpha}{2}}=t_{13,.975}=2.16037$

$\therefore \mu \in (45.26363, 49.02217)$

النتائج باستخدام Minitab:

One Sample T: 1

Variable	N	Mean	StDev	95% C.I
1	14	47.1429	3.2548	(45.2636, 49.0221)

③ $n=185$, $\bar{X}=141$ → معلومات عن العينة

population is normal, $\sigma^2=3025 \Rightarrow \sigma=55$ → معلومات عن المجتمع

(*) 90% C.I for μ ?!

$\mu \in \bar{X} \pm Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$, $(1-\alpha)100=90 \Rightarrow 1-\alpha=.9 \Rightarrow \alpha=.1 \Rightarrow Z_{1-\frac{\alpha}{2}}=Z_{.95}=1.645$

$\therefore \mu \in (134.34815, 147.65185)$

النتائج باستخدام Minitab:

One Sample Z

The assumed standard deviation = 55

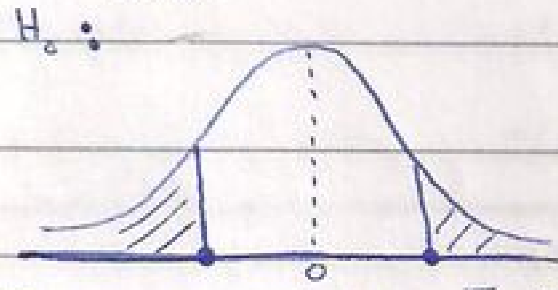
N	Mean	90% C.I
185	141	(134.349, 147.651)

✓

(**) $H_0: \mu = 130$ vs $H_1: \mu \neq 130$, $\alpha = .1$, $\mu_0 = 130$?!

the statistic: $Z = \dfrac{\bar{X} - \mu_0}{\sigma / \sqrt{n}} \sim N(0,1) \Rightarrow Z = 2.72029$

R.R and A.R of H_0:



$Z_{\frac{\alpha}{2}} = -Z_{1-\frac{\alpha}{2}} = -1.645$ $\quad Z_{1-\frac{\alpha}{2}} = Z_{.95} = 1.645$

decision: as $Z = 2.72029 > 1.645$, so we reject H_0

النتائج باستخدام Minitab :

نلاحظ أن p-value < α
وهذا يدعم القرار الذي توصلنا إليه --

One sample Z

test of mu = 130 vs not = 130

The assumed standard deviation = 55

N	Mean	90% C.I	Z	P
185	141	(134.349, 147.651)	2.72	.007

(4) $\sigma^2 = 5$. ← معلومات عن المجتمع

99% C.I , width = 1

the sample size n ?!

$n = \left(\dfrac{Z_{1-\frac{\alpha}{2}}}{d} \right)^2 \sigma^2$.

$(1-\alpha)100 = 99 \Rightarrow 1-\alpha = .99 \Rightarrow \alpha = .01 \Rightarrow Z_{1-\frac{\alpha}{2}} = Z_{.995} = 2.57$

width = 1 , width = 2d $\Rightarrow d = \frac{1}{2} = .5$

$\therefore n = 132.098 \simeq 133$

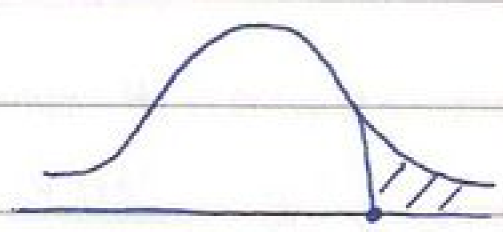
For P

(3.2) $n = 86$, $a = 74$, $r = \frac{a}{n} = .8605$ ← معلومات عن العينة

(a) $H_0: P = .75$ vs $H_1: P > .75$, $\alpha = .05$, $P_0 = .75$?!

the statistic: $Z = \dfrac{r - P_0}{\sqrt{\frac{P_0(1-P_0)}{n}}} \sim N(0,1) \Rightarrow Z = 2.3658$

R.R and A.R of H_0:



$Z_{1-\alpha} = Z_{.95} = 1.645$

decision: as $Z = 2.3658 > 1.645$, so we reject H_0

Test and CI for one proportion						:Minitab
Test of $p = .75$ Vs $p > .75$						
sample	X	N	sample p	95% lower bound	Z-Value	P-Value
1	74	86	.860465	.799006	2.37	.009

نلاحظ أن p-Value < α وهذا يدعم القرار الذي توصلنا إليه.

ⓑ 99% C.I for P ?!

$p \in r \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{r(1-r)}{n}}$, $(1-\alpha)100 = 99 \Rightarrow 1-\alpha = .99 \Rightarrow \alpha = .01 \Rightarrow Z_{1-\frac{\alpha}{2}} = Z_{.995} = 2.57$

$\therefore p \in (.7642, .9567)$

النتائج باستخدام Minitab:

Test and CI for one Proportion						
Test of $p = .75$ Vs p not $= .75$						
Sample	X	N	sample p	99% CI	Z-Value	P-Value
1	74	86	.860465	(.764221, .956710)	2.37	.018

ⓒ 99% C.I , width $= .1$

the sample size $n = ?!$

$n = \left(\frac{Z_{1-\frac{\alpha}{2}}}{d}\right)^2 r(1-r)$

$(1-\alpha)100 = 99 \Rightarrow 1-\alpha = .99 \Rightarrow \alpha = .01 \Rightarrow Z_{1-\frac{\alpha}{2}} = Z_{.995} = 2.57$

width $= .1$, width $= 2d \Rightarrow d = \frac{.1}{2} = .05$

we have nothing about r, so use $r = .5$

$\therefore n = 660.49 \simeq 661$

For σ^2

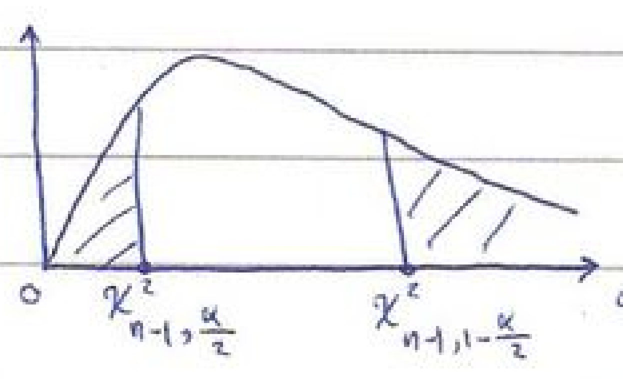
(2.43) $n = 54$, $\bar{X} = 177.6$, $s = 3.6$ → معلومات عن العينة

Population is normal → معلومات عن المجتمع

ⓐ $H_0: \sigma^2 = 20$ vs $H_1: \sigma^2 \neq 20$, $\alpha = .05$, $\sigma_0^2 = 20$?!

the statistic : $\chi^2 = \frac{(n-1)S^2}{\sigma_0^2} \sim \chi^2_{n-1} \Rightarrow \chi^2 = 34.344$

R.R and A.R of H_0:



$\chi^2_{n-1,\frac{\alpha}{2}} = \chi^2_{53,.025} = \chi^2_{50,.025} + \frac{53-50}{60-50}\left[\chi^2_{60,.025} - \chi^2_{50,.025}\right]$

$= 32.357 + \frac{3}{10}\left[40.482 - 32.357\right] = 34.7945$

✓

and $\chi^2_{n-1,1-\frac{\alpha}{2}} = \chi^2_{53,.975} = \chi^2_{50,.975} + \frac{53-50}{60-50}\left[\chi^2_{60,.975} - \chi^2_{50,.975}\right]$

$= 71.420 + \frac{3}{10}\left[83.298 - 71.420\right] = 74.9834$

decision: as $\chi^2 = 34.344 < 34.7945$, so we reject H_0

(b) 95% C.I for σ^2?!

$$\frac{(n-1)s^2}{\chi^2_{n-1,1-\frac{\alpha}{2}}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{n-1,\frac{\alpha}{2}}}$$

$(1-\alpha)100 = 95 \Rightarrow 1-\alpha = .95 \Rightarrow \alpha = .05 \Rightarrow \chi^2_{n-1,\frac{\alpha}{2}} = \chi^2_{53,.025} = 34.7945$ and $\chi^2_{n-1,1-\frac{\alpha}{2}} = \chi^2_{53,.975} = 74.9834$

$\therefore \sigma^2 \in (9.1604, 19.7411)$

two Population

For $\mu_1 - \mu_2$

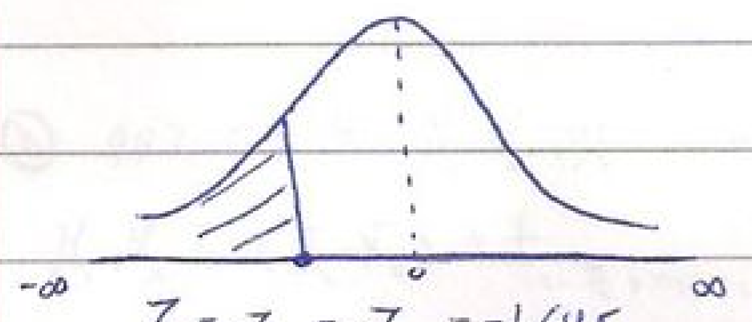
(5) $n_1 = 28$, $\bar{X}_1 = 5.3$
$n_2 = 24$, $\bar{X}_2 = 8.8$ → معلومات عن العينتين المستقلتين

Populations are normal, $\sigma_1^2 = 2$, $\sigma_2^2 = 3$ → معلومات عن المجتمعين

(a) $H_0: \mu_1 = \mu_2 \cong \mu_1 - \mu_2 = 0$ Vs $H_1: \mu_1 < \mu_2 \cong \mu_1 - \mu_2 < 0$, $\alpha = .05$?!

the statistic: $Z = \dfrac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0,1) \Rightarrow Z = -7.897$

R.R and A.R of H_0:



$Z_\alpha = -Z_{1-\alpha} = -Z_{.95} = -1.645$

decision: as $Z = -7.897 < -1.645$, so reject H_0

(b) 90% CI for $\mu_1 - \mu_2$?!

$\mu_1 - \mu_2 \in (\bar{X}_1 - \bar{X}_2) \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

$(1-\alpha)100 = 90 \Rightarrow 1-\alpha = .9 \Rightarrow \alpha = .1 \Rightarrow Z_{1-\frac{\alpha}{2}} = Z_{.95} = 1.645$

$\therefore \mu_1 - \mu_2 \in (-4.2291, -2.7709)$

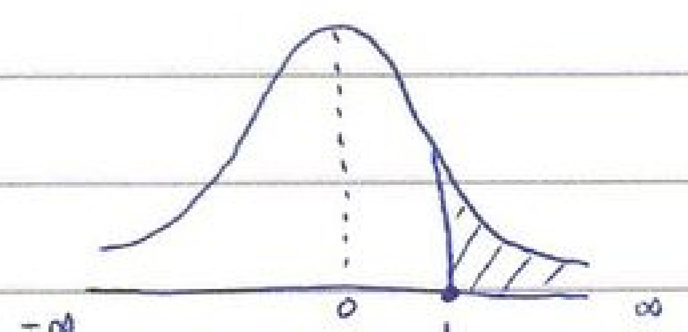
(6) $n_1 = 10$, from the first sample we find $\bar{X}_1 = 94.645$ and $S_1 = .50302$
$n_2 = 10$, from the second sample we find $\bar{X}_2 = 91.340$ and $S_2 = .48293$ → معلومات عن العينتين المستقلتين

Population are normal, $\sigma_1^2 = \sigma_2^2 = ?$ → معلومات عن المجتمعين

(a) $H_0: \mu_1 = \mu_2 \cong \mu_1 - \mu_2 = 0$ Vs $H_1: \mu_1 > \mu_2 \cong \mu_1 - \mu_2 > 0$, $\alpha = .01$?!

The statistic: $T = \dfrac{\bar{X}_1 - \bar{X}_2}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim T_{n_1+n_2-2}$, $S_p = \sqrt{\dfrac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}} \Rightarrow T = 14.98784$, $S_p = .49308$

R.R and A.R of H_0:



$t_{1-\alpha, n_1+n_2-2} = t_{.99, 18} = 2.552$

the decision: as $T = 14.98784 > 2.552$, so reject H_0

✓

```
two-Sample T-test and CI: 1, 2
Two-Sample T For 1 vs 2
     N      Mean      StDev
1   10    94.645      .503
2   10    91.34       .483
Difference = mu(1) - mu(2)
Estimate for difference : 3.30500
99% lower bound for difference : 2.74217
T-test of difference = 0 Vs (>) : T-value = 14.99  P-value = 0  DF = 18
Both use Pooled StDev = .4431
```

ستخدام Minitab :

نجد أن p-value < α

وهذا يدعم القرار الذي توصلنا إليه.

ⓑ 99% C.I for $\mu_1 - \mu_2$?!

$$\mu_1 - \mu_2 \in (\bar{X}_1 - \bar{X}_2) \pm t_{1-\frac{\alpha}{2}, n_1+n_2-2} \, S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$(1-\alpha)100 = 99 \Rightarrow 1-\alpha = .99 \Rightarrow \alpha = .01 \Rightarrow t_{1-\frac{\alpha}{2}, n_1+n_2-2} = t_{.995, 18} = 2.8784$$

$$\therefore \mu_1 - \mu_2 \in (2.67, 3.94)$$

الناتج باستخدام Minitab :

```
two-sample T-test and C.I : 1, 2
two-sample T For 1 vs 2
     N      mean      StDev
1   10    94.645      .503
2   10    91.34       .483
Difference = mu(1) - mu(2)
Estimate for difference : 3.30500
99% C.I for difference : (2.67027, 3.93973)
T-test of difference = 0 Vs (not =) : T-value = 14.94   P-value = 0   DF = 18
Both use pooled StDev = .4431
```

✓

(7) عينتين مرتبطة , population is not normal → المجتمع

, 95% C.I for $\mu_1 - \mu_2 = \mu_D$?!

$\mu_D \in \bar{D} \pm Z_{1-\frac{\alpha}{2}} \frac{S_D}{\sqrt{n}}$

	Before	after	D = Before - after
1	148	78	70
2	145	78	67
3	123	80	43
4	140	81	59
5	129	87	42
6	119	70	49
7	151	94	57
8	122	79	43
9	120	75	45
10	150	89	61
11	102	70	32
12	154	130	24
13	114	60	54
14	129	70	59
15	148	70	78
16	113	60	53
17	117	120	-3
18	122	81	41
19	149	95	54
20	109	67	42
21	137	63	74
22	154	83	71
23	110	70	40
24	107	80	27
25	143	72	71
26	134	71	63
27	151	76	75
28	129	61	68
29	131	89	42
30	129	60	69
31	108	71	37

$(1-\alpha)100 = 95 \Rightarrow 1-\alpha = .95 \Rightarrow \alpha = .05 \Rightarrow Z_{1-\frac{\alpha}{2}} = Z_{.975} = 1.96$

$n = n_1 = n_2 = 31$

From the sample D we find:

$\bar{D} = 51.83871$ and $S_D = 17.96496$

$\therefore \mu_D \in (45.51457, 58.16285)$

↙

لا ينفع استخدام الكمبيوتر الا اذا

كان لدينا جميع بيانات كل الأفراد النتيجة الآتية

$\mu_D \in (46.1298, 60.1283$

(7)

(8)

عينتين مرتبطة

	Method A	Method B	D = Method A - Method B
1	27	23	4
2	37	28	9
3	31	30	1
4	38	32	6
5	29	27	2
6	35	29	6
7	41	36	5
8	37	31	6

From the sample D we find:

$\bar{D} = 4.875$, $S_D = 2.53194$

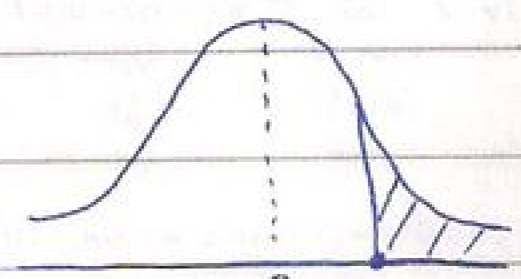
$n = n_1 = n_2 = 8$

Population is normal $\longrightarrow$ معلومات عن المجتمع

$H_0: \mu_A = \mu_B \equiv \mu_A - \mu_B = 0 \equiv \mu_D = 0$ vs $H_1: \mu_A > \mu_B \equiv \mu_A - \mu_B > 0 \equiv \mu_D > 0$, $\alpha = .05$?!

The statistic: $T = \dfrac{\bar{D}}{S_D / \sqrt{n}} \sim T_{n-1} \Rightarrow T = 5.45$

R.R and A.R of H_0:



0 $\quad t_{n-1,1-\alpha} = t_{7,.95} = 1.8946$

decision: as $T = 5.45 > 1.8946$, so reject H_0

النتائج باستخدام Minitab :

Paired T-test and CI: 1,2

Paired T for 1-2

	N	Mean	StDev
1	8	34.375	4.8679
2	8	29.5	3.8173
difference	8	4.875	2.53194

95% lowerbound for mean difference: 3.17902

T-test of mean difference = 0 vs (>): T-value = 5.45 p-value = 0

نجد أن p-value < α

وهذا يدعم القرار الذي توصلنا إليه

✓

(232) $n_1 = 16$, from the first sample we find $\bar{X}_1 = 2.3063$ and $S_1 = .2516$ → عن عينتين مستقلتين

$n_2 = 16$, from the second sample we find $\bar{X}_2 = 2.5063$ and $S_2 = .2839$

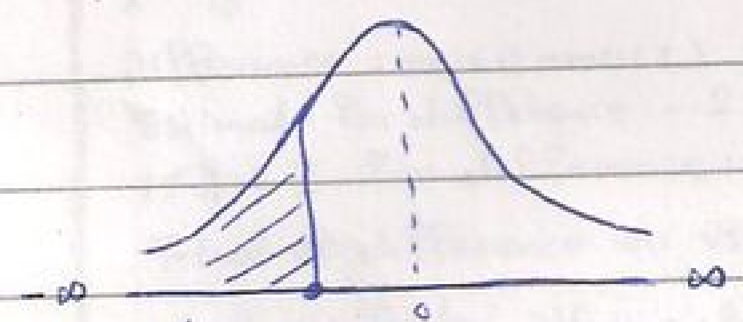
populations are normal, $\sigma_1^2 = \sigma_2^2 = ?$ → متساويتين لكن مجهولتين

ⓐ $H_0: \mu_1 = \mu_2 \equiv \mu_1 - \mu_2 = 0$ Vs $H_1: \mu_1 < \mu_2 \equiv \mu_1 - \mu_2 < 0$, $\alpha = .1$?!

the statistic : $T = \dfrac{\bar{X}_1 - \bar{X}_2}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim T_{n_1+n_2-2}$, $S_p = \sqrt{\dfrac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}}$

$\Rightarrow T = -2.0419$, $S_p = .26823$

R.R and A.R of H_0 :



$t_{n_1+n_2-2, \alpha}$

$= -t_{n_1+n_2-2, 1-\alpha}$

$= -t_{30, .9} = -1.31042$

decision : as $T = -2.0419 < -1.31042$, so reject H_0

النتائج باستخدام Minitab :

Two-sample T-test and CI : 1, 2

Two-sample T for 1 vs 2

	N	mean	StDev
1	16	2.306	.063
2	16	2.506	.071

Difference = mu(1) - mu(2)

Estimate for difference : -.2

90% upper bound for difference : -.075719

T-test of difference = 0 vs (<) : T-value = -2.11 P-value = .022 DF = 30

Both use Pooled StDev = .2683

نلاحظ أن p-value < α

وهذا يدعم القرار الذي توصلنا إليه

✓

(b) 95% C.I for $\mu_1-\mu_2$?!

$\mu_1-\mu_2 \in (\bar{X}_1-\bar{X}_2) \pm t_{n_1+n_2-2,1-\frac{\alpha}{2}} \, S_p \sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$

$(1-\alpha)100=95 \Rightarrow 1-\alpha=.95 \Rightarrow \alpha=.05 \Rightarrow t_{n_1+n_2-2,1-\frac{\alpha}{2}}=t_{30,.975}=2.04227$

$\therefore \mu_1-\mu_2 \in (-.39368 \quad , -.00632 \quad)$

النتائج باستخدام Minitab :

Two-sample T-test and CI : 1,2

two-sample T for 1 vs 2

	N	mean	StDev
1	16	2.306	.063
2	16	2.506	.071

Difference = mu(1) − mu(2)

Estimate for difference : −.2

95% C.I for difference : (−.393691, −.006309)

T-test of difference = 0 vs (not =) : T-Value = −2.11 P-Value = .043 DF = 30

Both use pooled StDev = .2683

(2.34) $n_1=54$, from the first sample we find $\bar{X}_1=67.7$ and $S_1=38$

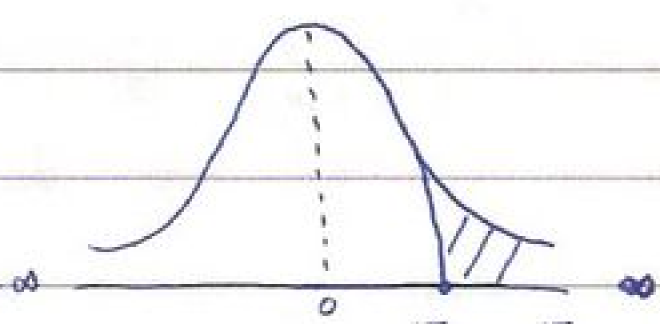
$n_2=53$, from the second sample we find $\bar{X}_2=28.7$ and $S_2=15.7$ معلومات عن عينتين مستقلتين

Populations are normal, $\sigma_1^2 \neq \sigma_2^2$, $\sigma_1^2=?$, $\sigma_2^2=?$, $\alpha=.05 \longrightarrow$ معلومات عن المجتمعين

(a) $H_0: \mu_1=\mu_2 \equiv \mu_1-\mu_2=0$ Vs $H_1: \mu_1>\mu_2 \equiv \mu_1-\mu_2>0$?!

Statistic: $Z=\dfrac{\bar{X}_1-\bar{X}_2}{\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}} \sim N(0,1) \Rightarrow Z=6.97$

R.R and A.R of H_0:



$-\infty$ 0 ∞

$Z_{1-\alpha}=Z_{.95}=1.645$

decision: as $Z=6.97>1.645$, so reject H_0

النتائج باستخدام Minitab :

في البرنامج نختار 2-sampleT, مع عدم تفعيل مساواة التباينين, والهدف الوصول إلى النتيجة الرفض الذي توصلنا إليه في الطريقة التقريبية السابقة .

two-sample T-test and CI : 1, 2

two-sample T for 1 vs 2

	N	Mean	StDev
1	54	67.7	38
2	53	28.7	15.7

Difference = Mu(1) − Mu(2)

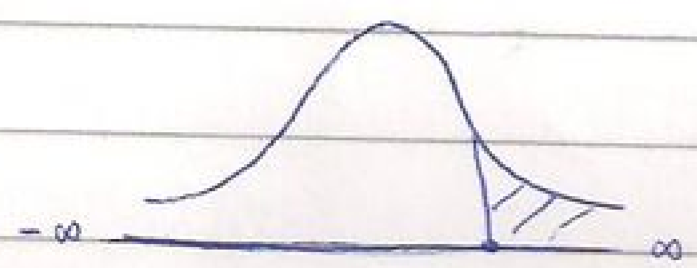
Estimate for difference : 39.0238

95% lower bound for difference : 29.6845

T-test for difference = 0 Vs (>) : T-value = 6.97 P-value = 0 DF = 70 statistic: $T = \dfrac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \sim T_v$, $v = \dfrac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}}$

$\Rightarrow T = 6.97$, $v = \dfrac{985.4259871}{13.90778655} = 70.85426452 \simeq 70$

R.R and A.R of H_0:



$t_{70, 1-\alpha} = t_{70, .95} = 1.6669$

decision: as $T = 6.97 > 1.6669$, so reject H_0

(b) C.I for $\mu_1 - \mu_2$?!

$$\mu_1 - \mu_2 \in (\bar{X}_1 - \bar{X}_2) \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

$\alpha = .05 \Rightarrow Z_{1-\frac{\alpha}{2}} = Z_{.975} = 1.96$

$\therefore \mu_1 - \mu_2 \in (28.01849, 49.98151)$

النتائج باستخدام Minitab :

Two-sample T-test and CI : 1, 2

two-sample T for 1 vs 2

	N	mean	stDev
1	54	67.7	38
2	53	28.7	15.7

Difference = mu(1) − mu(2)

Estimate for difference : 39.0238

95% CI for difference : (27.8495, 50.1980)

T-test of difference = 0 vs (not =) : T-value = 6.97 P-value = 0 DF = 70

في البرنامج نختار 2-sample T مع عدم تفعيل مساواة التباينين والهدف معرفة ان فترة الثقة الناتجة تكون الى حدما قريبة من فترة الثقة التقريبية السابقة.

For $\frac{\sigma_1^2}{\sigma_2^2}$

(8.46) rate 6.7 : 1 $\Rightarrow n_1 = 10, \bar{X}_1 = 37.82, S_1 = 4.89$ → معلومات عن العينتين المستقلة

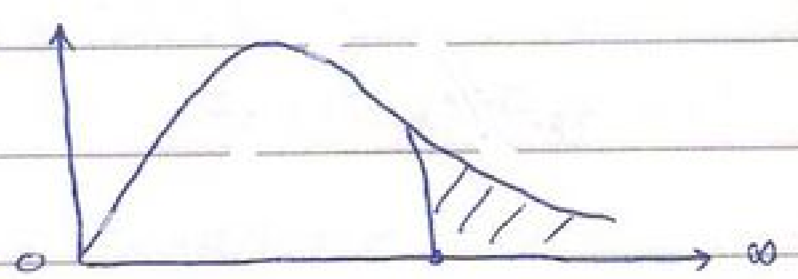
rate 8.97 : 2 $\Rightarrow n_2 = 10, \bar{X}_2 = 36.39, S_2 = 1.6$

populations are normal → معلومات عن المجتمعين

(a) $H_0: \sigma_1^2 = \sigma_2^2 \equiv \frac{\sigma_1^2}{\sigma_2^2} = 1$ vs $H_1: \sigma_1^2 > \sigma_2^2 \equiv \frac{\sigma_1^2}{\sigma_2^2} > 1$, $\alpha = .05$?!

statistic : $F = \frac{S_1^2}{S_2^2} \sim F_{n_1-1, n_2-1} \Rightarrow F = 9.341$

R.R and A.R of H_0 :



$F_{n_1-1, n_2-1, 1-\alpha} = F_{9,9,.95} = 3.18$

decision : as $F = 9.341 > 3.18$, so reject H_0

(b) 90% C.I for $\frac{\sigma_1^2}{\sigma_2^2}$?!

$$\frac{\frac{S_1^2}{S_2^2}}{F_{n_1-1, n_2-1, 1-\frac{\alpha}{2}}} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{\frac{S_1^2}{S_2^2}}{F_{n_1-1, n_2-1, \frac{\alpha}{2}}}$$

$(1-\alpha)100 = 90 \Rightarrow 1-\alpha = .9 \Rightarrow \alpha = .1 \Rightarrow F_{n_1-1, n_2-1, \frac{\alpha}{2}} = F_{9,9,.05} = \frac{1}{F_{9,9,1-.05}} = \frac{1}{F_{9,9,.95}} = \frac{1}{3.18} = .31447$

$F_{n_1-1, n_2-1, 1-\frac{\alpha}{2}} = F_{9,9,.95} = 3.18$

$\therefore \frac{\sigma_1^2}{\sigma_2^2} \in (2.9371, 29.7012)$

$$\mu_1-\mu_2 \in (\bar{X}_1-\bar{X}_2) \pm t_{v,1-\frac{\alpha}{2}} \sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}} \quad , v=70$$

$$\alpha=.05 \Rightarrow t_{.975,70} = 1.9945$$

$$\therefore \mu_1-\mu_2 \in (27.82519 \, , \, 50.17481)$$

{for P_1-P_2}

(3.7) $n_1=200$, $r_1=.47$, $a_1=n_1 r_1=94$
$n_2=200$, $r_2=.9$, $a_2=n_2 r_2=180$ → معلومات عن العينتين المستقلة

ⓐ 95% C.I for P_1-P_2 ?!

$$P_1-P_2 \in (r_1-r_2) \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{r_1(1-r_1)}{n_1}+\frac{r_2(1-r_2)}{n_2}}$$

$(1-\alpha)100=95 \Rightarrow 1-\alpha=.95 \Rightarrow \alpha=.05 \Rightarrow Z_{1-\frac{\alpha}{2}}=Z_{.975}=1.96$

$\therefore P_1-P_2 \in (-.5107, -.3443)$

النتائج باستخدام Minitab :

Test and CI for two proportions

Sample	X	N	Sample P
1	94	200	.47
2	180	200	.9

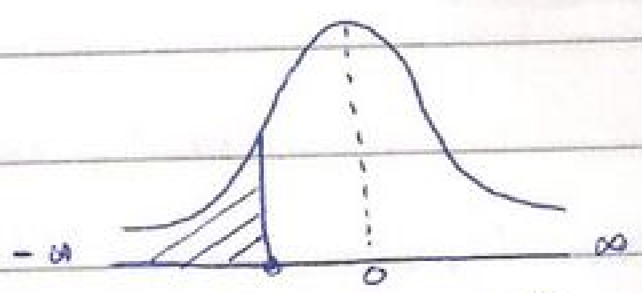
Difference = P(1) - P(2)
Estimate for difference : -.43
95% C.I for difference : (-.510704, -.349296)
Test for difference = 0 Vs (not = 0) : Z = -9.26 P-value = 0

تم تفعيل استخدام النسبة المشتركة $\hat{r}$ في إعدادات الإختبار

ⓑ $H_0: P_1=P_2 \cong P_1-P_2=0$ Vs $H_1: P_1<P_2 \cong P_1-P_2<0$, $\alpha=.05$?!

statistic : $Z=\dfrac{r_1-r_2}{\sqrt{\hat{r}(1-\hat{r})(\frac{1}{n_1}+\frac{1}{n_2})}} \sim N(0,1)$, $\hat{r}=\dfrac{a_1+a_2}{n_1+n_2} \Rightarrow Z=-9.257$, $\hat{r}=.685$

R.R and A.R of H_0 :



$Z_\alpha=-Z_{1-\alpha}=-Z_{.95}=-1.645$

decision : as $Z=-9.257<-1.645$, so reject H_0

to find p-value $=P(Z<-9.257)=P(Z<-9.26)=0$

النتائج باستخدام Minitab :

Test and CI for two proportions

Sample	X	N	Sample P
1	94	200	.47
2	180	200	.9

Difference = P(1) - P(2)
Estimate for difference : -.43
95% upper bound for difference : -.36227
Test for difference = 0 Vs (<) : Z = -9.26 P-value = 0

نجد أن p-value < α
وهذا يدعم القرار الذي توصلنا إليه.