واجب – 1 -

1 - اثبتي أن التكلفة الحدية تساوي التكلفة المتوسطة عندما تكون الأخيرة عند نهايتها الصغرى . افترضي أن كل حالة c تمثل التكاليف الكلية بينما x تمثل الإنتاج .

1 – $(Ac) = \frac{C}{X} = 2 + X\ \ell nX$

$$(\ Ac) = \frac{C}{X} = 2 + X\ \ell nX$$

$$C = [Ac \times X] = 2X + X^2\ \ell nX$$

$$Mc = \frac{dc}{dx} = 2 + X^2\ \frac{1}{X} + 2X\ \ell nX$$

$$Mc = \frac{dc}{dx} = 2 + X\ \ + 2X\ \ell nX$$

$$\frac{d(\ Ac)}{dX} = \ell nX + X\ \frac{1}{X} = 0$$

$$\frac{d(\ Ac)}{dX} = \ell nX + 1 = 0$$

$$\ell nX = -1$$

$$X = e^{-1}$$

$$X = \frac{1}{e}$$

$$e = 2.7182$$

$$X = .3679$$

$$\ Ac) = \frac{C}{X} = 2 + (.3679)\ \ell n(.3679) = 1.6321$$

$$Mc = \frac{dc}{dx} = 2 + (.3679)\ \ + 2(.3679)\ \ell n(.3679) = 1.6321$$

$$2- \qquad (Ac)= \frac{C}{X} = 0.2\,e^{x} + e^{-X}$$

$$(Ac)= \frac{C}{X} = 0.2\,e^{x} + e^{-x}$$

$$C = [Ac \times X] = 0.2X\,e^{x} + Xe^{-x}$$

$$Mc = \frac{dc}{dx} = 0.2\,X\,e^{x} + 0.2\,e^{x} - X\,e^{-x} + e^{-x}$$

$$Mc = \frac{dc}{dx} = 0.2\,X\,e^{x} + 0.2\ e^{x} - X\,e^{-x} + e^{-x}$$

$$Mc = \frac{dc}{dx} = 0.2\ e^{x}(1+X) + e^{-x}(1-X)$$

$$\frac{d(\ Ac)}{dX} = 0.2\,e^{x} - e^{-x} = 0$$

$$0.2\,e^{x} = e^{-x}$$

$$0.2\,e^{x} = \frac{1}{e^{x}}$$

$$e^{x} = \frac{1}{\frac{e^{x}}{0.2}}$$

$$e^{x} = \frac{5}{e^{x}}$$

$$e^{2X} = 5$$

$$2X\,\ell ne = \ell n5$$

$$2X = 1.6094$$

$$X = 0.805$$

$$Ac)= \frac{C}{X} = 0.2\,e^{x} + e^{-x} = 0.894$$

$$Mc = \frac{dc}{dx} = 0.2\ e^{x}(1+X) + e^{-x}(1-X)$$

$$Mc = \frac{dc}{dx} = 0.2\ e^{x}(1+0.805) + e^{-x}(1-0.805) = 0.894$$

2 -اثبتي العلاقة بين الايراد الحدي والمرونة لكل من دوال الطلب الآتية (حيث y تمثل السعر و x تمثل الكمية)

$$Y = 550 - 3X + 6X^2$$

$$\eta = \frac{dX}{dY} \times \frac{Y}{X}$$
$$TR = Y \times X$$

$$TR = (550 - 3X + 6X^2)X$$
$$TR = (550X - 3X^2 + 6X^3)$$
$$MR = \frac{d(\ TR)}{dX} = 550 - 6^2 + 18X^3$$
$$MR = Y\left(1 + \frac{1}{\eta}\right)$$
$$\frac{1}{\eta} = \frac{dY}{dX} \times \frac{X}{Y}$$
$$\frac{dY}{dX} \times \frac{X}{Y} = (-3 + 12X) \times \frac{X}{550 - 3X + 6X^2}$$
$$\frac{dY}{dX} \times \frac{X}{Y} = \frac{(-3X + 12X2)}{550 - 3X + 6X^2}$$

$$MR = (550 - 3X + 6X^2)\left[1 + \frac{1}{\frac{550 - 3X + 6X^2}{-3X + 12X^2}}\right]$$
$$MR = (550 - 3X + 6X^2)\left(1 + \frac{(-3X + 12X^2)}{(550 - 3X + 6X^2)}\right)$$
$$MR = (550 - 3X + 6X^2)\left(\frac{(550 - 3X + 6X^2) + (-3X + 12X^2)}{(550 - 3X + 6X^2)}\right)$$
$$MR = (550 - 6X + 18X^2)$$

$$TR = 100X - 6X^3$$

$$MR = 100 - 18X^2$$

$$Y = \frac{TR}{X} = 100 - 6X^2$$

$$\frac{1}{\eta} = -12X \quad \times \frac{X}{100 - 6X^2}$$

$$\frac{1}{\eta} = \frac{-12X^2}{100 - 6X^2}$$

$$MR = Y\left(1 + \frac{1}{\eta}\right)$$

$$MR = 100 - 6X^2\left(1 + \frac{-12X^2}{100 - 6X^2}\right)$$

$$MR = 100 - 6X^2\left(\frac{100 - 6X^2 - 12X^2}{100 - 6X^2}\right)$$

$$MR = 100 - 18X^2$$